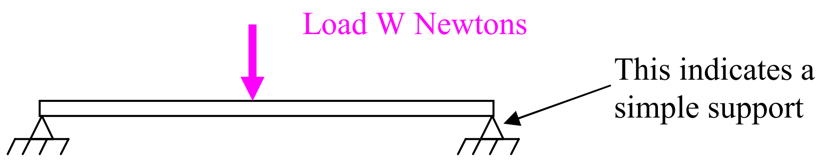
SHEAR FORCE AND BENDING MOMENT DIAGRAMS

Shear Forces in Beams with Concentrated or Point Loads

A beam is used in a variety of engineering situations for supporting and carrying loads. Forces, or loads, which are at 90^0 to the beam cause a SHEAR STRESS in the beam material. The beam must be able to withstand the shear stress or failure may occur.

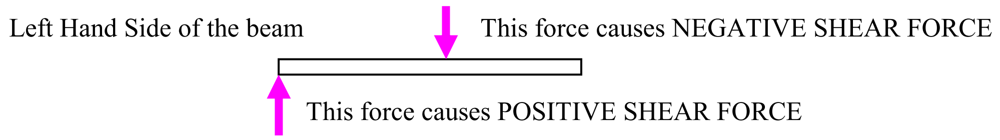
To determine the shear stress, a **SHEAR FORCE DIAGRAM** can be drawn which shows the distribution of shear force (SF) along the length of the beam.

Consider a simply supported beam with a single central concentrated load W Newtons:



The load and reaction forces at the supports cause shear forces in the beam.

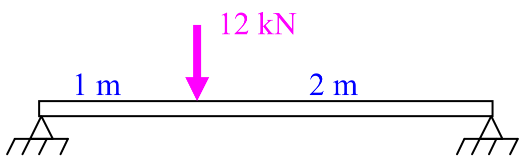
A **SIGN CONVENTION** is needed for positive and negative shear force. The usual convention is to work from the Left Hand Side (LHS) of the beam and therefore **UPWARD FORCES** cause **POSITIVE SHEAR FORCE** and **DOWNWARD FORCES** cause **NEGATIVE SHEAR FORCE.**



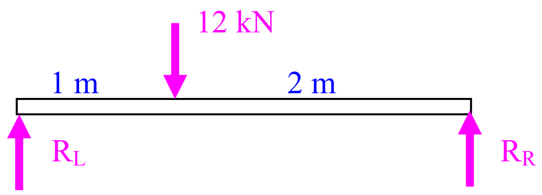
Values of SF can be found at different distances along the length of the beam and if necessary used to plot a graph.

WORKED EXAMPLE

For the beam shown, find the values of shear force at 0.5 m intervals and use these to plot a Shear Force Diagram.



The first problem is to find the support reactions at the Left Hand and Right Hand sides. The easiest way is usually to take moments of forces about the LHS.



Taking moments about R_L:

$$\text{Clockwise Moments} = \text{Anticlockwise Moments}$$
$$12\text{ kN} \times 1\text{ m} = R_R \times 3\text{ m}$$
$$R_R = \frac{12}{3}$$
$$R_R = \underline{\mathbf{4\ kN}}$$

Then use Equilibrium of Forces is zero:

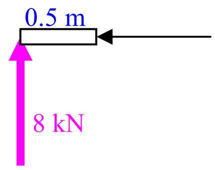
$$\text{Upward acting forces} = \text{Downward acting forces}$$
$$R_L + R_R = 12$$
$$R_L + 4 = 12$$
$$R_L = 12 - 4$$
$$R_L = \underline{\mathbf{8\ kN}}$$

Now values of SF can be found starting at the LHS and using 0.5 m intervals.

At the LHS, the distance is zero.

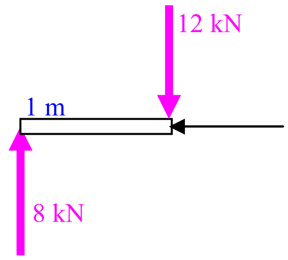
Therefore the SF = +8 kN

At a distance of 0.5 m from the LHS:



The shear force at this position is SF = +8 kN

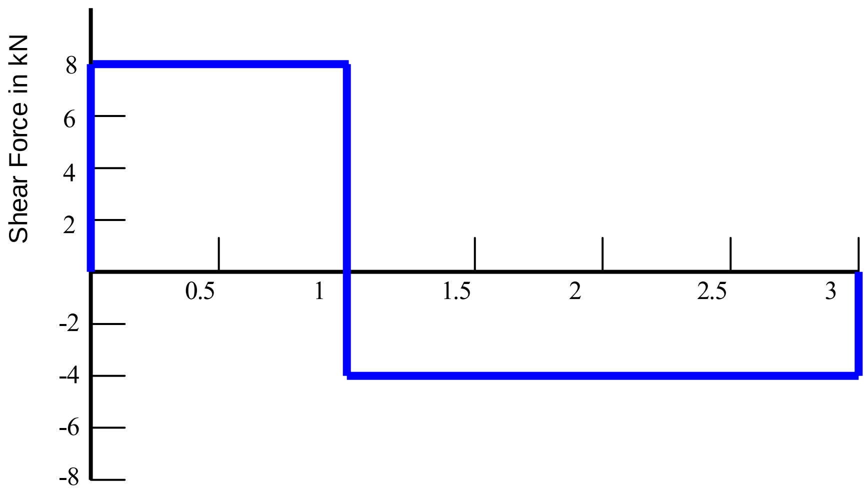
At a distance of 1 m from the LHS:



The shear force at this position has two values.
Just before the 12 kN load the SF = +8 kN.
Just after the 12 kN load the SF = +8 kN –12 kN = - 4 kN

Continuing to the end of the beam at the RHS, the SF will remain constant at – 4 kN.

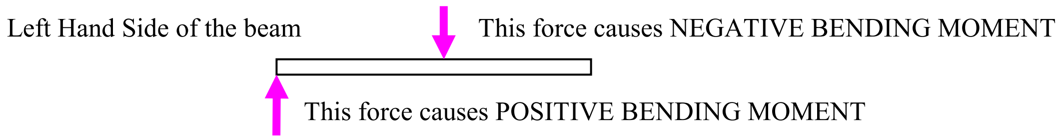
A graph of SF values against distance along the beam will be:



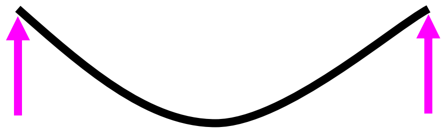
Bending Moments in Beams with Concentrated or Point Loads

As well as trying to shear the beam, forces also bend it and cause BENDING MOMENTS (BM) along its length. Again, values of BM can be found at different positions along the beam length and if necessary a graph can be drawn.

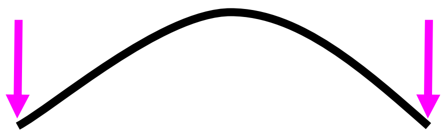
As with shear forces, a **SIGN CONVENTION** is needed for positive and negative bending moments. Once more, the usual convention is to work from the Left Hand Side (LHS) of the beam and therefore **UPWARD FORCES** cause **POSITIVE BENDING MOMENT** and **DOWNWARD FORCES** cause **NEGATIVE BENDING MOMENT.**



If a **POSITIVE BENDING MOMENT** is acting on the beam it causes **SAGGING**.



A **NEGATIVE BENDING MOMENT** acting on the beam causes **HOGGING**.



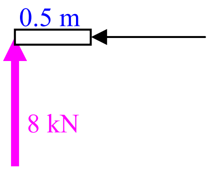
WORKED EXAMPLE

Using the beam from the previous example, values of Bending Moment can be found starting at the LHS and using 0.5 m intervals.

