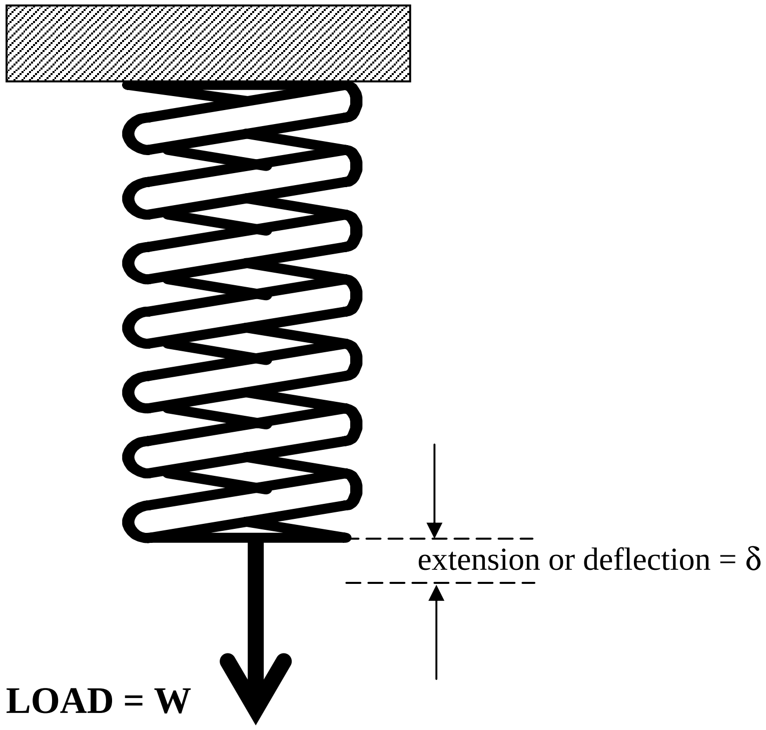
SPRINGS

The use of springs is common within many engineering applications. They provide a controlled force that will assist sealing (in clutches, and safety valves), limit movement (in governors), and can be easily adjustable. Springs are also able to store energy, and return it to the system (in engine cylinder valves).
The types of springs that will be studied with this section are closed coiled helical springs.



As the coils are closely pitched, then the axial load will produce a twist on the spring material that equates to the moment applied by the load. Thus the load W will apply a twist or torque of WR, where R is the radius of the spring.

From the standard torsion equation $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$

J for the spring = J for any solid bar = $\frac{\pi d^4}{32}$

r = the radius of the coil wire, not the spring, as we are examining the stress caused at the wire by an external torque, so r = $\frac{d}{2}$.

Note the notation we are using, the small letters relate to the wire and the larger or capital letters refer to the coil or spring.

Rearranging the torsion equation gives $\tau = \frac{Tr}{J} = \frac{W \times R \times \frac{d}{2}}{\frac{\pi d^4}{32}} = \frac{16WR}{\pi d^3}$

The deflection of the spring under load will be δ. We shall investigate the work done on the spring by the external load, with the work done to twist the spring.

So external work applied by the load W = internal work required to twist the coil

So average force x distance = Average torque x twist

$\frac{1}{2}(W+0)\delta = \frac{1}{2}(T+0)\theta$ or $W\delta = T\theta$

From the torsion equation $\frac{T}{J} = \frac{G\theta}{L}$ so $\frac{TL}{JG} = \theta = \frac{WRL}{JG}$

The length of the spring will be approximately the circumference of each coil times the number of coils. So L = 2πRN, where N is the number of coils.

Equating the work done and substituting gives

$$\delta = \frac{T\theta}{W} = \frac{WR \times WRL}{W \times JG} = \frac{W^2 R^2 2\pi RN}{W \frac{\pi d^4}{32} G} = \frac{64WR^3 N}{Gd^4}$$

Finally we also need to know the equation for the stiffness of a spring, as this parameter is widely used in calculations.

From Load = stiffness x deflection, so stiffness k = $\frac{W}{\delta} = \frac{WGd^4}{64WR^3N} = \frac{Gd^4}{64R^3N}$

To assist your memory recall of these equations:

Student example

A closed coiled spring is made with the following dimensions:

Spring diameter	=	80mm
Wire diameter	=	6mm
No of coils	=	7

When subjected to a load of 66N, calculate

a) the deflection of the spring
b) the shear stress in the spring

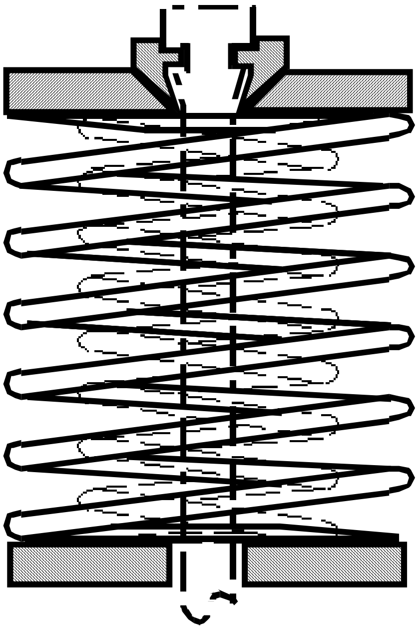
Take the Modulus of Rigidity of the spring as 80GN/m^2

From $\delta = \dfrac{64WR^3N}{Gd^4}$ then $\delta = \dfrac{64 \times 66 \times 0.04^3 \times 7}{80 \times 10^9 \times 0.006^4} = 18.25\text{mm}$

From $\tau = \dfrac{16WR}{\pi d^3} = \dfrac{16 \times 66 \times 0.04}{\pi 0.006^3} = 62.25\text{MN/m}^2$

Springs in parallel

What happens when springs are connected in parallel, which would be the standard case for the support of most diesel engine cylinder head valves?
Each spring will support part of the load, so $W_{total} = W_1 + W_2$



If we simplify the analysis by assumed in the deflection of both springs are equal then from $W_{total} = W_1 + W_2$ and as $W = k\delta$

then $k_{total}\,\delta = k_1\delta + k_2\delta$. Cancelling the common δ factor, then $k_{total} = k_1 + k_2$

Thus the spring arrangement will be stiffer by the addition of both individual spring stiffness.

All other relationships that were shown in the basic single spring hold for the combined springs. We shall now analyse a problem on springs to demonstrate the application of this theory.

Parallel Springs

Student example

A compound spring has 2 closed coiled helical springs having exactly the same length when unloaded. The outer spring has 16 coils of 12mm diameter bar coiled to a mean diameter of 125mm. The inner spring has 24 coils with a mean diameter of 75mm. The working stress in each spring is to be the same.
Calculate
a) The diameter of the wire bar for the inner spring
b) the stiffness of the compound spring

Take G = 70GN/m^2

The question states that the shear stress in each spring is the same, so we will equate these.

$$\tau = \frac{16W_1R_1}{\pi d_1^{\,3}} = \frac{16W_2R_2}{\pi d_2^3} \text{ so } \frac{W_1 \times 0.0625}{0.012_1^{\,3}} = \frac{W_2 \times 0.0375}{d_2^3} \text{ so } W_1 = \frac{W_2 \times 1.037 \times 10^{-6}}{d_2^3}$$

Also from $\delta = \dfrac{64WR^3N}{Gd^4}$, we also know that the deflection of each spring must be the same, as they were the same lengths before being compressed.

So equating the deflection for each spring gives $\frac{64W_1 R_1{}^3 N_1}{Gd_1{}^4} = \frac{64W_2 R_2{}^3 N_2}{Gd_2{}^4}$

so $\frac{W_1 \times 0.0625^3 \times 16}{G \times 0.012^4} = \frac{W_2 \times 0.0375^3 \times 24}{G \times d_2{}^4}$ so $W_1 = \frac{W_2 \times 6.718 \times 10^{-9}}{d_2{}^4}$

Combining the two equations with W_1 gives $\frac{W_2 \times 6.718 \times 10^{-9}}{d_2{}^4} = \frac{W_2 \times 1.037 \times 10^{-6}}{d_2^3}$

$\frac{6.718 \times 10^{-9}}{1.037 \times 10^{-6}} = \frac{d_2^4}{d_2^3} = d_2 = 6.48\text{mm}$

Now that the wire diameter has been found we can find the stiffness of each spring, so stiffness $k_1 = \frac{Gd^4}{64R^3N} = \frac{70 \times 10^9 \times 0.012^4}{64 \times 0.0625^3 \times 16} = 5.81\text{kN/m}$

Stiffness $k_2 = \frac{Gd^4}{64R^3N} = \frac{70 \times 10^9 \times 0.00648^4}{64 \times 0.0375^3 \times 24} = 1.52\text{kN/m}$

So the combined stiffness is $k_1 + k_2 = k_{total} = 5.81 + 1.52 = 7.33$kN/m

<u>Typical examination question</u>
Two closed coiled helical springs are concentrically fitted in parallel to vertically support an engine exhaust valve of mass 100 kg.

	Wire diameter	No of coils	Mean coil diameter
Outer spring coil	16mm	8	350mm
Inner spring coil	16mm	10	250mm

The outer spring is compressed 5mm more than the inner spring.

