KINEMATICS SECTION c)

VELOCITY DIAGRAMS FOR MECHANISMS

Angular Velocity of a line

Consider a line or link AB of fixed length moved across a plane to any new position as shown in Figure 1. After a small interval of time d*t* let the line AB move to A'B' and let AB make a small angle dθ rads with A'B'.

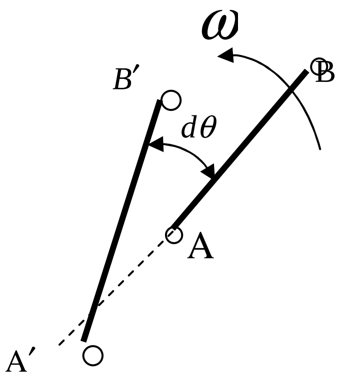


Figure 1.

Then the *angular velocity* ω of the line AB is defined as:

$$\omega = \frac{d\theta}{dt}$$

and is measured in radians per second (rad/s).

MOTION OF A BODY IN A PLANE

A rigid body forming part of a mechanism will always be of fixed length whatever its motion. Any two points A and B on the body will therefore remain at a fixed distance apart. Since there is no stretch along the line AB, there will be no velocity of B relative to A along AB, Figure 2. However, since the body may rotate, B may

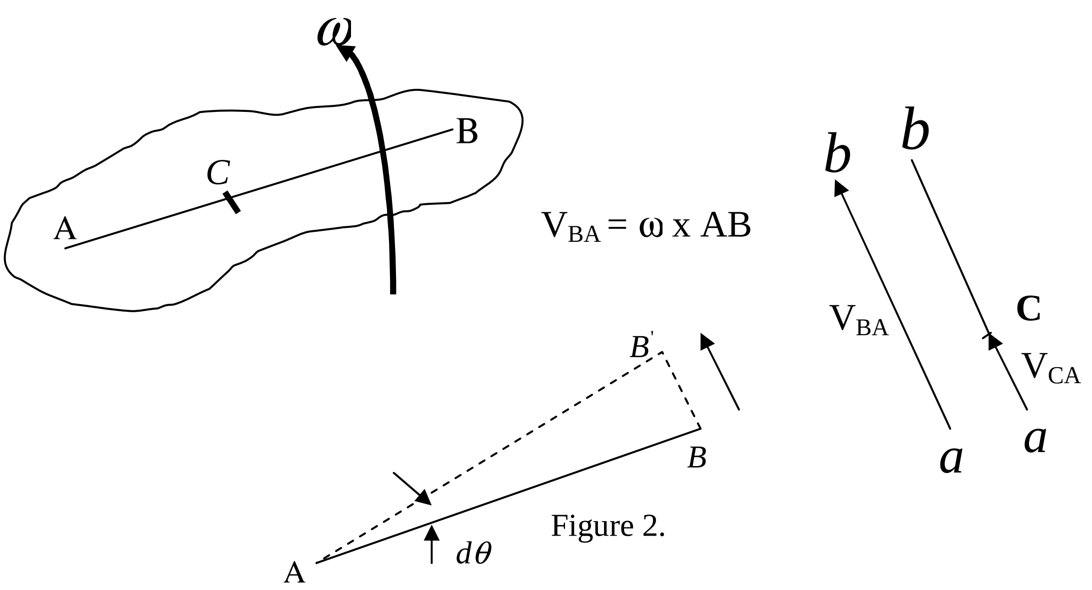


Figure 2.

have a velocity relative to A by rotation about A, as if A were fixed. The motion of B relative to A *can occur only in a direction perpendicular to the line AB.* This must be so whatever the motion of A. If B rotates about A with angular velocity ω then, after a small time d*t*, let AB′ be the position of AB, where $\angle BAB' = d\theta$ (Fig.1)

Distance travelled by B = BB'

Therefore velocity of B normal to AB $= v_{BA} = \dfrac{BB'}{dt}$

Since $d\theta$ is small, $BB' = AB \times d\theta$

Therefore $v_{BA} = AB \times \dfrac{d\theta}{dt}$

$= AB \times \omega$, since $\omega = \dfrac{d\theta}{dt}$

hence $\omega = \dfrac{V_{BA}}{AB}$

$= \dfrac{velocity\ of\ B\ relative\ to\ A}{length\ AB}$

The relative velocity V_{BA} would be represented by a vector **ab**, of length ωAB drawn perpendicular to AB, the sense of the vector corresponding to the motion of B relative to A, figure 2. In the same way for any point C on the line AB we may write.