At the LHS, the distance is zero.

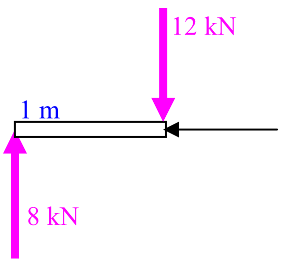
Therefore the BM = 0

At a distance of 0.5 m from the LHS:



The bending moment at this position is BM = +8 kN × 0.5 m
= 4 kNm

At a distance of 1 m from the LHS:



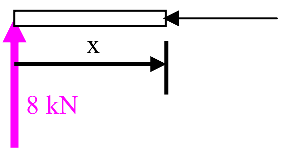
The bending moment at this position is BM = 8 × 1
= 8 kNm

Very often an equation is needed to find the BM at any value of x (distance along the beam). Once the equation has been established, any value of x can be substituted and the BM found. The difficulty is that a new equation is needed every time there is a load change.

In this example, there is no load change between the LHS and up to a distance of 1 m. Therefore the limits of the equation are

$0 < x < 1$ i.e. only values of x between 0 and 1 m can be used.

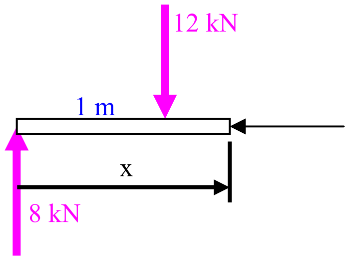
To determine the equation, a section of the beam is drawn between 0 and 1 m:



The bending moment at this position is BM = 8 × x
= 8x kNm

Now any value of x between 0 and 1 m can be substituted into 8x to give the bending moment at that position.

For distance between 1 m and 3 m the diagram would be:



The bending moment at this position is BM = 8x – 12(x-1) kNm

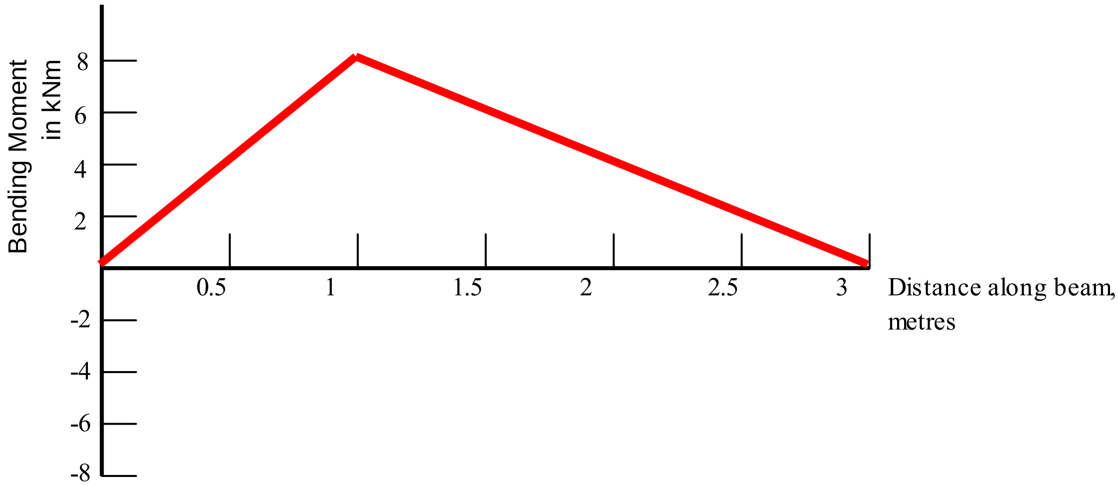
and the limits are $1 < x < 3$

So what is the BM at a distance of 2.6 m from the LHS?

Substituting x = 2.6 into the equation gives

BM = $8 \times 2.6 - 12(2.6 - 1)$ = **<u>1.6 kNm</u>**

Once the appropriate BM values have been calculated, a graph of BM against distance along the beam can be plotted.

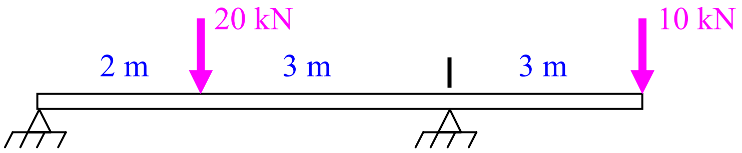


In this example all the bending moment diagram is positive, which means the beam will be sagging.

Also notice where the **MAXIMUM BM** occurs. It is at the same point where the SF crosses the zero axis from +8 to –4 kN (look at the SF diagram drawn earlier). This is usually the case. The maximum BM occurs where the SF is zero.

WORKED EXAMPLE

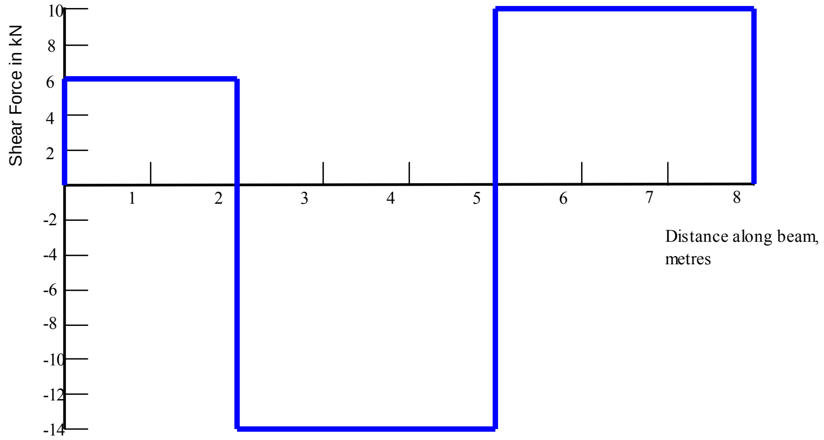
For the simply supported beam shown, draw the shear force and bending moment diagrams.



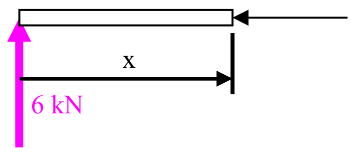
Try to calculate the reactions at the Left Hand and Right Hand Supports yourself. The values you should obtain are:

$$R_L = 6 \text{ kN and } R_R = 24 \text{ kN}$$

The shear force diagram will follow the loads from the LHS and the complete diagram will be:

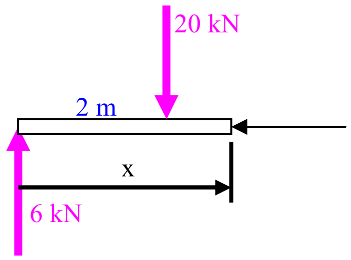


For bending moments, a diagram is needed between each load change. The THREE diagrams for this example will be:



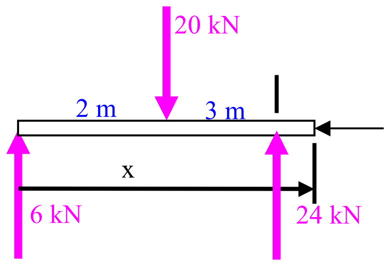
The bending moment at this position is BM = 6x

and the limits are $0 < x < 2$



The bending moment at this position is BM = 6x – 20(x-2) kNm

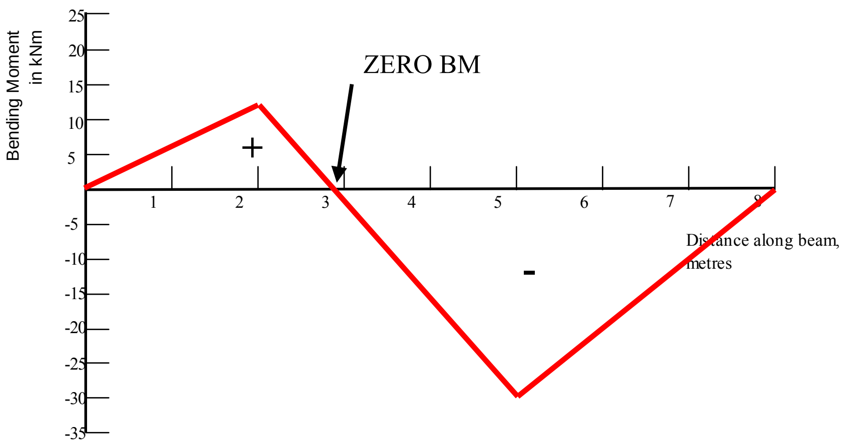
and the limits are $2 < x < 5$



The bending moment at this position is
BM = 6x – 20(x-2) +24(x – 5) kNm

and the limits are $5 < x < 8$

Substituting values for x and plotting the complete diagram gives:



Again, the maximum positive BM of +12 kNm and the maximum negative BM of –30 kNm occur where the SF is zero.

Notice that there are now Positive and Negative areas. This means the beam changes from sagging to hogging. The point where this occurs is at the position where the **BM is ZERO** and is termed a **POINT OF CONTRAFLEXURE**.

To find the exact position on the beam where this occurs, either draw the BM diagram to scale and measure the distance on the graph or use the BM equation which applies to that section of the beam.
i.e.

$$BM = 6x - 20(x - 2)$$

The BM is zero at the point of contraflexure therefore

$$0 = 6x - 20(x - 2)$$

$$0 = 6x - 20x + 40$$

$$14x = 40$$

$$x = \frac{40}{14}$$

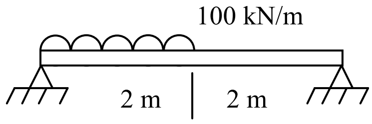
x = **2.857 m from the LHS** Answer

Shear Forces and Bending Moments in Beams with Uniformly Distributed Loads (UDL)

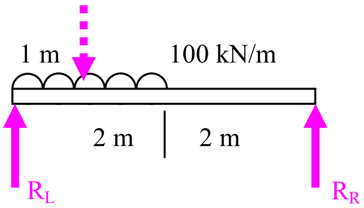
In this case the load is spread evenly along either the whole length of the beam or over only a section of its length. It assumed that the total load caused by the UDL acts through the centre of the UDL.

WORKED EXAMPLE

For the beam shown, draw the SF and BM diagram.



The first problem is to find the support reactions. Remember it is assumed the load produced by the UDL acts through the centre of the UDL.

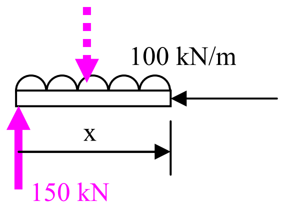


Taking moments about the Left Hand Side:

$$\text{Clockwise Moments} = \text{Anticlockwise Moments}$$
$$(100\ \text{kN/m} \times 2\ \text{m}) \times 1\ \text{m} = R_R \times 4\ \text{m}$$
$$200 = 4R_R$$
$$R_R = \underline{\mathbf{50\ kN}}$$

$$\text{Equilibrium of forces}$$
$$\text{Upward forces} = \text{Downward forces}$$
$$R_L + R_R = 100\ \text{kN/m} \times 2\ \text{m}$$
$$R_L + 50 = 200$$
$$R_L = \underline{\mathbf{150\ kN}}$$

With a UDL it is easier to establish equations for the shear force as was done in the previous section for bending moments.



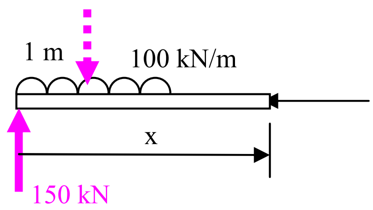
The shear force at this position is $SF = 150 - 100x$

The bending moment at this position is $BM = 150x - 100x \times \frac{x}{2}$

$$= 150x - 50x^2$$

and the limits are $0 < x < 2$

For the section 2 m to 4 m:



The shear force at this point is $SF = 150 - 100 \times 2$

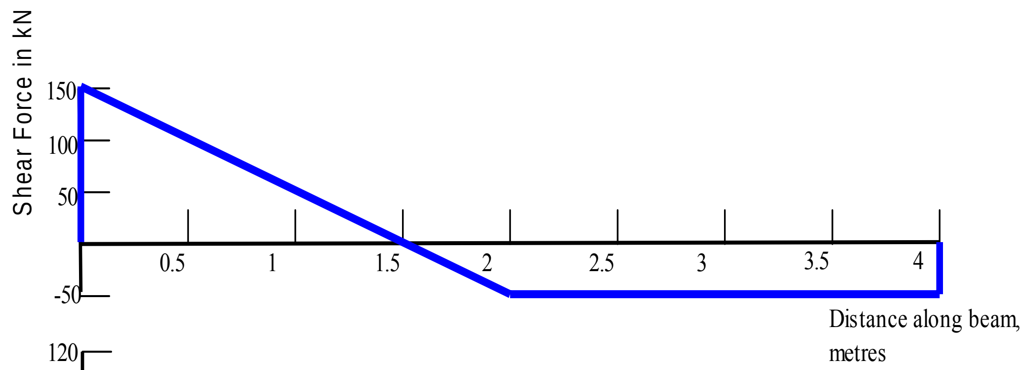
$= -50$ kN

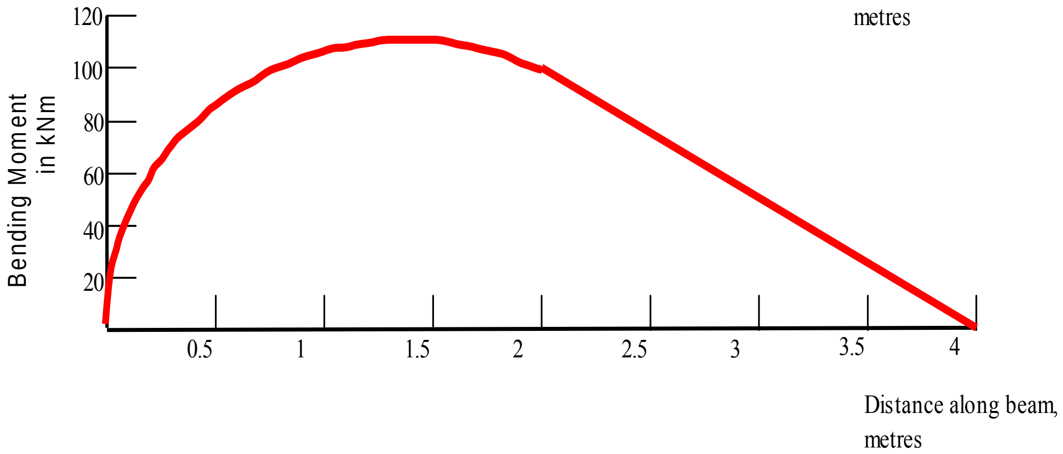
The bending moment at this point is $BM = 150x - (100 \times 2)(x - 1)$

$= 150x - 200(x - 1)$

The limits are $2 < x < 4$

Using these equations, making sure that the values of x lie between the limits of the equation, both the shear force and bending moment diagrams can be drawn.





