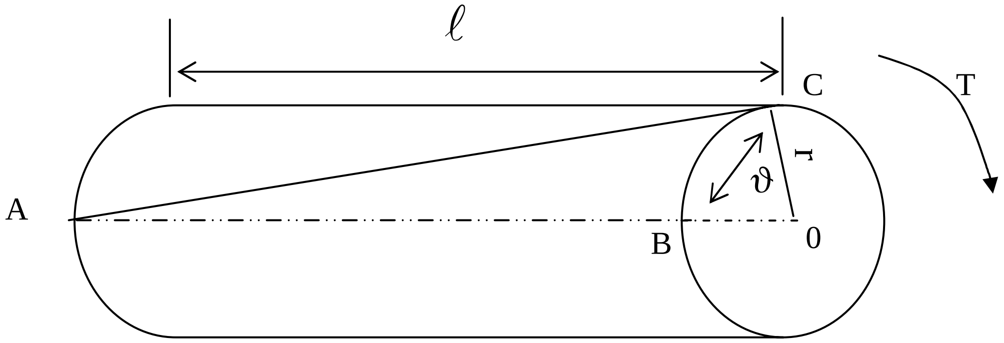
TORSION

Torsion of Shafts

Angle of Twist



When torque T is applied to a shaft:

Shear strain $= \dfrac{BC}{\ell}$

but, BC $= r\vartheta$

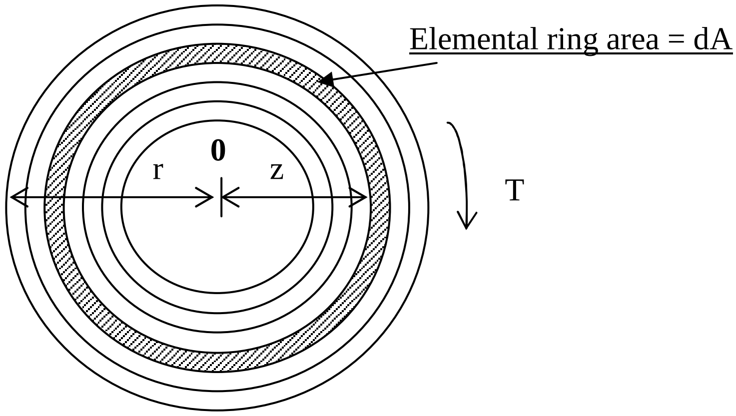
thus shear strain $= \dfrac{r\vartheta}{\ell}$

Modulus of rigidity (G) $= \dfrac{\text{shear stress}}{\text{shear strain}}$

$\therefore$ G $= \dfrac{\tau}{\dfrac{r\vartheta}{\ell}}$

Thus $\dfrac{\tau}{r} = \dfrac{G\vartheta}{\ell}$

Stress due to Twisting



It is assumed that shear stress due to torsion is directly proportional to radius. Hence if τ = shear stress at surface of shaft

then $\tau \times \frac{z}{r} =$ shear stress at radius z.

For the elemental ring of material:

shear stress = $\tau \times \frac{z}{r}$

shear force = $\tau \times \frac{z}{r} \times dA$

moment of force about 0 = $\tau \times \frac{z}{r} \times dA \times z$

$= \frac{\tau}{r} \times z^2 \times dA$

For the total cross-section area of the shaft:

Total moment of resistance about 0 = $\frac{\tau}{r} \int z^2 \times dA$ (Note T, r are constants)

But $\int z^2$ dA is the second moment of the area about 0, denoted by "J"

Total moment of resistance, i.e. the Torque $T = \frac{\tau}{r} \times J$ or $\frac{\tau}{r} = \frac{T}{J}$

Fundamental torsion equation

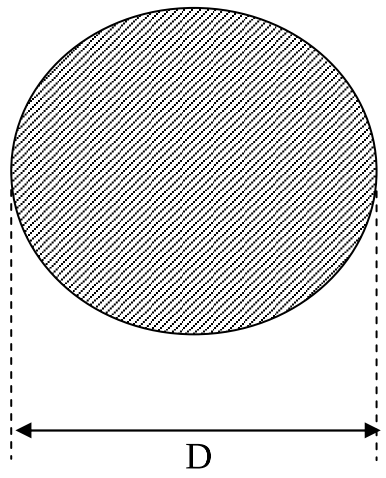
This is derived from a combination of the two previous equations and forms the fundamental torsion equation. This equation is used whenever twisting or torsion is present, and allows us to find the shear stress produced by this torque.

$$\frac{T}{J} = \frac{G\theta}{\ell} = \frac{\tau}{r}$$

Where:

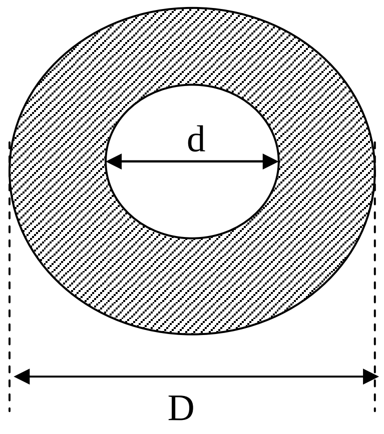
T	=	externally applied torque (Nm)
J	=	polar second moment of area of shaft section (m^4)
τ	=	shear stress at radius r(N/ ^{2})
r	=	radius at which stress is to be calculated (m)
G	=	modulus of rigidity of shaft material (N/m^2)
ϑ	=	angle of twist over length ℓ (radians)
ℓ	=	length of shaft subject to twisting (m)

Second moment of area for a Solid circular shaft



$$J = \frac{\pi D^4}{32}$$

Second moment of area for a Hollow circular shaft



$$J = \frac{\pi\left(D^4 - d^4\right)}{32}$$

Student example

A shaft of 200mm diameter operates at 200 rev/min. If the maximum stress due to shear is 140MN/m^2, what is the maximum power the shaft can deliver.

The second moment of area (we shall use J for the rest of this unit) = $\frac{\pi d^4}{32}$

= $\frac{\pi 0.2^4}{32}$ = 157 x 10^{-6} m^4. Note the units are metres^4

For ALL shafts the maximum shear stress will occur at the extreme or maximum radius of the shaft, so r = 0.1m

As τ has been given as140MN/m^2, then the torsion equation can now be used.

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}, \text{ so } T = \frac{J\tau}{r} = \frac{157 \times 10^{-6} \times 140 \times 10^6}{0.1} = 219.8\text{kNm}$$

The speed of the shaft rotation is given in rev/min. We can use either the relationship of P = ωT or P = $\frac{2\pi NT}{60}$ where N = speed in rev/min.

So Power = $\frac{2\pi \times 200 \times 219.8 \times 10^6}{60} = 4.6\text{MW}$

Student example

A hollow shaft of external diameter 260mm and internal diameter 180mm is to be replaced by a solid shaft . Both shafts are capable of transmitting 360kW at
200 rev/min. If the maximum shear stress in both the shafts is the same, calculate

a) the shear stress due to torsion

b) the diameter of the solid shaft

J for the hollow shaft is $\frac{\pi\left(D^4 - d^4\right)}{32} = \frac{\pi\left(0.26^4 - 0.18^4\right)}{32} = 0.346$ x 10^{-3} m^4

from P = ωT, then T = $\frac{P}{\omega} = \frac{360 \times 10^3}{2\pi \frac{200}{60}} = 17.19\text{kNm}$

From $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$, then $\tau = \frac{Tr}{J} = \frac{17189 \times 0.13}{0.346 \times 10^{-3}} = 6.47\text{MN/m}^2$

From $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$, then $\frac{r}{J} = \frac{\tau}{T}$

Notice how I have re-arranged the torsion equation to place all the unknowns on one side of the equation.

So resolving the unknown value $\frac{r}{J} = \frac{\frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16}{\pi d^3}$. Look closely at this term, to ensure you can see how it was produced. If you have any difficulties with this, read Section Maths 1 with special regard to indices.

So $\frac{16}{\pi d^3} = \frac{\tau}{T}$ or $\frac{16T}{\pi\tau} = d^3 = \frac{16 \times 17189}{\pi 6.47 \times 10^6} = 13.54$ x 10^{-3} m^3

So the diameter of the solid shaft will be 238.3mm

Note that the maximum shear stress is each case will be at the outside of the shaft and equals 6.47MN/m^2

So what will be the shear stress at the inside diameter of the shaft. Well this will be smaller as the diameter at this point is smaller,

so $\tau = \frac{Tr}{J} = \frac{17189 \times 0.09}{0.346 \times 10^{-3}} = 4.48$MN/m^2.

SAQ
The torque exerted on a solid shaft is 600 kNm when transmitting 8 MW. If the stress in the shaft is 50 MN/m^2 find its diameter and speed of rotation in rev/min.
Ans: 394 mm and 127.5 rev/min

SAQ
A solid shaft of 300 mm diameter transmits a certain power when running at 180 rev/min and the shear stress induced is 40 MN/m^2. What would be the stress induced in a solid shaft of 200 mm diameter running at 450 rev/min when transmitting the same power?
Ans: 54 MN/ m^2

SAQ
A steel shaft 3m long is transmitting 1MW at 240 rev/min. The working conditions that should be satisfied are:

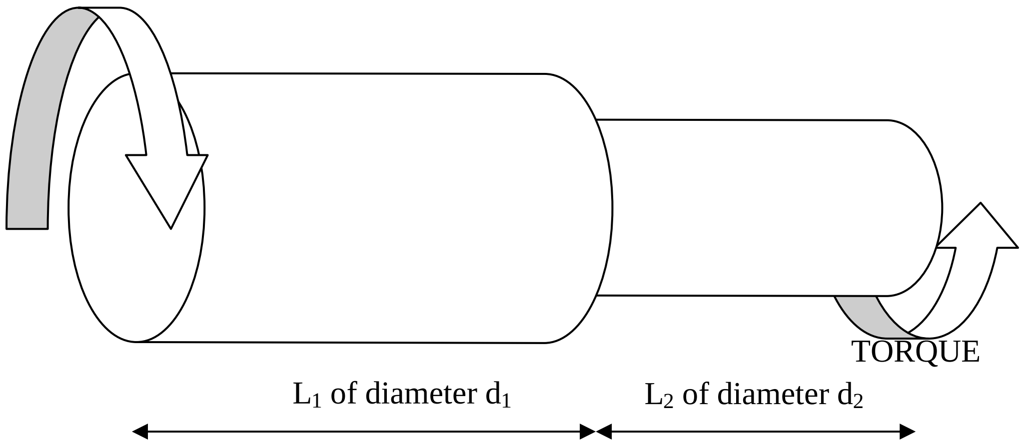
- Shaft must not twist more than 0.02 radian on a length of 10 diameters
- The working stress must not exceed 60MN/m^2

If the modulus of rigidity of steel is 80GN/m^2, what is
a) the diameter of the shaft required
b) the actual working stress
c) the angle of twist over a 3m length
ANSWERS a) 150mm b) 60MN/m^2 c) 0.030 radian

Note the important of using radians for ALL angles. Always convert and use radians whenever calculations are required, and this should avoid mistakes.

The study of the basic shaft arrangement can be used to analyse the stress within shaft arrangement when they are arranged in parallel and series. Both types of shaft could be found within the marine application.

Shafts in series



Consider two shafts that are aligned in series. The first shaft is rigidly attached to the second shaft, and both shafts are subjected to the torque T. Hence the TORQUE IS UNIFORM over the combined length of the shaft. This type of shaft could be fitted on Controllable Pitch Propeller installations, where the larger shaft is hollow.

So $T_{total} = T_1 = T_2$

and $\theta_{total} = \theta_1 + \theta_2$

Analysing the basic torsion equation $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$ gives

$T = \frac{G\theta_1 J_1}{L_1} = \frac{\tau_1 J_1}{r_1} = \frac{G\theta_2 J_2}{L_2} = \frac{\tau_2 J_2}{r_2}$ as the torque is constant.

Note I have used the same G throughout, as usually shafts aligned in series are continuous and the same material.

Student example

A steel transmission shaft is 510mm total length and 50mm external diameter. For part of its length it is bored to a diameter of 25mm and for the rest to 38mm diameter.
Find the maximum power that can be transmitted at a speed of 210 rev/min if the shear stress is not to exceed 70MN/m^2.
If the angle of twist in the length of the 25mm bore is equal to that in the length of the 38mm bore, find the length of the 38mm bore section

As we have stated earlier because the shafts are aligned in series, then the torque transmitted by each section must be the same.

So the torque $T = \frac{G\theta_1 J_1}{L_1} = \frac{\tau_1 J_1}{r_1} = \frac{G\theta_2 J_2}{L_2} = \frac{\tau_2 J_2}{r_2}$

J_1 for the 25mm bore section $= \frac{\pi\left(0.05^4 - 0.025^4\right)}{32} = 0.575 \text{ x } 10^{-6}\text{ m}^4$

J_2 for the 38mm bore section $= \frac{\pi\left(0.05^4 - 0.038^4\right)}{32} = 0.409 \text{ x } 10^{-6}\text{ m}^4$

The section that will be submitted to the highest stress is usually that with the lowest J value, but it is wise to check both sections. So I will now work out the permissible torque that can be carried for both sections.

$$T = \frac{\tau_1 J_1}{r_1} = \frac{70\times10^6\times0.575\times10^{-6}}{0.025} = 1610\text{Nm}$$

$$T = \frac{\tau_2 J_2}{r_2} = \frac{70\times10^6\times0.409\times10^{-6}}{0.025} = 1145.2\text{Nm}$$

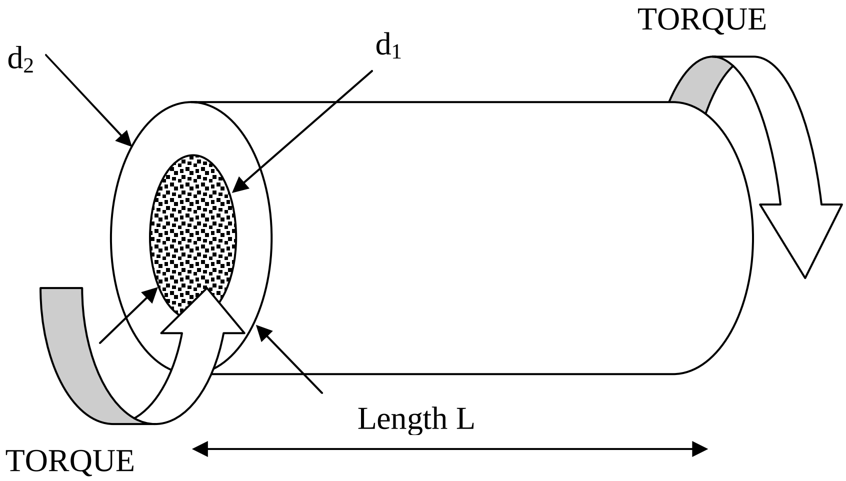
So the maximum torque that can be carried is 1145.2Nm and is limited by the 38mm bore section.