$$V_{CA} = \omega.AC$$

thus

$$\frac{V_{CA}}{V_{BA}} = \frac{\omega AC}{\omega AB}$$

i.e.

$$\frac{ac}{ab} = \frac{AC}{AB}$$

Therefore, if point c is located on **ab** such that

$$\frac{ac}{ab} = \frac{AC}{AB}$$

then the velocity of C relative to A is given by **ac.**
Vector **ab** is called the *velocity image* of the line AB. When the velocity image of a link has been obtained, therefore, the relative velocity between any two points on the link (or an extension of the link) is given by the length between the corresponding points on the image. This is a most useful fact to remember when dealing with mechanisms.

Velocity Triangle for a Rigid Link. Application to Mechanisms

Suppose now that AB represents a link in a mechanism and that V_A, the velocity of A, is known completely whereas V_B the velocity of B is known in direction only (Figure 3).

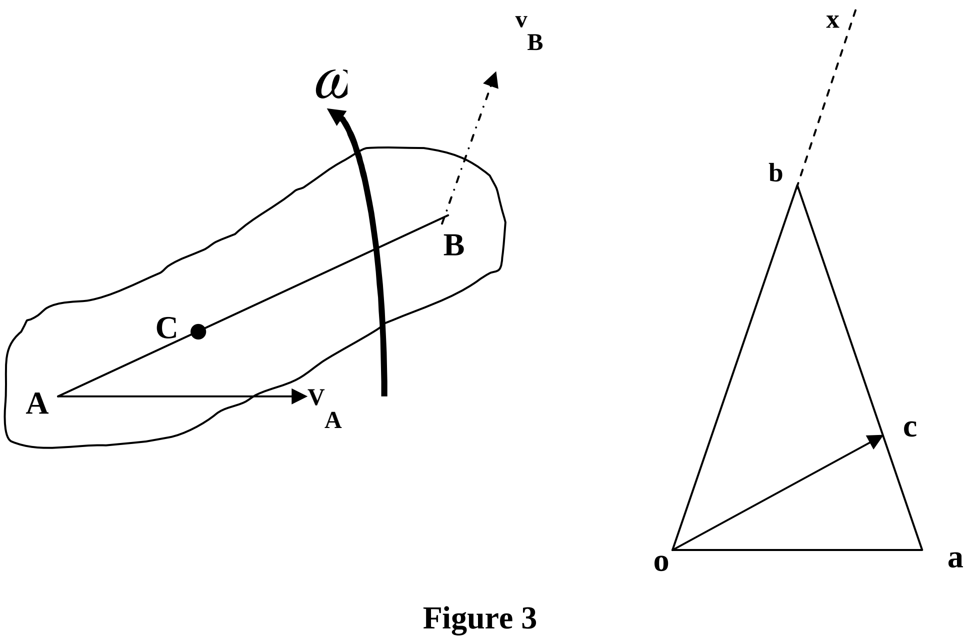


Figure 3

The problem is to find the magnitude V_B of the angular velocity of the link. To do this we draw a *velocity triangle* for the link as follows –

> Draw **oa** to represent $V_{\mathbf{A}}$ in magnitude direction and sense
> Through **o** draw a line ox parallel to the given direction of V_B.
> Through **a** draw a line *perpindicular to the link* to cut **ox** in **b**.
> Then ob represents V_B and **ab** is the velocity image of AB.
> **oab** is the velocity triangle for link AB.

To find the velocity of any point C on AB first locate point **c** on the velocity image **ab** such that.

$$\frac{ac}{ab} = \frac{AC}{AB}$$

then $$V_c = oc$$

The angular velocity of the link is

$$\omega = \frac{V_{BA}}{AB} = \frac{ab}{AB}$$

Note that **oac** is the velocity triangle for link AC and.

$$\omega = \frac{V_{CA}}{AC} = \frac{ac}{AC}$$

and that all absolute velocities are measured from **o.**

The velocity triangle is most useful when dealing with the problem of finding the velocities of points in mechanisms. Each link is taken in turn and the velocity diagram obtained before proceeding to the next link in the chain. The method is shown in the following examples.