The maximum bending moment again occurs where the shear force is zero. The diagram shows that the shear force is zero between 0 and 2 m. Therefore, using the correct equation for shear force the exact position of maximum bending moment can be calculated.

SF = 150 – 100x and SF = 0

0 = 150 – 100x

$x = \frac{150}{100} = 1.5$ m from the LHS

Using this value in the correct equation for the bending moment will give the value of the maximum bending moment.

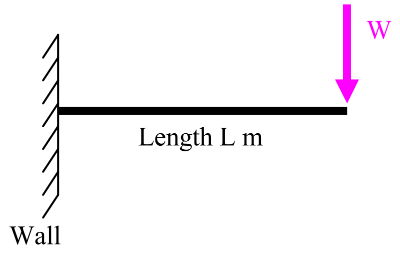
$BM = 150x = 50x^2$

$BM_{MAX} = 150 \times 1.5 - 50 \times 1.5^2$

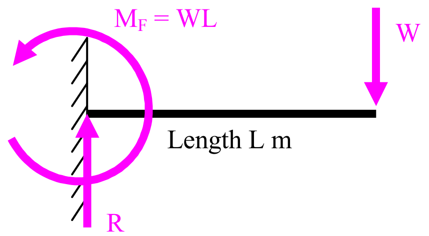
$BM_{MAX} =$ **112.5 kNm**

Cantilever Beams

A cantilever beam is where one end of the beam is considered to be built into a solid surface such as a wall or bulkhead.

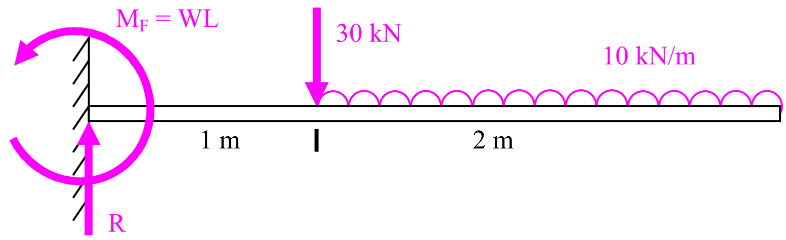


At the wall, there must be a vertical reaction force to support the load W. In addition there is a **FIXING MOMENT** which prevents rotation of the beam caused by the moment **WL**.



WORKED EXAMPLE

For the cantilever shown, calculate the reaction at the wall and the fixing moment. Also draw the SF and BM diagrams.



To calculate the wall reaction, use equilibrium of forces:

Upward forces = Downward forces

$$R = 30 + 10 \times 2$$

R = **50 kN** Answer

To calculate the fixing moment, take moments about the LHS:

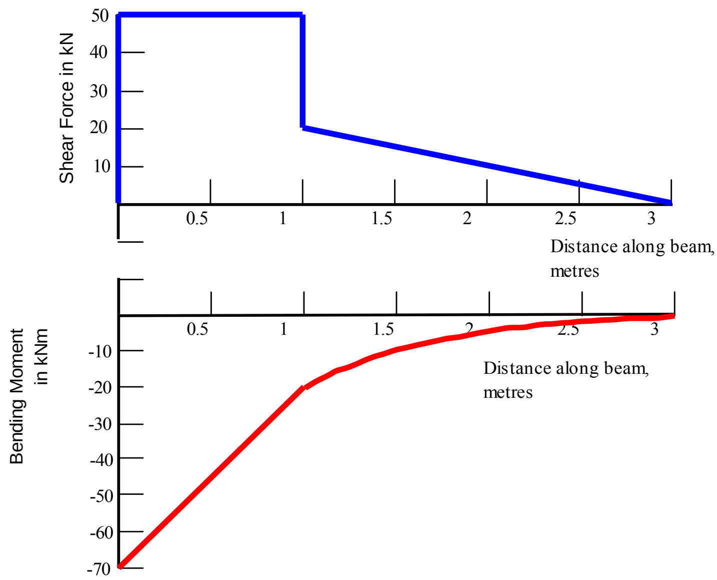
Clockwise moments = anticlockwise moments

$$30 \times 1 + (10 \times 2) \times 2 = M_F$$

(Remember the UDL is assumed to act at its centre, therefore the distance to the wall is 1 m + 1 m = 2 m)

M_F = **70 kNm** Answer

The SF and BM diagrams will be:



STUDENT EXAMPLES

1. A beam 7 m long is simply supported at 1 m from each end. It carries a uniformly distributed load of 12 kN/m over its entire length and concentrated loads of 15 kN at each end. Sketch the SF and BM diagrams and determine also
(a) The BM at mid-span.
(b) Where zero BM occurs.

((a) 16.5 kN (b) 1.84 m from each end)

2. A beam AB is 5 m long and simply supported at A and another point C which is 4 m from A. Between A and C, at intervals of 1 m, it supports concentrated loads of 20 kN, 30 kN and 20 kN respectively. At end B there is a concentrated load of F and the bending moment at 3 m from end A is zero. Determine the magnitude of the force F and the reactions at the supports. Sketch also the SF and BM diagrams.

(F = 46.7 kN; R_A = 23.3 kN; R_C = 93.4 kN)

3. A beam 6 m long rests on two supports 3.4 m apart overhanging the left hand support by 1.6 m. It carries concentrated loads of 50 kN at the left hand end, 70 kN midway between the supports, 30 kN at the right hand end, and a uniformly distributed load of 30 kN/m between the two supports. Find the BM midway between the two supports and the position of the points of contraflexure.

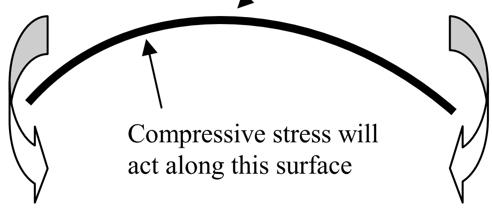
(47.75 kNm, 0.465 m in from the right hand support; point of contraflexure is 0.922 m in from the left hand support)

4. A beam of 18 m overall length is simply supported at its left hand end and at 12 m from this end. It carries concentrated loads of 80 kN, 40 kN and 50 kN at distances of 4 m; 6 m and 14 m respectively from the left hand end. In addition, a uniformly distributed load of 10 kN/m acts along its entire length.
(a) Sketch the SF and BM diagrams.
(b) Find the maximum bending moment.
(c) Determine the position of the point of contraflexure from the left hand end.

((b) 360 kNm (c) 9.665 m from the left hand end)

BENDING STRESSES CAUSED BY BENDING MOMENTS

In the previous section, it was shown that loads acting on a beam cause bending moments. These moments cause the beam to deflect (hogging and sagging) and this in turn causes stresses to be set up in the beam material.



It can be shown that an equation relating Bending Moment M to Bending Stress σ is given by:

$$\frac{\mathbf{M}}{\mathbf{I}} = \frac{\boldsymbol{\sigma}}{\mathbf{y}} = \frac{\mathbf{E}}{\mathbf{R}}$$

This is called the Bending Formula and will have to be remembered. The symbols are:

M = Bending Moment, Nm.
Very often this is the MAXIMUM bending moment acting on the beam since this will give the maximum bending stress. Its value can be found from a bending moment duagram.

σ = Bending Stress, N/m^2.
This will be tensile on one side of the material and compressive on the other side. Therefore, as the stress is measured through the depth of the material its value will become smaller and at some position will be ZERO. This is called the NEUTRAL AXIS.