So maximum power is $\omega T = \frac{2\pi 210\times1145.2}{60} = 25.2\text{kW}$

From $\frac{G\theta_1 J_1}{L_1} = \frac{G\theta_2 J_2}{L_2}$ we are told that the angle of twist is the same, and also as the material is the same then $L_1 = \frac{L_2 J_1}{J_2} = \frac{L_2\times0.575\times10^{-6}}{0.409\times10^{-6}} = 1.406\ L_2$

So as the total length L = L_1 + L_2, then 510 = 1.406L_2 + L_2, so L_2 = 212mm

Shafts in parallel



d_1 = external diameter of the inner shaft
d_2 = external diameter of the outer shaft

Consider two shafts that are aligned in parallel. The outer shaft is rigidly attached to the inner shaft, and both shafts are subjected to the torque T. In this case the LENGTH IS THE SAME , and the ANGLE OF TWIST IS THE SAME for both lengths of the shaft. This type of shaft would be fitted where protection was required, such as a bronze liner over a steel shaft.

So $L_1 = L_2$

and $\theta_1 = \theta_2$

Also $T_{total} = T_1 + T_2$

Analysing the basic torsion equation $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$ gives

$\frac{T_1}{G_1 J_1} = \frac{\tau_1}{G_1 r_1} = \frac{T_2}{G_2 J_2} = \frac{\tau_2}{G_2 r_2}$ as L and θ are the same.

Note I have used a different G for each shaft as usually the inner and outer shafts are different materials.

Student example

A steel shaft of diameter 260mm is enclosed in layer of bronze "x" mm thick to provide corrosion protection. The shaft must transmit 150kW at 514 rev/min. If the steel shaft transmits 80% of the total torque, calculate

a) the thickness "x" of the bronze sleeve
b) the stress due to torsion in the steel shaft
c) the stress due to torsion in the bronze liner

Assume The Modulus of Rigidity of the steel is 80GN/m^2
The Modulus of Rigidity of the bronze is 44GN/m^2

The torque carried by the steel and the bronze will equate to the total torque. From P = ωT,

then total torque = $\dfrac{150\times10^3}{2\pi\dfrac{514}{60}} = 2786.76\text{Nm}$

J for a shaft $= \dfrac{\pi d^4}{32}$

For the steel shaft J $= \dfrac{\pi 0.26^4}{32} = 0.449$ x 10^{-3} m^4

The bronze liner is a hollow shaft

so J $= \dfrac{\pi\left(D^4 - 0.26^4\right)}{32}$ = 0.0982D^4 – 0.449 x 10^{-3} m^4

From $\dfrac{T_1}{G_1J_1} = \dfrac{\tau_1}{G_1r_1} = \dfrac{T_2}{G_2J_2} = \dfrac{\tau_2}{G_2r_2}$ then $\dfrac{T_1}{G_1J_1} = \dfrac{T_2}{G_2J_2}$ or $\dfrac{T_1}{T_2} = \dfrac{G_1J_1}{G_2J_2}$

But as $\dfrac{T_1}{T_2} = \dfrac{0.8}{0.2} = 4$ from the relationship given in the question stem, then

$$4 = \frac{80\times10^9\times0.449\times10^{-3}}{44\times10^9\times\left(0.0982D^4 - 0.449\times10^{-3}\right)}$$

so $\left(0.0982D^4\right) = 0.204\times10^{-3} + 0.449\times10^{-3}$

or $D = \sqrt[4]{\dfrac{0.653\times10^{-3}}{0.0982}}$ so D = 285.56mm

So thickness of bronze liner = $\dfrac{285.56 - 260}{2} = 12.78\text{mm}$

$$\tau_S = \frac{Tr}{J} = \frac{(0.8\times2786.76)\times\dfrac{0.26}{2}}{0.449\times10^{-3}} = 645.5 \text{ kN/m}^2.$$

$$\tau_B = \frac{Tr}{J} = \frac{(0.2\times2786.76)\times\dfrac{0.28556}{2}}{\dfrac{\pi\left(0.28556^4 - 0.26^4\right)}{32}} = 389.5 \text{ kN/m}^2.$$

Student example

A circular bar 4m long with an external radius of 25mm is solid over half its length and bored to an internal radius of 12mm over the other half. If a torque of 120Nm is applied at the centre of the shaft, the two ends being fixed, determine the maximum shear stress set up in the surface of the shaft, the angle of common twist, and the work done by the torque in producing this stress.