Example 1

The crank OA of the engine mechanism shown (Figure 4) rotates at 3,600 rev/min anticlockwise. OA = 100mm, and the connecting-rod AB is 200 mm long. Find *(a)* the piston velocity; (b) the angular velocity of AB; *(c)* the velocity of point C on the rod 50 mm from A.

Solution Angular Velocity of crank OA

$$\omega = \frac{2\pi \times 3{,}600}{60} = 376.8 rad/s$$

velocity of A $V_A = \omega OA = 376.8 \times 0.1$

$$= \mathbf{37.68\ m/s}$$

Point A is moving in a circular path and oa is therefore drawn at right angles to OA to represent $V_{A.}$ The velocity of B is unknown but its **direction** is horizontal. Hence in the velocity diagram a line of indefinite length is drawn horizontally through **o**

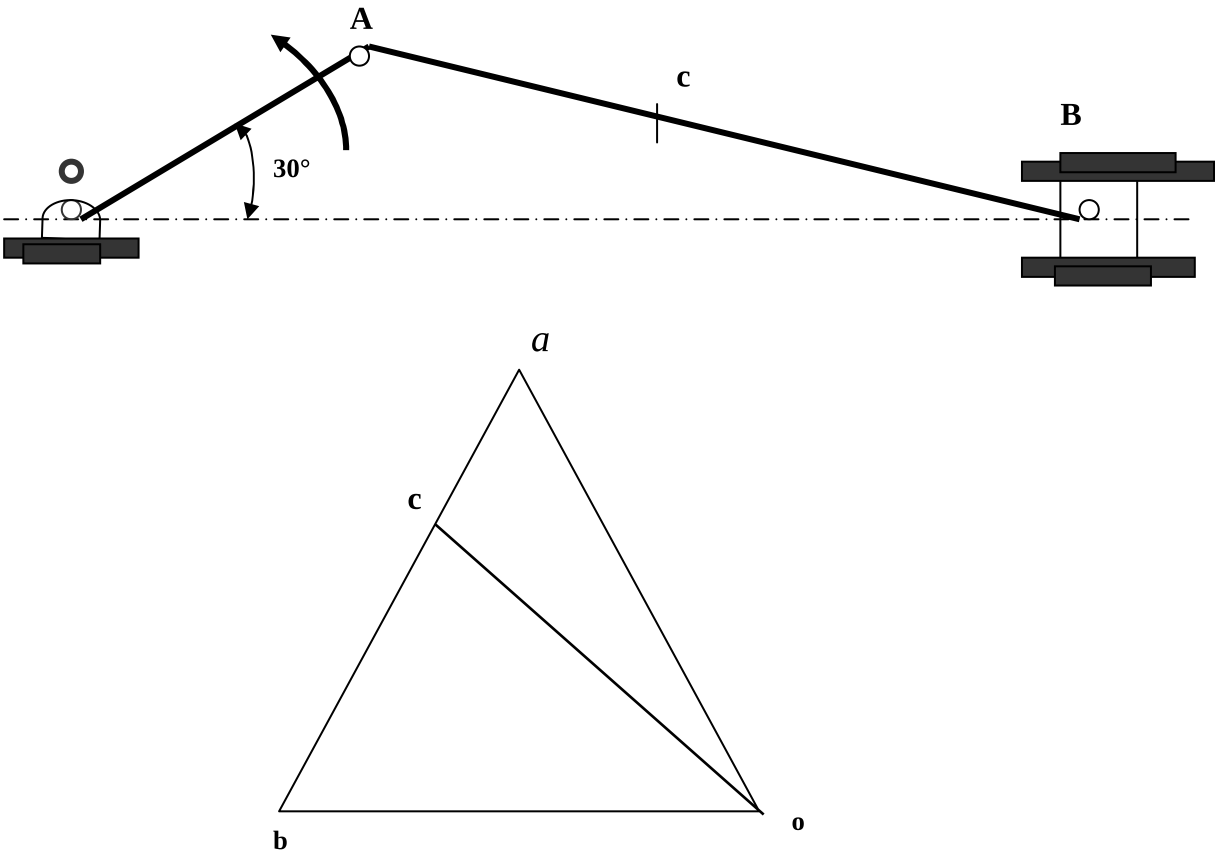


Figure 4

The velocity of B *relative* to A must be *perpendicular* to AB; therefore the velocity triangle for the link AB is completed by drawing a line **ab** perpendicular to AB. The lines **ab**, **ob** cut at point **b** which determines the magnitude of the velocity $V_{B.}$ of the piston B. From the diagram.

$$V_B = \mathbf{ob} = \mathbf{27\ m/s}$$
$$V_{BA} = \mathbf{ab} = \mathbf{33.6\ m/s}$$

$$\text{Angular velocity of AB} = \frac{velocity\ of\ B\ relative\ to\ A}{length\ of\ AB}$$

$$= \frac{33.6}{0.2}$$
$$= \mathbf{168\ rad/s}$$

To find the velocity of C, then since C is on AB mark off **ac** on the velocity image **ab** such that:

$$\frac{ac}{ab} = \frac{AC}{AB}$$

Then **oc** represents in magnitude and direction the velocity of C, the sense being from **o** to **c.** Thus

$$V_C = oc = \mathbf{32.4\ m/s}$$

Example 2
Figure 5 shows a three-bar mechanism OABQ for a wrapping machine. OA amd QB rotate about fixed points O and Q 400 mm apart. OA = 75 mm, AB = 300 mm, BQ = 100 mm. At the instant considered OA is at 30° to OQ and is rotating clockwise at 12 rev/s. Find *(a)* the velocity and direction of motion of point C on AB, 100 mm from A; *(b)* the angular velocity of the link AB.