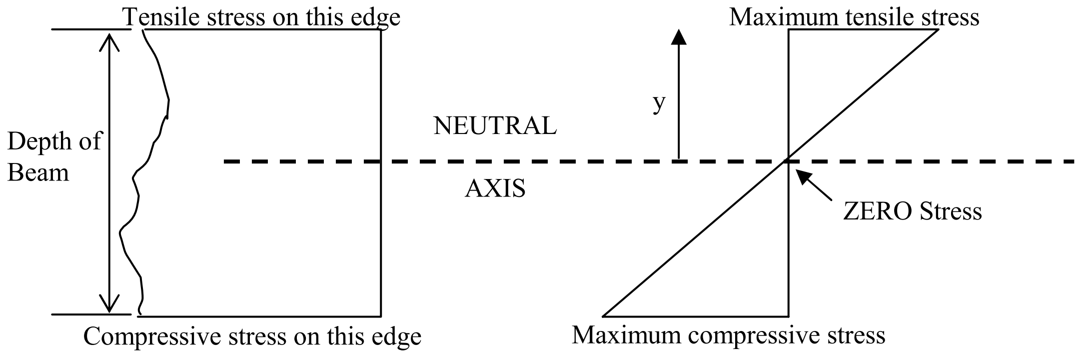
E = Young's Modulus, N/m^2.

I = Second Moment of Area of the beam cross section, m^4.
This is a measure of how easy it is to bend the beam. The larger the value the more difficult it is to bend the beam.

R = Radius of curvature of the beam as it is bent, metres.
Usually this is a very large value (perhaps several hundred metres) and often is not important in engineering situations.

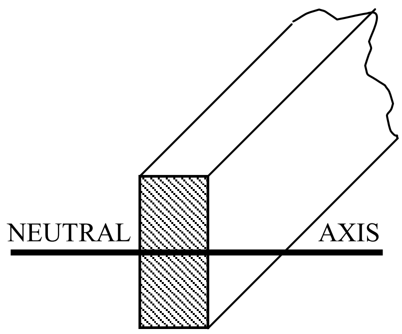
y = distance, metres, measured from the Neutral Axis to the position where the stress is to be determined.

Again, this is usually the distance to the surface of the beam since that is where the largest stress will occur. A diagram of stress distribution through the depth of the beam will be:



For symmetrical or simple shapes, the position of the Neutral Axis is easy to find since it passes through the CENTROID of the shape.

For example a beam which has a rectangular cross section:

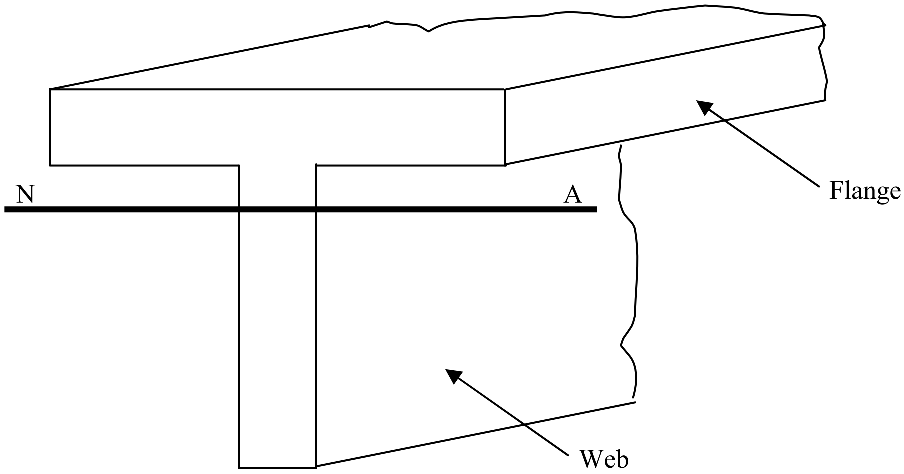


For more complicated shapes the position needs to be found using **FIRST MOMENTS OF AREA**. An example could be a beam which has a T for the cross sectional shape.

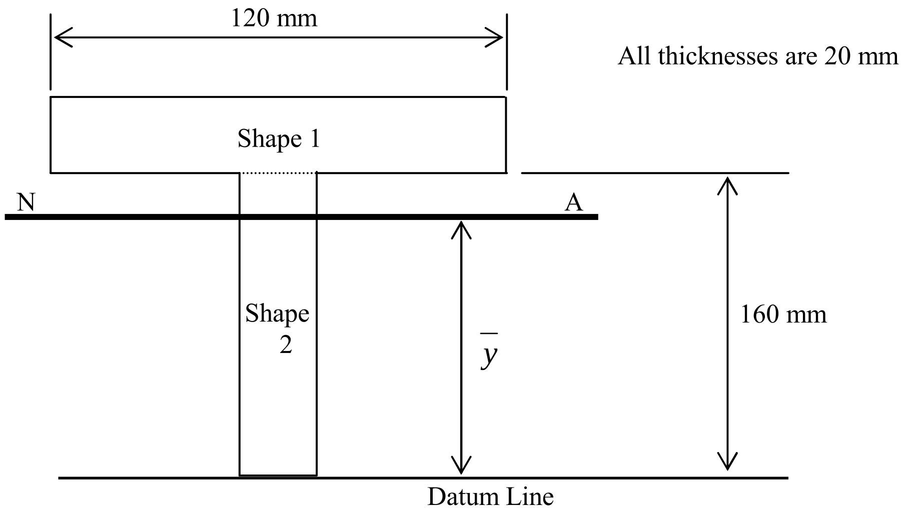
EXAMPLE

A beam has the cross section shown. Calculate the position of the Neutral Axis (NA).

The flange has dimensions 120 mm wide and 20 mm thickness. The web has dimensions 20 mm wide and depth 160 mm.



A datum line is needed about which to take moments. This is usually the base line of the cross section. The cross section is then subdivided into simple shapes whose area and own neutral axis can be found. In this example, the cross section is subdivided into two rectangles.



Taking First moments of area about the datum line:

$$A_1 y_1 + A_2 y_2 = (A_1 + A_2)\bar{y}$$

where A is the area of each shape and y is the distance from its own neutral axis to the datum line.

Rearranging the equation gives

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\Sigma Ay}{\Sigma A}$$

For determining the values of areas and distances, a table can be used to reduce possible calculation errors.

Shape Number	**Area A mm^2**	**Distance to Datum y mm**	**First moment of area Ay mm^3**
1	120×20 = **2400**	170	2400×170 = **408000**
2	160×20 = **3200**	80	3200×80 = **256000**
	Sum of areas ΣA = **5600**		Sum of moments ΣAy = **664000**

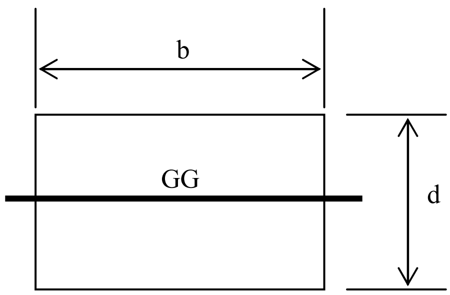
The distance to the neutral axis of the total cross section from the datum line is now given by:

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{664000}{5600} = \underline{\mathbf{118.6\ mm}}$$

The next problem is to find the value of I, the SECOND MOMENT OF AREA m^4, about the neutral axis.