From $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$, and knowing that the bar MUST be twisted equally as the torque is applied in the centre of the bar. As the bar is solid and therefore the same material, then both G and L are equal for this particular application. However as the torque is placed at the centre, this torque will be shared equally between the two sections. This differs when the torque is input at one end, and an equal torque resists at the other. Thus when the torque is applied to the centre of the bar, the situation of shared torque means that this is a parallel bar situation
as $T_{total} = T_1 + T_2$

Re-arranging the basic torsion equation gives the stress in the solid section

$$\tau = \frac{Tr}{J} = \frac{60 \times 0.025}{\frac{\pi 0.05^4}{32}} = 2.44\text{MN/m}^2$$

For the stress in the hollow section

$$\tau = \frac{Tr}{J} = \frac{60 \times 0.025}{\frac{\pi\left(0.05^4 - 0.024^4\right)}{32}} = 2.58\text{MN/m}^2$$

As expected the shaft with the least material to resist the torque will have the highest shear stress.
We measure this resistance by the Second Moment of Area or J.

In the angular sense work done is Torque x Angle of Twist = Tθ

The angle of twist from $\frac{T}{J} = \frac{G\theta}{L}$ gives $\frac{TL}{JG} = \theta$

$$= \frac{60 \times 2}{\frac{\pi 0.05^4 \times 80 \times 10^9}{32}} = 0.00243 \text{ radian}$$

Thus work done by an applied torque = strain energy = $\frac{1}{2}T\theta$ = 60 x 0.00243 = 0.147Nm

Typical examination question

A compound shaft consists of a bronze sleeve shrunk onto a 350mm diameter steel shaft. The proportion of the power transmitted by the bronze sleeve is limited to 10% of the total shaft power of 8MW at 140 rev/min.

Calculate EACH of the following
a) The outside diameter of the bronze sleeve (8)
b) The maximum shear stress due to torsion in both the steel shaft and bronze sleeve (6)
c) The angle of twist over a 3 metre length of compound shaft (2)

The modulus of rigidity of the steel material = 82 GN/m^2
The modulus of rigidity of the bronze material = 40 GN/m^2

Study the question stem. Obviously there are two different materials here, and as one material surrounds another, then this is a parallel shaft. Make sure that you correctly identify what type of shaft the question relates to, or you will be unable to apply the correct relationships.

With the parallel shaft arrangements the lengths and angle of twist are equal so

$$\frac{T_1}{G_1 J_1} = \frac{\tau_1}{G_1 r_1} = \frac{T_2}{G_2 J_2} = \frac{\tau_2}{G_2 r_2}$$

In this question we can find the torque transmitted by the steel shaft and bronze liner.

$$T_{total} = \frac{8 \times 10^6}{2\pi \times \frac{140}{60}} = 545.67\text{kNm}$$

T_{steel} = 0.9 x T_{total} = 491.1kNm

T_{bronze} = 0.1 x T_{total} = 54.57kNm

$$J_{steel} = \frac{\pi 0.35^4}{32} = 1.47 \text{ x } 10^{-3} \text{ m}^4$$

From $\frac{T_1}{G_1 J_1} = \frac{\tau_1}{G_1 r_1} = \frac{T_2}{G_2 J_2} = \frac{\tau_2}{G_2 r_2}$ then

$$\frac{T_b J_s}{T_s} = J_b = \frac{54.57 \times 1.47 \times 10^{-3}}{491.1} = 0.163 \text{ x } 10^{-3} \text{ m}^4$$

$$J_b = \frac{\pi\left(D_b{}^4 - 0.35^4\right)}{32} = 0.163 \text{ x } 10^{-3} \text{ so } D_b = 359.3\text{mm}$$

From $\frac{T_1}{G_1 J_1} = \frac{\tau_1}{G_1 r_1} = \frac{T_2}{G_2 J_2} = \frac{\tau_2}{G_2 r_2}$ then