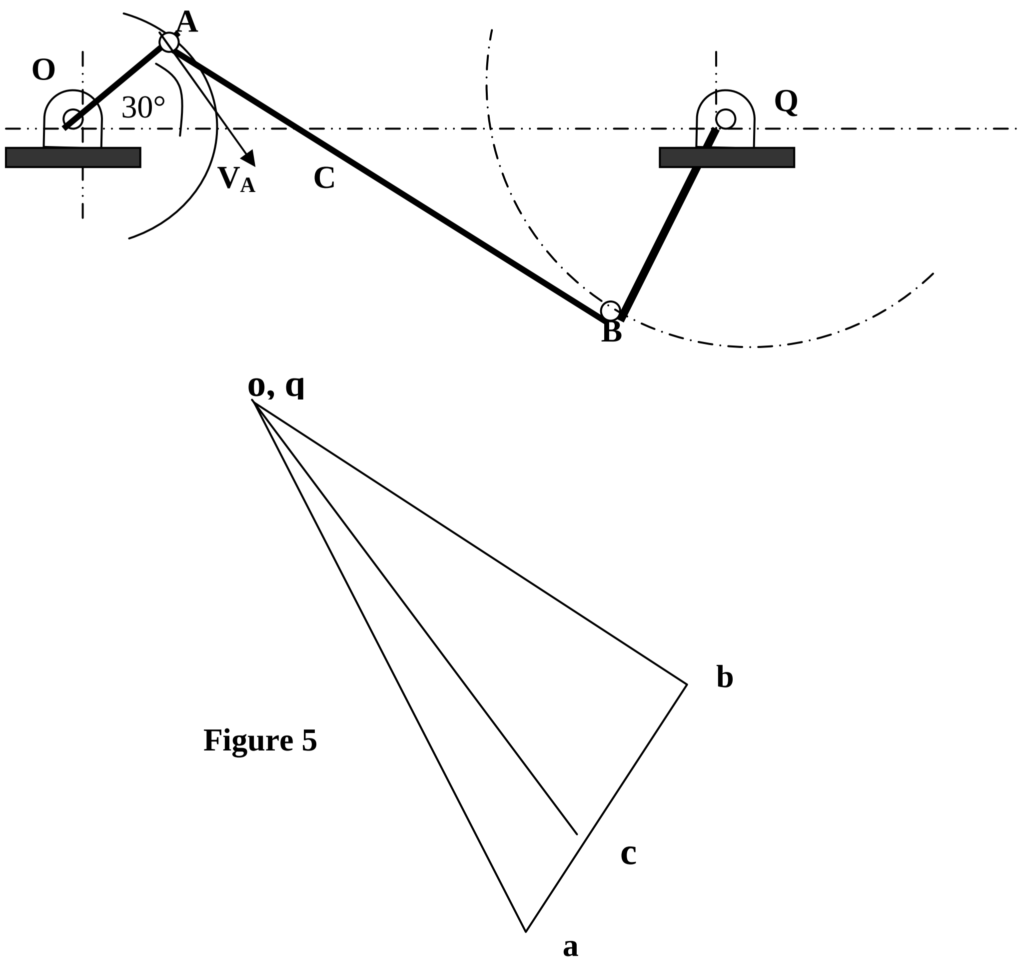


Figure 5

Solution

Angular Velocity of OA

$$\omega = 2\pi \times 12 = 75.4\, rad/s$$

So linear velocity of A

$$V_{\mathbf{A}} = 75.4 \times 0.075 = 5.66 \text{ m/s}$$

First draw the mechanism to scale.
Set off **oa** to represent the velocity of A, 5.66 m/s. Note the sense of **oa** since OA rotates clockwise.

Draw **ab** of idefinite length perpendicular to AB, and draw **ob** perpendicular to QB: these two lines meet at point **b**. **ob** represents the velocity of B.

(a) Divide **ab** at **c** such that

$$\frac{ac}{ab} = \frac{AC}{AB} = \frac{100}{300}$$

Then **oc** represents the velocity of C in magnitude and direction. From the diagram, $V_C =$ **oc** = **5.25 m/s** at **53°** to the horizontal in the sense **o** to **c.**

(b) The velocity of B relative to A

$$V_{BA} = \mathbf{ab}$$

$$= 2.4 \text{ m/s, from the diagram}$$

$$\text{angular velocity of AB} = \frac{ab}{AB} = \frac{2.4}{0.3} = \mathbf{8\ rad/s}$$

Example.3

Figure 6 shows a four-bar mechanism OABQ with a link CD attached to C the mid-point of AB. The end D of link CD is constrained to move vertically. OA = 1m, AB = 1.6 m, QB = 1.2 m, OQ = 2.4 m, and CD = 2m. At the position shown the angular velocity of crank OA is 60 rev/min clockwise; find:

(a) the velocity of D;
(b) the angular velocity of CD;
(c) the angular velocity of BQ.