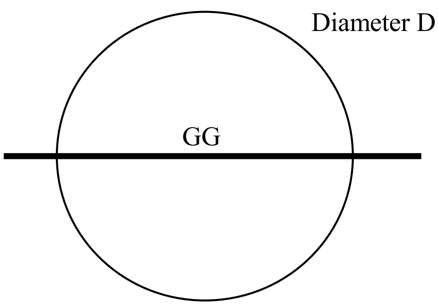
Again, for simple shapes the value of I **about its own neutral axis, the axis through the centre of gravity,** is known. Some example are:

Rectangular Cross Section



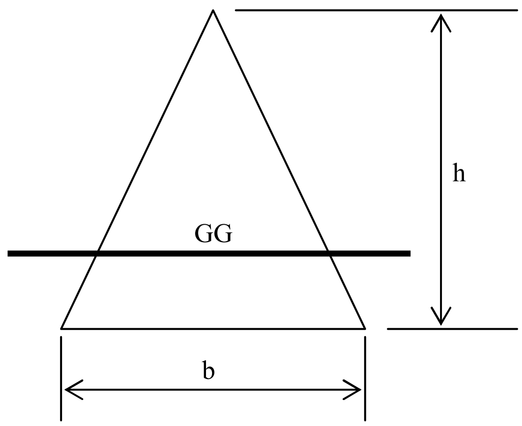
$$I_{GG} = \frac{bd^3}{12}\ \text{m}^4$$

Circular Cross Section



$$I_{GG} = \frac{\pi D^4}{64} \quad \text{Units m}^4$$

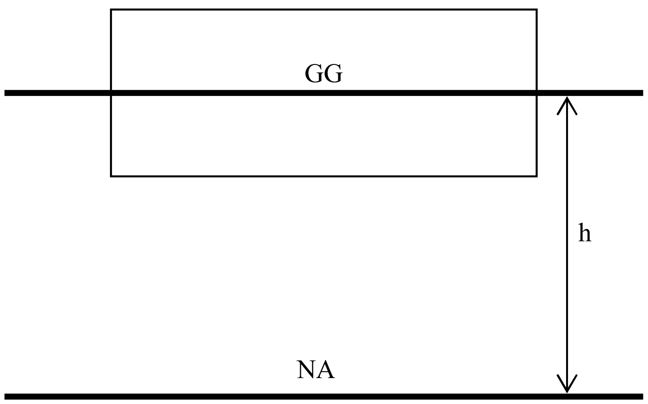
Triangular Cross Section



$$I_{GG} = \frac{bh^3}{36}$$

For more complex shapes, such as the T section in the previous example, the I value about the neutral axis of the shape itself is required. This involves the use of the

Parallel Axis Theorem



The value of I_{GG} is known depending on the shape. If it were a rectangle then $I_{GG} = \frac{bd^3}{12}$.

To find the value of I about a parallel axis such as NA, then

$$\mathbf{I_{NA} = I_{GG} + Ah^2}$$

A table is most useful for determining the values of I_{GG}, A and h. The value of I_{NA} can then be found by addition of the values.

EXAMPLE

For the T section in the previous example, find the value of I about its neutral axis.

Shape Number	**I_{GG} mm^4**	**Area, A mm^2**	**h^2 mm^2**	**Ah^2 mm^4**	**$I_{GG} + Ah^2$ mm^4**
1	$\frac{bd^3}{12}$ $= \frac{120 \times 20^3}{12}$ = **80000**	2400	$(170 - 118.6)^2$ = 2642	**6341000**	**6421000**
2	$\frac{bd^3}{12}$ $= \frac{20 \times 160^3}{12}$ = **6827000**	3200	$(118.6 - 80)^2$ = 1490	**4768000**	**11600000**
					$\Sigma I_{GG} + Ah^2$ =18020000

The value of I_{NA} for the T section about its own neutral axis is

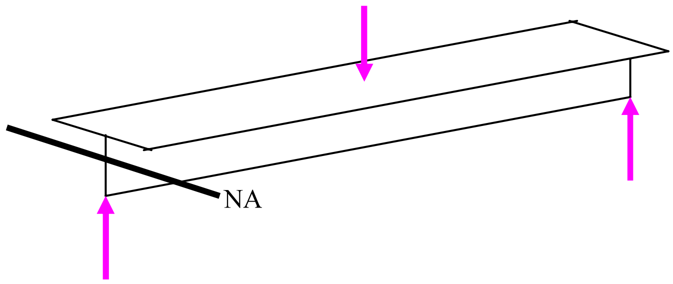
$18020000\ mm^4$

$= 18.02 \times 10^6\ mm^4$

$= \underline{\mathbf{18.02 \times 10^{-6}\ m^4}}$

Suppose the T-section beam is 2 m long, simply supported at each end and carries a single concentrated load of 10 kN at its mid-position. The flange is placed uppermost.

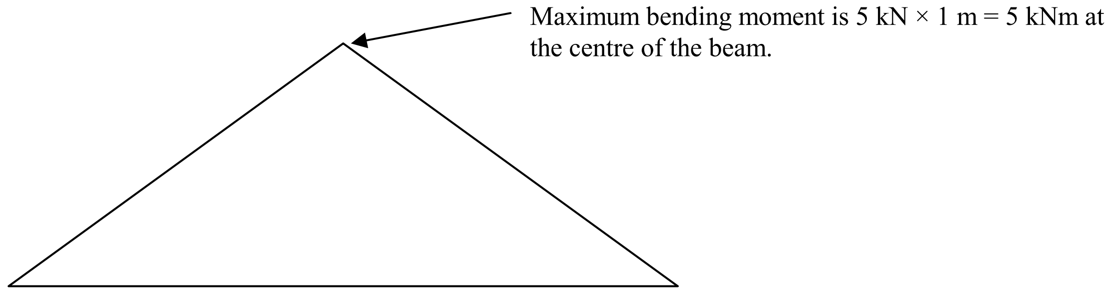
(a) What are the support reactions?



Since the loading is symmetrical each reaction will be **5 kN** Answer

(b) What is the maximum bending moment and where on the beam is it located?

See if you can draw the bending moment diagram for this beam. It should be a positive triangle indicating that the beam is sagging. This means there will be compressive stress along the top edge of the flange and tensile stress along the bottom edge of the web.



(c) What are the maximum compressive and tensile stresses?

For the compressive stress, the maximum value occurs on the top edge of the flange in the centre of the beam. To find its value, the bending formula is used:

$$\frac{M}{I} = \frac{\sigma}{y}$$

Therefore

$$\sigma_C = \frac{My}{I} \text{ where y is the distance from the NA}$$

$$\text{to the top edge of the flange } (180 - 118.7 = 61.3 \text{ mm})$$

$$\sigma_C = \frac{5 \times 10^3 \times 0.0613}{18.02 \times 10^{-6}}$$

$$\sigma_C = \underline{\mathbf{17.01\ MN/m^2}} \text{ Answer}$$

For the tensile stress, the maximum value occurs on the bottom edge of the web in the centre of the beam. In this case the value of y is 118.7 mm.

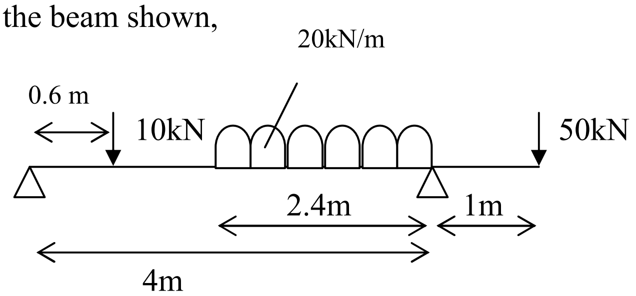
$$\sigma_T = \frac{My}{I}$$

$$\sigma_T = \frac{5\times10^3\times0.1187}{18.02\times10^{-6}}$$

$$\sigma_T = \underline{\mathbf{32.94\ MN/m^2}}\ \text{Answer}$$

Typical examination question

For the beam shown,



Calculate EACH of the following

a) The maximum shear force and its position (6)
b) The maximum bending moment and its position (4)
c) The position of the point of contraflexure (6)

The first task is to find the force at each of the reactions at the simple supports. The left hand support will be nominated A, and the right hand support nominated B.