Calculate EACH of the following

b) The compression required on the springs to support the valve (8)

b) The stress in each spring when they are opened by the application of a 400N force (8)

Take the modulus of rigidity of the spring material as 80 GN/m^2

For this question we will again quote the formulas we derived earlier, and use the information in the question stem to assist us.

deflection $\delta = \dfrac{64WR^3N}{Gd^4}$. We have also been informed that $\delta_o = \delta_I + 0.005$

So equating the deflection for each spring

gives $\delta_o = \dfrac{64W_oR_o{}^3N_o}{Gd_o{}^4} = \dfrac{64W_o 0.175^3 \times 8}{80\times10^9 \times 0.016^4} = W_o \times 0.523\times10^{-3}\, m$

and $\delta_i = \dfrac{64W_iR_i{}^3N_i}{Gd_i{}^4} = \dfrac{64W_i 0.125^3 \times 10}{80\times10^9 \times 0.016^4} = W_i \times 0.238\times10^{-3}\, m$

So equating these two deflection gives $0.523W_o = 0.238W_i + 5$

I have omitted the term 10^{-3} on all terms for clarity and to make the calculation slightly easier. You are advised to simplify equations if possible, as that will reduce the number of calculations required, and hence you should make fewer mistakes.

We also know the load both springs have to support which is 100kg or 981N

So $W_o + W_I = 981$, thus $W_o = 981 - W_i = \dfrac{0.238W_i}{0.523} + \dfrac{5}{0.523}$

so $981 - W_I = 0.455W_I + 9.56$

So $W_i\,(1 + 0.455) = 981 - 9.56$

thus $W_i = 667.7$N, and therefore $W_o = 313.3$N

From $\delta_o = W_o \times 0.523 \times 10^{-3}$ thus δ_o = 163.9mm
$\delta_i = W_i \times 0.238 \times 10^{-3}$ thus δi = 158.9mm

The check of course is that δ_o is 5mm larger than δi which is correct

The springs are now loaded by an additional 400N force. BUT this force must be split between the two springs. The only thing we know is that the additional deflection of the two springs must be equal.

I will use the relationship that we used earlier
So $\delta_o = W_o \times 0.523 \times 10^{-3}$ and $\delta_i = W_i \times 0.238 \times 10^{-3}$ where δ_o now equals δ_i

So $W_o + W_i = 400$, thus $W_o = 400 - W_i = \dfrac{0.238 W_i}{0.523}$
So 400 = W_i (1 + 0.455), thus the additional force placed on the inner spring is 274.9N, and the additional load placed on the outer spring is 125.1N

Hence the total load carried by the inner spring is the initial load + additional load = 667.7 + 274.9 = 942.6N

Total load carried by the outer spring is the initial + additional load
= 313.3 + 125.1 = 438.4N
From the formula shear stress $\tau = \dfrac{16WR}{\pi d^3}$

$$\tau_{outer} = \frac{16WR}{\pi d^3} = \frac{16 \times 438.4 \times 0.175}{\pi 0.016^3} = 95.4\text{MN/m}^2$$

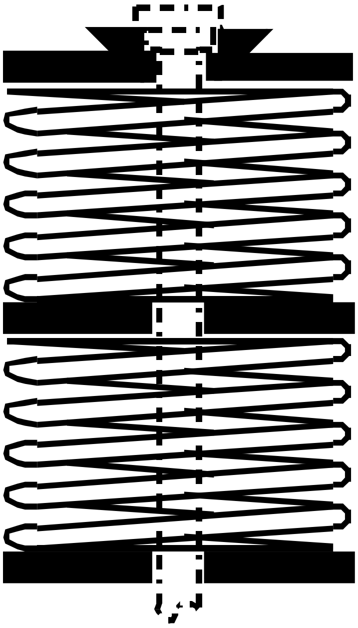
$$\tau_{inner} = \frac{16WR}{\pi d^3} = \frac{16 \times 942.6 \times 0.125}{\pi 0.016^3} = 146.5\text{MN/m}^2$$

Springs in series

In this configuration, springs are connected in series so that the springs are connected end to end. Hence the load on each spring is common
so $W_{total} = W_1 = W_2$

However the deflection on each spring must be added to find the total deflection
so $\delta_{total} = \delta_1 + \delta_2 = \dfrac{W}{k_1} + \dfrac{W}{k_2} = \dfrac{W(k_1 + k_2)}{(k_1 \times k_2)}$

Thus the composite stiffness of the spring arranged in series is $\dfrac{(k_1 \times k_2)}{(k_1 + k_2)}$



Series Springs

Alternately we can show that the combined stiffness k can be found from

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Student example

A composite spring has two closed coiled helical springs connected in series. Each spring has 12 coils at a mean diameter of 25mm. If the diameter of the wire in one spring is 2.5mm, and the combined stiffness is 700N/m, then calculate the diameter of wire of the second spring. Take G = 70GN/m^2

Calculate the load that can be carried by the composite spring, and the total extension when the maximum shear stress is 180MN/m^2

The stiffness of the first spring can be calculated from
stiffness k = $\dfrac{W}{\delta} = \dfrac{WGd^4}{64WR^3N} = \dfrac{Gd^4}{64R^3N}$.