Solution

Angular velocity of OA

$$\omega = \frac{2\pi \times 60}{60} = 6.28\, rad\ / s$$

velocity of A, $V_A = \omega OA = 6.28 \times 1 = 6.28 m/s$

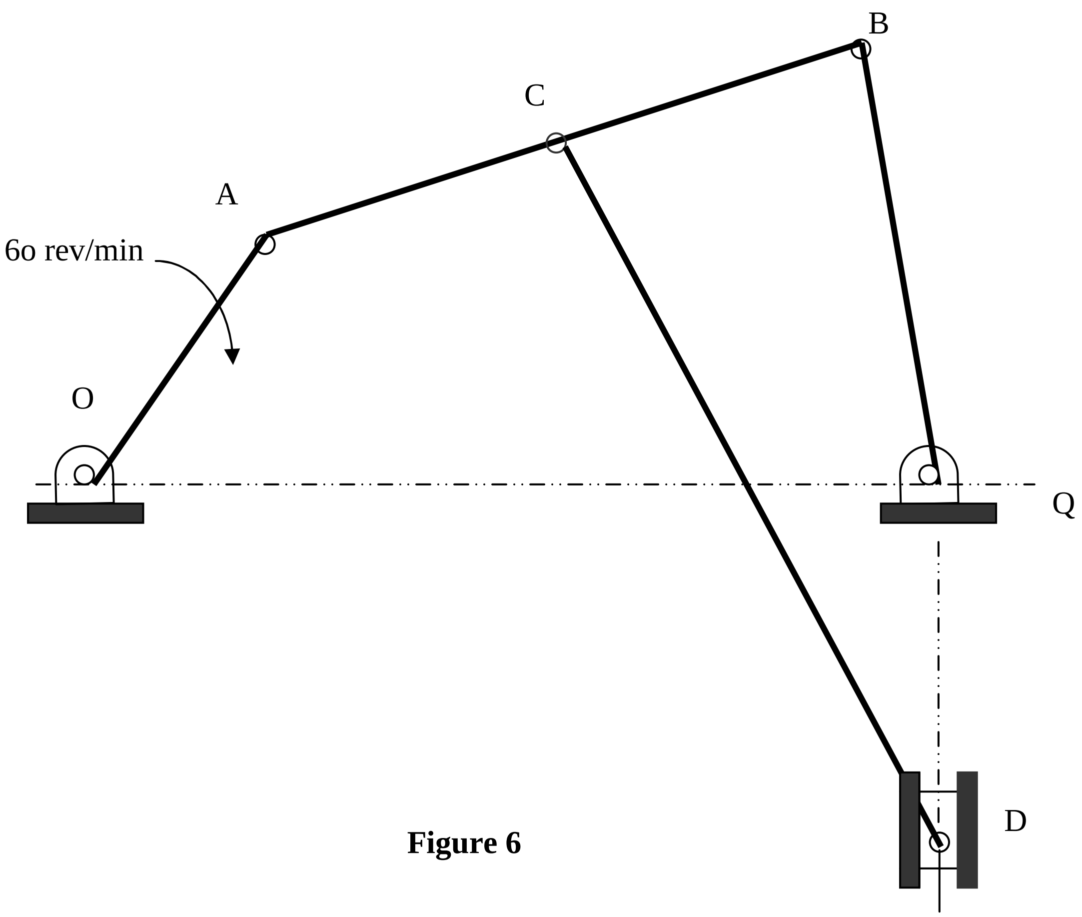
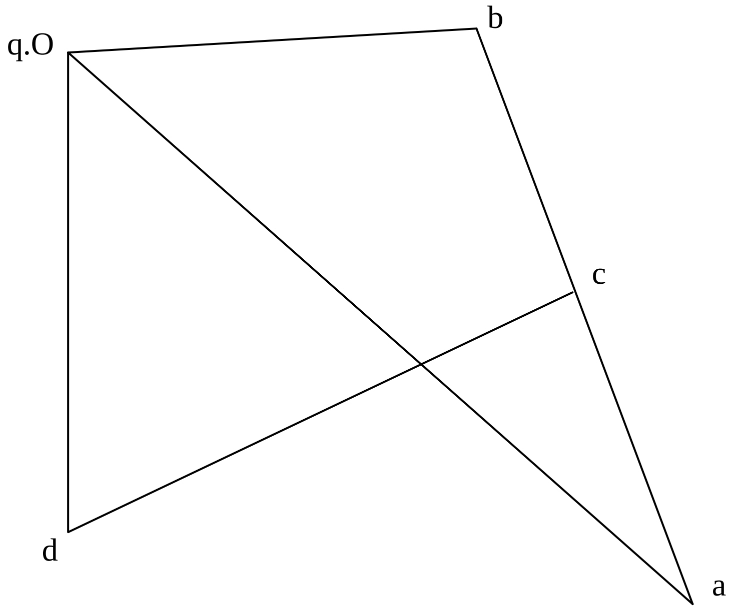


Figure 6

Points O and Q are at rest. Points A and B are moving in circular paths, thus the directions of their velocities are known.

Draw **oa** normal to OA to represent v_A, 6.28 m/s.
Draw through **q** (coincident with **o**) normal to QB a line **qb** of indefinite length to represent v_B, the magnitude of which is unknown.
Draw through **a** a line perpindicular to AB to represent v_{BA}, the velocity of B relative to A; thus point **b** is located. Since C is the mid-point of AB, **c** is the mid-point of the velocity image **ab** in the velocity diagram.
The velocity of D is vertical , therefore draw **od** vertically through **o,** this line being of indefinite length.
The velocity of D relative to C is normal to DC , therefore draw **cd** from c perpindicular to CD.
The intersection **d**, of **od** and **cd**, completes the diagram.