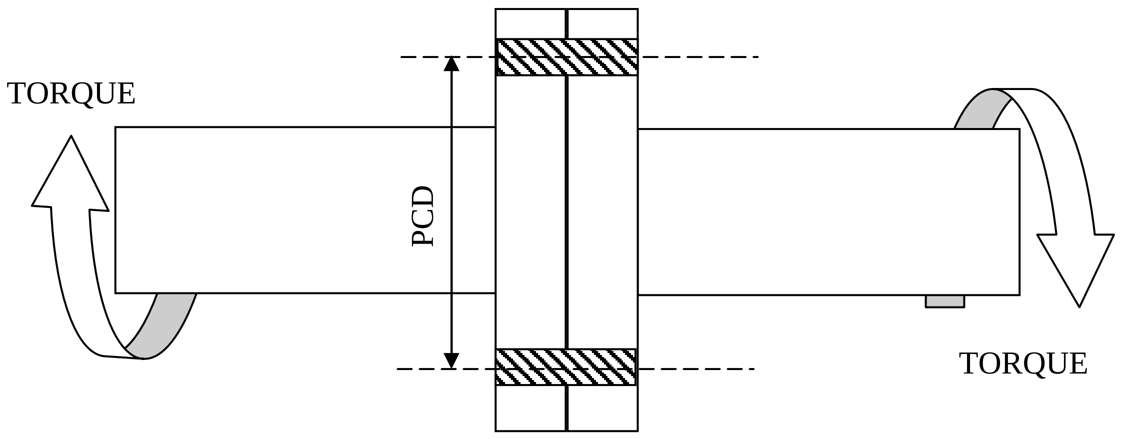
$$\frac{T_b r_b}{J_b} = \tau_b = \frac{54.57\times10^3 \times \frac{0.3593}{2}}{0.163\times10^{-3}} = 60.14\text{MN/m}^2$$

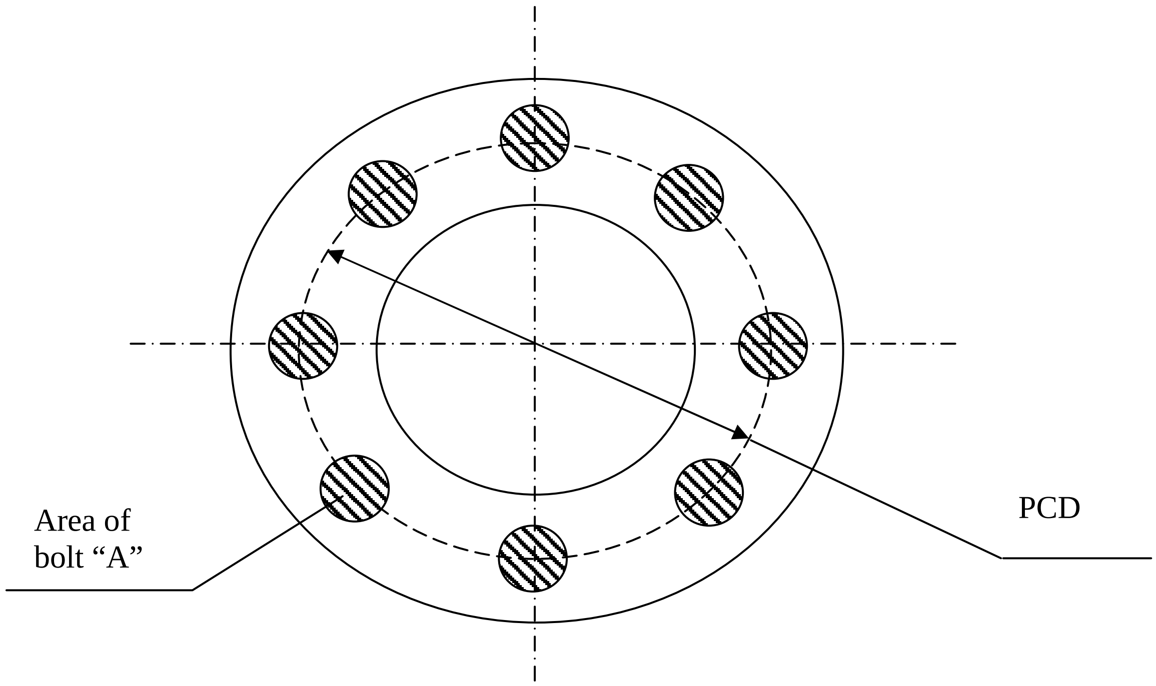
$$\frac{T_s r_s}{J_s} = \tau_s = \frac{491.1\times10^3 \times \frac{0.35}{2}}{1.47\times10^{-3}} = 58.46\text{MN/m}^2$$

As the angle of twist will be the same for both shafts I will calculate for only one material, so from the common torsion equation $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$, then

$$\theta = \frac{L\tau}{Gr} = \frac{3\times58.46\times10^6}{80\times10^9\times\frac{0.35}{2}} = 0.0125 \text{ radians}$$

Transmission of torque by bolted couplings





The sketches show a bolted flanged coupling popular in transmission shafting. The shafts transmit a torque of “T”Nm which must be carried by the bolts in the connection. It is assumed that the bolts can carry the full torque, although in practice the friction grip present between the flanges must contribute towards this torque.

When a torque is present, then each bolt will be placed in shear. The area under shear is shown as “A”.
Study the sketch, what will be the total area of bolts under shear?

This will depend on two factors, the area of each bolt, and the number of bolts in the coupling. So for this shaft and coupling, the area under shear will be 8 x A,

or $\frac{N\pi d^2}{4}$ where N is the number of bolts, and d is the diameter of the bolt.

From Torque = Force x radius, so the torque = shear force x shear radius

This shear force will produce a shear stress in the bolt, so the shear force can be re-written as shear stress x shear area.