Note how the standard formula for individual springs can be applied.

so $k_1 = \dfrac{70\times10^9\times0.0025^4}{64\times0.0125^3\times12} = 1823\text{N/m}$

As the combined stiffness k can be found from $\dfrac{1}{k} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$, then $\dfrac{1}{700} = \dfrac{1}{1823} + \dfrac{1}{k_2}$, so k_2 = 1136.3N/m

Thus stiffness $k_2 = \dfrac{Gd^4}{64R^3N} = \dfrac{70\times10^9\times d^4}{64\times0.0125^3\times12} = 1136.3$.

Hence d_2 = 2.22mm

From $\tau = \dfrac{16WR}{\pi d^3}$ then for the given maximum shear stress, the load can be found for either spring. For this application the radius R (of the spring) and the shear stress is common, but the spring diameters are different. Hence it would be wise to calculate both spring possible loads for the maximum shear stress criteria to ensure that the correct solution is found.

So for the first spring $\tau = \dfrac{16WR}{\pi d^3} = \dfrac{16W\times0.0125}{\pi 0.0025^3} = 180\times10^6$
Hence limiting load W_1 = 44.2N

For the second spring $\tau = \dfrac{16WR}{\pi d^3} = \dfrac{16W\times0.0125}{\pi 0.00222^3} = 180\times10^6$
Hence limiting load W_2 = 30.9N

So the highest load that can be carried is 30.9N

The extension of the combined spring is $\dfrac{W}{k} = \dfrac{30.9}{700} = 44.1\text{mm}$

The extension can also be found from the individual extensions of each spring, so as $\delta = \dfrac{64WR^3N}{Gd^4}$ then $\delta_1 = \dfrac{64\times30.9\times0.0125^3\times12}{70\times10^9\times0.0025^4} = 16.95\text{mm}$

then $\delta_2 = \dfrac{64\times30.9\times0.0125^3\times12}{70\times10^9\times0.00222^4} = 27.26\text{mm}$

So total extension is 16.95 + 27.26 = 44.21mm, which is similar to that found from the combined stiffness.

SAQ
The mean diameter of a close coiled helical spring is 8 times the wire diameter and the energy expended in stretching the spring 250 mm is 2 kJ. If the maximum shear stress is not to exceed 200 MN/m^2 determine (a) the wire diameter (b) the mean coil diameter (c) the number of free or active coils.
G = 80 GN/m^2

Ans: (a) 40.4 mm (b) 323.2 mm (c) 12.3

SAQ
A close coiled helical spring is deflected 20 mm when subjected to an axial force F. The mean coil diameter is 40 mm and the wire diameter 5 mm. If the shear stress in the spring material is not to exceed 240 MN/m^2 determine the deflecting force F and the number of coils in the spring.
G = 100 GN/m^2

Ans: 295 N and 8.25 free coils

SAQ
A close coiled helical spring with a wire diameter of 10 mm and mean coil diameter of 100 mm with 12 effective coils is not to carry a shear stress greater than 250 MN/m^2 when subjected to an axial force. For these conditions determine (a) the maximum axial force that can be exerted on the spring (b) the corresponding deflection (c) the strain energy stored when the maximum force is applied.
G = 80 GN/m^2

Ans: (a) 982 N (b) 117.5 mm (c) 57.5 J

SAQ
The control spring for a relief valve is required to have a stiffness of 90 kN/m and not more than 6 free coils in its length. If the mean coil diameter is five times the wire diameter determine the wire diameter.
G = 90 GN/m^2

Ans: 6 mm

SAQ
A pair of governor springs is placed co-axially and concentric with each other, the details of the two springs being as follows:

	N	d	R
Spring A:	6	3 mm	18 mm
Spring B :	10	5 mm	25 mm

In the free position Spring B is 12 mm longer than Spring A.
Determine
(i) the axial force applied to Spring B if Spring A is just on the point of being compressed
(ii) the deflection of Spring B when an axial force of 120 N is applied to the compound spring.

G = 100 GN/m^2 (both springs) Ans: (a) 75 N (b) 16.5 mm