Take moments about the left hand support, with clockwise (CW) moments being positive

$(10 \times 0.6) + (20 \times 2.4 \times 2.8) + (50 \times 5) - (B \times 4) = 0$

$6 + 134.4 + 250 - 4B = 0$

So B = 97.6 kN

Take moments about the right hand support, CW moments +ve

$(50 \times 1) + (A \times 4) - (10 \times 3.4) - (20 \times 2.4 \times 1.2) = 0$

$50 + 4A - 34 - 57.6 = 0$

So A = 10.4 kN

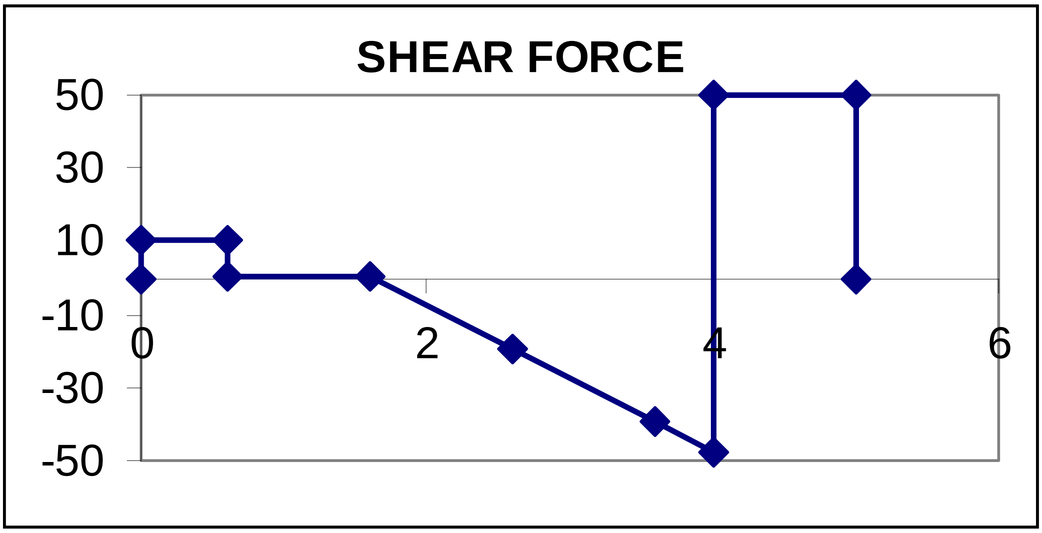
We can check that we have obtained the correct value of the support forces by adding them and equating them to the total force acting down.

A + B = 97.6 + 10.4 = 108 kN

Total forces acting down = $10 + 50 + (20 \times 2.4) = 108$ kN

The shear force and bending moment diagrams can now be drawn to find the required answers.

From the diagram the maximum shear force is +50 kN, and this occurs at the RHS support.



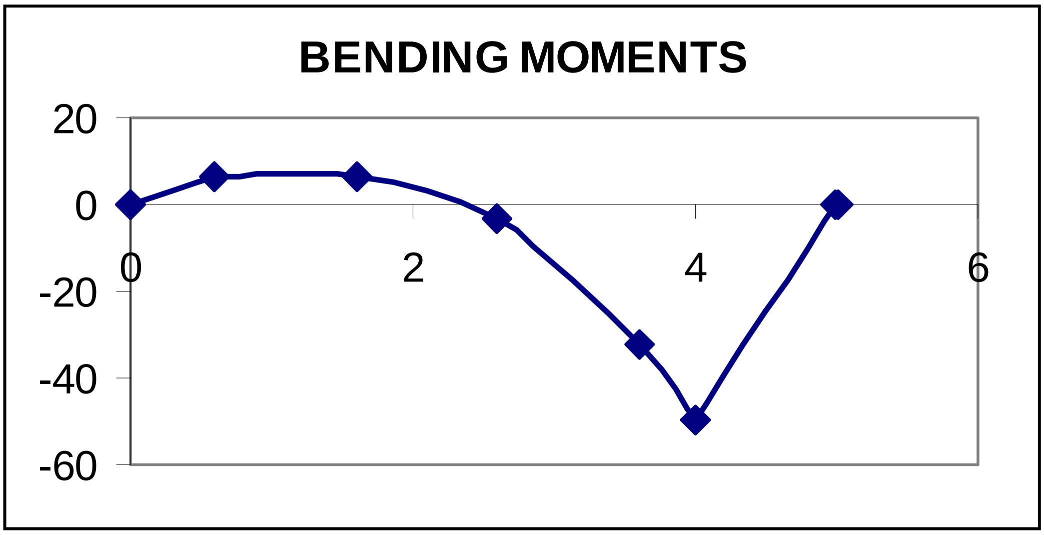
The maximum bending moment can also be found. The maximum bending moment will ALWAYS be at the point where the shear force is zero, so we only need to find the bending moment at two points.

I have drawn the bending moment diagram just to explain the answer, but this would not be required for this question.
So the shear force is zero at the RHS support and just past the LH edge of the uniform distributed load (UDL).

BM at the LH edge of the UDL is (10.4 x 1.6) – (10 x 1) = 6.64 kNm

BM at the RHS support is 50 x 1 = 50 kNm

Hence the maximum bending moment is 50 kNm at the RHS support. Note it is important that the position of the maximum bending moment is important, and should be quoted even if not specifically asked for.



The final part of the question asks for the point of contraflexure. i.e. where the Bending Moment is zero. From a rough sketch of the BM diagram, this will occur within the UDL part of the beam.

Stating the bending moment equation for that part of the beam, where d is the distance from the LH edge of the beam, gives:
$(10.4 \times d) - (10 \times (d - 0.6)) - 20\,(d - 1.6)(d - 1.6) \times 0.5 = 0$

Thus $10.4d - 10d + 6 - 10d^2 + 32d - 25.6 = 0$

so $10d^2 - 32.4\,d + 19.6 = 0$

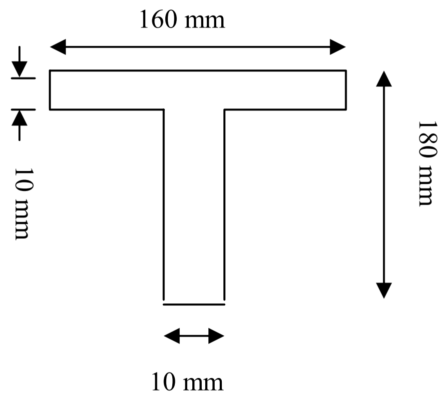
or $d^2 - 3.24d + 1.96 = 0$

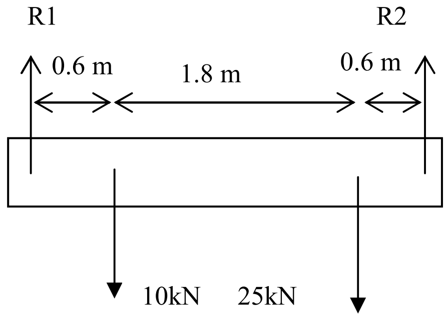
solving using the quadratic equation solution gives an answer of 2.44 m or 0.8 m.

As only the 2.44 m answer can be correct (as it is the only dimension inside the UDL and hence would make the original equation valid), then this will be the answer for the point of contraflexure.