The shear radius in this instance is half the Pitch Circle Diameter or half the PCD, as shown on the sketch.

Combining these to equate the torque carried within the shaft to the shear stress produced within the bolts gives

$$T = \tau_{bolts} \times \frac{\pi d^2}{4} \times N \times \frac{PCD}{2}$$

<u>Student example</u>

Calculate the shear stress in a six bolt flanges that carries 500kW when rotating at 250 rev/min. The bolts are arranged on a PCD of 400mm, and are 18mm in diameter.

Torque T = $\frac{P}{\omega} = \frac{500 \times 10^3}{2\pi \frac{250}{60}} = 19.1\text{kNm}$

From $T = \tau_{bolts} \times \frac{\pi d^2}{4} \times N \times \frac{PCD}{2}$

then $\tau_{bolts} = \frac{8T}{\pi d^2 N \times PCD} = \frac{8 \times 19098}{\pi 0.018^2 \times 6 \times 0.4} = 62.54\text{MN/m}^2$

SAQ
Two hollow shafts, 350 mm outside diameter and 150 mm inside diameter, are connected by a coupling having 9 bolts of 60 mm diameter on a pitch circle diameter of 600 mm. When transmitting a certain power the stress in the shafts is 40 MN/m^2. Find the stress in the bolts.

Ans: 42.7 MN/ m^2

Typical examination question

An engine transmission shaft is required to transmit 6MW at 100 rev/min. The shaft comprises of two shafts joined at a bolted coupling. The coupling is designed to transmit the shaft torque by either coupling friction, or coupling bolt shear.
The coupling has 8 bolts on a Pitch Circle Diameter of 400mm, and each bolt has a length of 180mm. The coefficient of friction of the coupling faces is 0.15.
The maximum bolt shear stress is 120 MN/m^2, and the maximum bolt tensile stress is 200 MN/m^2.
Assume that the ratio of coupling area to bolt area is sufficiently large to prevent significant coupling deformation under compression when loaded by the bolts.

Calculate EACH of the following
a)
i) The require bolt diameter to satisfy the bolt shear stress limit (4)
ii) The bolt temperature increase required during the fitting procedure that will produce the correct bolt tension upon cooling to provide the same friction torque as achieved by the shear stress on the bolts. (8)
b) Explain why the coupling friction grip alone should not be used to transmit the shaft torque based on these design calculations (4)

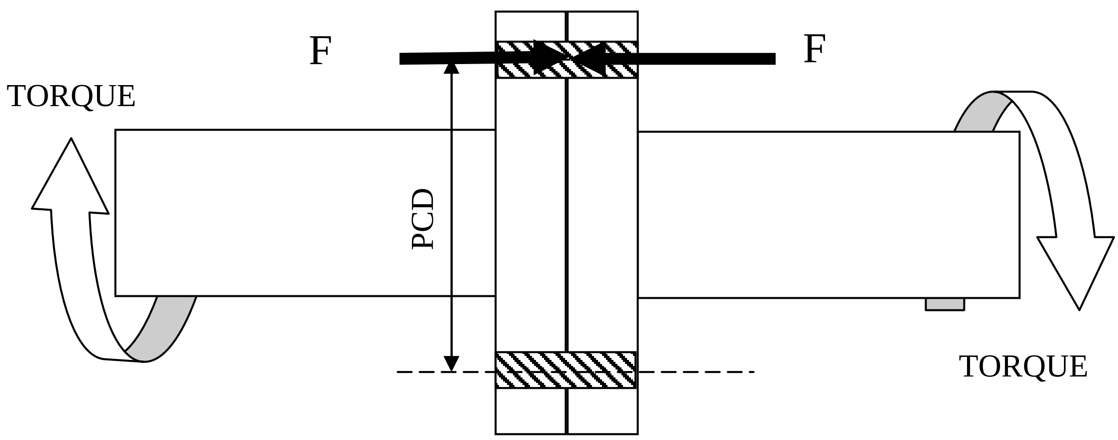
The Modulus of Elasticity of the bolt material is 210 GN/m^2
The coefficient of expansion of the bolt material is 12 x 10^{-6}/^{o}C

This question is a mixture of the coupling bolt and differential expansion theory that you will have studied during this particular unit of the Mechanics module.

The first part of the question uses the coupling theory, so we will apply that now.

From $T = \tau_{bolts} \times \frac{\pi d^2}{4} \times N \times \frac{PCD}{2}$ where T = $\frac{6 \times 10^6}{2\pi \frac{100}{60}} = 573\text{kNm}$

then $d = \sqrt{\frac{8T}{\tau_{bolts} \pi \times N \times PCD}} = \sqrt{\frac{8 \times 573 \times 10^3}{120 \times 10^6 \times \pi \times 8 \times 0.4}} = 61.6\text{mm}$



The drawing shows the effect of the bolt tension at the coupling, by producing a force that will cause friction interaction or grip between the flanges.