VELOCITY DIAGRAM



From the diagram,

V_D = od = **od** = 4 m/s in direction **o** to **d**

Angular velocity of CD = $\dfrac{velocity\ of\ D\ relative\ to\ C}{length\ of\ CD}$

$= \dfrac{cd}{CD}$

$= \dfrac{4.21}{2}$

= **2.1 rad/s**

Angular velocity of BQ = $\dfrac{velocity\ of\ B\ relative\ to\ Q}{BQ}$

$= \dfrac{qb}{BQ}$

$= \dfrac{3.02}{1.2}$

= **2.51 rad/s**

Self Assessed Questions for you to try. (Answers given).

1. The engine mechanism of Figure 7 has a crank 200 mm long and connecting rod 500 mm long. If the crank speed is 50 rev/s clockwise, find for the position shown:

(a) the piston velocity;

(b) the angular velocity of the connecting rod;

(c) the velocity of a point C on the rod 200 mm from the crankpin.

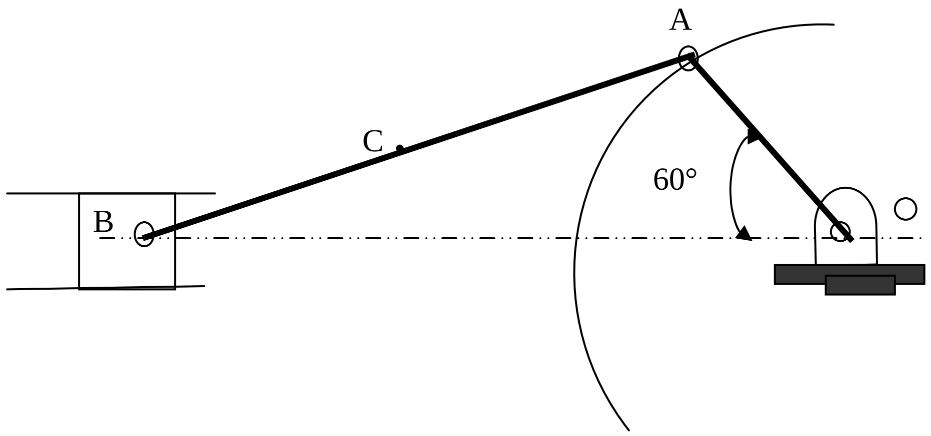


Figure 7

(ANS 66 m/s; 66 rad/s anticlockwise; 61.8 m/s)

2. The crank OA in the mechanism shown, Figure 8, rotates anticlockwise at 5 rev/s and is 300 mm long. The link AB is 600 mm long and the end B moves in horizontal guides.

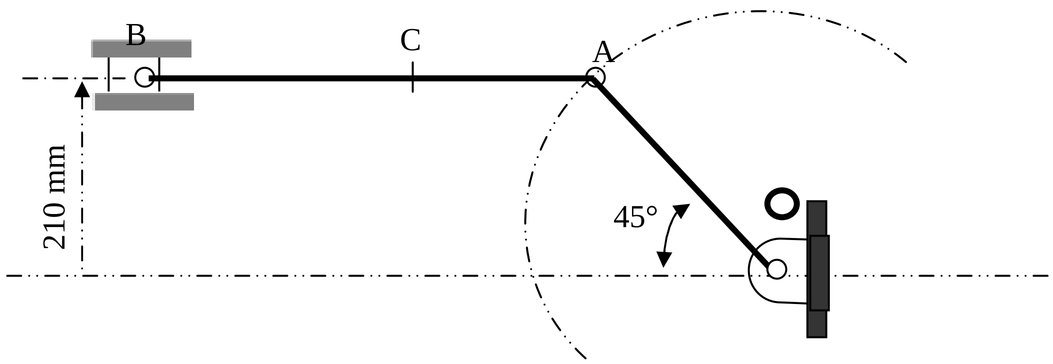


Figure 8

Find for the position shown:

(a) the velocity of the B;

(b)the velocity of point C, the mid-point of AB;

(c) the angular velocity of AB.

(6.7 m/s; 7.5 m/s; 11.1 rad/s clockwise)