Typical examination question

A lifting beam of cross section is used as shown.





Calculate EACH of the following:

a) The tension in cables R1 and R2 (4)

b) The maximum stress due to bending in the lifting beam, stating its position and whether it is tensile or compressive (12)

To find the tension in the cables R_1 and R_2, w shall take moments about each cable in turn:

Taking moments about R_1 (clockwise moments are +ve)
$(10 \text{ x } 0.6) + (25 \text{ x } 2.4) – (R_2 \text{ x } 3) = 0$

Hence $R_2 = 22$ kN

Taking moments about R_2 (clockwise moments are +ve)
$(R_1 \text{ x } 3) - (10 \text{ x } 2.4) + (25 \text{ x } 0.6) = 0$

Hence $R_1 = 13$ kN
To calculate the stresses within the lifting beam, then the maximum bending moment and the second moment of area are required.

As before the maximum bending moment will be found at the point where the shear force is zero. By observation (or even by drawing a rough shear force diagram) we can say that the shear force crossing the zero axis at the point where the 25 kN load is applied. Thus the maximum bending moment will be at this point.

So $BM_{max} = R_2 \text{ x } 0.6 = 22 \text{ x } 0.6 = 13.2$ kNm

As the beam cross section is not uniform about the x axis, then the neutral axis should be calculated.

$$y = \frac{(160\times10\times5)+(170\times10\times95)}{(160\times10)+(170\times10)} = 51.36 \text{ mm}$$

From the parallel axis theorem

Shape Number	I_{GG}	Area	h^2	Ah^2	$I_{GG} + Ah^2$
1	$\frac{bd^3}{12} = \frac{160\times10^3}{12}$ = **13333.3**	160 x 10 = 1600	$(51.36 - 10)^2$ = 1710.6	**2736960**	**2750293**
2	$\frac{bd^3}{12} = \frac{10\times170^3}{12}$ = **4094167**	10 x 170 = 1700	$(95 - 51.36)^2$ = 1904.4	**3237564**	**7331731**
					$\Sigma I_{GG} + Ah^2$ = **10082024** mm^4

Hence the Second moment of area for this beam section is 10082024 mm^4 or 10.08 x 10^{-6} m^4

If you are unable to see how we obtained these answers then carefully study the section of first and second moments of area again. It is important that you fully understand this concept, as without it you will be unable to tackle these questions.

Thus from $\frac{M}{I} = \frac{\sigma}{y}$ then the tensile and compressive stress due to bending can be calculated. Note the different values for the tensile and compressive "y" value due to the non-symmetrical shape (as the neutral axis is not in the centre of the beam)

For tensile stress $\sigma = \frac{My}{I} = \frac{13.2\times10^3\times(0.12-0.05136)}{10.08\times10^{-6}} = 168.46$ MN/m^2

For compressive stress $\sigma = \frac{My}{I} = \frac{13.2\times10^3\times0.05136}{10.08\times10^{-6}} = 67.26$ MN/m^2

Hence for the beam loaded as shown, the maximum stress due top bending is 168.46 MN/m^2 and is tensile in nature. The stress occurs at the point of the 25 kN load.

STUDENT EXAMPLES

1. A uniform cast iron beam is 4 m long and of solid square section of 180 mm side. It is slung at 240 mm from its mod length and balanced by a load suspended from one end. Find the force in the sling and the bending stress in the beam at the point of suspension. Density of cast iron is 7.25 tonne/m^3.
(10.49 kN; 5.88 MN/m^2)

2. A beam XABY is 13 m long and is simply supported at A and B. A is 1 m from X and B is 2 m from Y. It is loaded as follows:
Concentrated load of 20 kN at 3 m from X.
Concentrated load of 40 kN at 10 m from X.
Concentrated load of 30 kN at Y.
A uniformly distributed load of 30 kN/m over section XA.
A uniformly distributed load of 15 kN/m commencing at 3 m from X and extending to B.

Draw the SF diagram and determine the dimensions of a solid rectangular section of depth three times the breadth if the maximum stress is not to exceed 120 MN/m^2.
(300 mm deep, 100 mm wide)

3. A cantilever 2 m long consists of a solid steel bar of circular section. If a vertical force of 500 N is applied at its free end, determine the minimum diameter required if the maximum stress due to bending is not to exceed 50 MN/m^2. Determine also the radius of curvature to which the loaded bar will bend.
E = 200 GN/m^2 and ignore the weight of the bar.
(59 mm; 118 m)

4. A main steam pipe is firmly fixed to a stop valve at one end whilst the other end is free to move vertically (i.e. it can be considered as a cantilever). The pipe is 3 m long, 150 mm outside diameter and 10 mm thickness. As the pipe heats up when steam is admitted, the end of the pipe deflects 8 mm vertically. This is equivalent to applying a concentrated load F N at the end of the pipe. Find the maximum stress induced in the pipe.
NOTE. The main steam pipe may be treated as a cantilever for which the deflection is given by $\frac{FL^3}{3EI}$ where F is the deflecting force and L is the length of the cantilever or pipe. Take E = 210 GN/m^2.
(42 MN/m^2)

5. A cantilever of reducing cross section is 1.4 m long and carries a vertical force F at its free end. At AA, a vertical cross section 80 mm from the fixed end, the section is of I form of total depth 300 mm, equal flanges 150 mm broad and metal thickness 40 mm throughout. At BB, which is 120 mm from the free end, the vertical cross section is again of I form but with dimensions only half those at section AA. If the maximum bending stress at BB is not to exceed 10 MN/m^2 find the magnitude of the force F and the stress at AA assuming this force to be acting.
(16.65 kN; 13.75 MN/m^2)

6. A shaft of uniform cross section is to be lifted by two slings. Determine the points of attachment for the slings, in terms of the shaft length, so that the stress due to bending in the shaft material is a minimum value.

(0.207L from each end)

7. A horizontal cantilever 1.4 m long is of T section with the overall depth 150 mm, breadth of flange 150 mm and metal thickness 15 mm throughout. It carries a uniformly distributed load along the full length of the flange, which is placed uppermost, and the maximum tensile and compressive stresses are not to exceed 30 MN/m^2 and 90 MN/m^2 respectively. Determine the greatest allowable spread load, inclusive of the effect of the beams own mass, given that $I = 9.1 \times 10^{-6}$ m^4 for the T section.

(6.36 kN/m)

8. A beam has an I section of overall depth 300 mm, equal flanges of 150 mm breadth, thickness of flanges 20 mm and thickness of web 10 mm. It is simply supported over a span of 8 m and the working stress due to bending is limited to 120 MN/m^2. Calculate the maximum uniformly distributed load the beam can carry, including the effect of its own mass.

(13.3 kN/m)

9. A steel bar of T section is employed as a simply supported beam over a span of 3 m. The flange of the bar is 120 mm broad, overall depth 150 mm and thickness of metal 20 mm throughout. The beam is set with the flange uppermost and carries concentrated loads of F N, one centrally placed and the others at quarter points of the span. If the maximum tensile stress due to bending is not to exceed 120 MN/m^2 determine the allowable values of F.

(8.3 kN)

10. Two channel bars of 80 mm × 220 mm section are placed back to back with their webs vertical to form an I beam of 8 m length, which is simply supported at its ends. The uniformly distributed load exerted due to their own masses is 500 N/m. Find the additional uniformly distributed load that can be carried over the central 4 m of the beam if the maximum stress is limited to 100 MN/m^2. For the channel bars: flange thickness is 10 mm and web thickness is 12.5 mm.

(7.217 kN/m)