This force F will cause a friction torque, where the effective friction force will be μF, so friction torque = $\mu F \times \frac{PCD}{2} \times N$.

We have already calculated the torque transmitted by the shaft, so the force F required at the bolt = $\frac{2 \times 573 \times 10^3}{0.15 \times 0.4 \times 8} = 2387.5\text{kN}$

As the question states that the bolt compression will not distort or compress the coupling, then we need to find the temperature changes that will produce a force of 2387.5kN.

From $E = \frac{\sigma}{\varepsilon} = \frac{FL}{Ax}$ and $x = \alpha L \Delta T$ from the expansion or contraction of a material, then equating the change in dimension "x" gives $x = \frac{FL}{AE} = \alpha L \Delta T$

Thus $\Delta T = \frac{F}{\alpha EA} = \frac{2387.5 \times 10^6}{12 \times 10^{-6} \times 210 \times 10^9 \times \frac{\pi 0.0616^2}{4}} = 317.9\ ^{o}\text{C}$

This means that the bolts should be heated up by 317.9 ^{o}C, then inserted into the coupling holes, and lightly nipped against the flanges. When the bolts cool down they will contract and apply the force required of 2387.5kN, and hence the torque required.

However the final part of the question queries why friction grip is not used. To examine this we will calculate the stress placed onto the bolt during this contraction.

From $\sigma = \frac{F}{A} = \frac{2387.5 \times 10^3}{\frac{\pi 0.0616^2}{4}} = 800\text{MN/m}^2$. This stress is 4x large than the design stress quoted in the question stem of 200MN/m^2. Hence the bolted coupling must use a combination of friction grip and shear to transmit the torque.

SAQ

The shafting arrangement for a controllable pitch propeller has a hollow shaft of 400mm external diameter coupled to a solid shaft of the same material and external diameter. The length of the hollow shaft is 4.2 metres. The maximum shear stress due to torsion in the hollow shaft is 30% greater than that of the solid shaft.

The rotational speed of the shafts is 160 rev/min.

Calculate EACH of the following

a) the maximum power of the shaft system if the maximum shaft twist is limited to 0.01 radians/metre (12)

b) the range of shear stress due to torsion across the hollow shaft cross-section (4)

Note: The Modulus of Rigidity of the shaft material is 82 GN/m^2

ANSWER 26.5MW , 163.7 to 113.5 MN/m^2 for the outside and inside diameters

SAQ

A solid shaft of 500 mm diameter is replaced by a hollow shaft of the same external diameter. The material strengths of the solid and hollow shafts are 400 MN/m^2 and 500 MN/m^2 respectively. When the replacement was made the speed was increased by 10% and the power transmission increased by 30%. Find the inside diameter of the hollow shaft allowing the same factor of safety in both cases.

Ans: 242 mm

SAQ
A vessel having a single propeller shaft of 300 mm diameter and running at 160 rev/min, is re-engined with turbines driving two equal propeller shafts at 750 rev/min and developing 60% more power. If the working stresses in the new shafts are 10% greater than the old shafts find their diameters.
Ans: 161 mm

SAQ
Vibrational losses cause an engine shaft to be changed from a solid section to a hollow section having a ratio between inside and outside diameters of 0.65 to 1.0. When developing 6.9 MW at 250 rev/min the maximum torque is 35% greater than the mean torque. Find suitable diameters for the hollow shaft if the maximum shear stress is not to exceed 63 MN/m^2.
Ans: 214 mm and 328 mm

SAQ
A steel shaft of 300 mm diameter is shrink fitted with a bronze liner of 340 mm outside diameter to form a compound shaft. If the total torque transmitted is 136 kNm find the maximum shear stress in each material.
$G_s = 90\ kN/m^2$ $G_b = 40\ kN/m^2$

Ans: τs : 19.8 MN/m^2 τb : 10 MN/m^2

SAQ
A composite shaft consists of a solid steel rod of 80 mm diameter surrounded by a closely fitting brass tube fixed firmly to it. Find the outside diameter of the tube so that when a torque is applied to the composite shaft it is equally shared by the two materials. Determine also the maximum shear stress in each material and the angle of twist over a length of 2 m when a torque of 15 kNm is applied to the compound shaft.
$G_s = 80\ GN/m^2$ $G_b = 40\ GN/m^2$

Ans: 105.2 mm τs 74.6 MN/m^2 τb 49.1 MN/m^2 and 2.678

SAQ
A tail shaft consists of a solid steel shaft of 400 mm diameter shrink fitted with a bronze liner of 460 mm outside diameter. The power transmitted is 18 MW at 120 rev/min. Find the maximum stress in each material land the angle of twist, in degrees, on a length of 5 m.
$G_s = 90\ GN/m^2$ $G_b = 45\ GN/m^2$

Ans: $\tau s = 83\ MN/m^2$ $\tau b = 47.69\ MN/m^2$ and 1.328