KINEMATICS SECTION b)

The following section is just to remind you about angular motion. You will need to know the relationship between linear and angular motion, and how to convert between revolutions, radians and degrees. If you are already familiar with this work, you may like to move on to the next section on Resultant and Relative velocity.

ANGULAR MOTION

May be measured in units of "DEGREES", RADIANS or REVOLUTIONS:

Note!

Arc length = Angle (in radians) *x* Radius

For a circle, the arc length is $2\pi R$ meters and the angle is 360^0.

$$360^0 \text{ Angle (in Radians)} = \frac{2\pi R \ \text{(m)}}{R \ \text{(m)}} = 2\pi \text{ radians}$$

Hence 1 radian is approximately 57.3^0. Note also that the metres on the top and bottom of this equation cancel out, so radians are dimensionless

Angular Displacement (θ) [theta]

Basic unit is the radian.

Angular Velocity (ω) [omega]

The basic units of angular velocity are radians/second (rad/s).

Often revolutions are used in either revs/min or revs/sec. Each revolution is worth 2π radians or 360^0

Note! For N revs/min

$$\text{Angular velocity } \omega = \frac{2\pi N}{60} \qquad \text{rads/s}$$

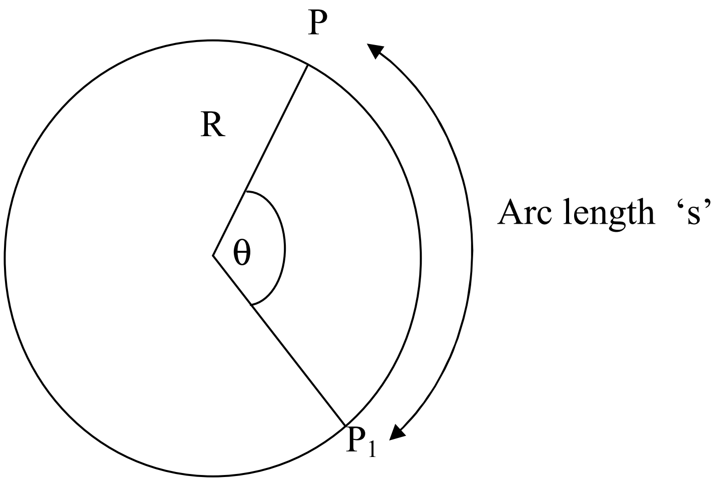
Angular Acceleration (α) [alpha]

This is the rate of change of angular velocity, where the units are radians per second^2 or rads/s^2.

LINEAR AND ANGULAR MOTION RELATIONSHIPS

DISPLACEMENT:-

For the point "P" moving in a circular path to point "P_1"



Arc length = angle in radians x Radius.

$$s = \theta R \text{ (m)} \qquad \text{(i)}$$

VELOCITY:-

If point "P" moved through arc "s" at a constant angular velocity:-

From Displacement = velocity *x* time

$$\theta = \omega \, x \, t \quad \text{substitute in (i)}$$

$$s = \omega \, x \, t \, x \, R$$

From:- Linear velocity = $\dfrac{\text{Linear Displacement}}{\text{time}}$

$$v = \frac{\omega \, x \, t \, x \, R}{t}$$

$$v = \omega R \qquad \text{(ii)}$$

ACCELERATION:-

If point "P" has a linear acceleration "a" (m/s^2)

$$a = \frac{v - u}{t}$$

$$a = \frac{\omega_2 R - \omega_1 R}{t}$$

$$= \frac{(\omega_2 - \omega_1)\,R}{t}$$

$$a = \alpha R \qquad \text{(iii)}$$

NOTE! IN EVERY CASE:- <u>LINEAR TERM = ANGULAR TERM X RADIUS</u>

Remember this**, the relationship between linear velocity and angular velocity is the radius**. We do not need to remember a whole new set of formulae for angular motion. They are essentially the same formulae used for linear motion (with uniform acceleration). Only the symbols change. Thus we have:

$\omega^2 = \omega_1 + \alpha t$ $\qquad$ $s = \theta R$

$\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$ $\qquad$ $v = \omega R$

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ $\qquad$ $a = \alpha R$

$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$

$\alpha = (\omega_2 - \omega_1)/t$

Examples

1.Express a rotational speed of 100 rev/min in rad/s.

$$\omega = \frac{2\pi N}{60} \text{ rad/s}$$

$$= \frac{2\pi \times 100}{60}$$

$\approx$ <u>10.5 rad/s</u>

2.Express a speed of 25 rad/s in rev/s.

Solution:

$$n = \frac{\omega}{2\pi} \text{ rev/s}$$

$$= \frac{25}{2\pi}$$

$\approx$ <u>4</u> <u>rev/s</u>

3. In an experiment, the time for a wheel to make 25 complete revolutions was found to be 14 s. Find the rotational speed of the wheel in rad/s.
What will be the angle turned through in 5 s ?
Solution:

$$\text{Rotational speed} = \frac{25}{14} \text{ revs/s}$$

$$\omega = \frac{25 \, x \, 2\pi}{14} \text{ rads/s}$$

$$= \underline{11.2 \text{ rad/s}}$$

Angle = Ang velocity *x* time

$= 11.2 \, x \, 5$

$= \underline{56.1 \text{ radian}}$ or 8.93 revolutions

4. A shaft is rotating at 40 r.p.m. when it is retarded at 0.017 rads/s^2 for a period of 30 seconds.
Determine its velocity at the end of this time.
If the speed is to be restored to 40 r.p.m. in 25 seconds calculate the acceleration required.
Solution:

$$\omega_2 = \omega_1 + \alpha.t$$

$$\omega_1 = \frac{40 \, . \, 2 \, . \, \pi}{60} = 4.189 \text{ rads/s}$$

$$\omega_2 = 4.189 - (0.017 \, x \, 30) = 3.679 \text{ rads/s}$$

$$N = \frac{3.679}{2\pi} = \underline{35.13 \text{ revs/min.}}$$

$$a = \frac{\omega_2 - \omega_1}{t}$$

$$\alpha = \frac{4.139 - 3.679}{25}$$

$$= \underline{0.0204 \text{ rads/s}^2}$$

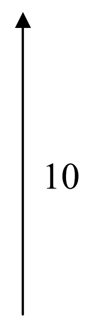
RESULTANT AND RELATIVE VELOCITY

Quantities dealt with in engineering can be placed into two broad groups, **"SCALAR"** quantities and **"VECTOR"** quantities.
A Scalar quantity is one that has magnitude only, e.g. speed and the base units of Length, Mass and Time.

A **Vector** quantity is one that possesses **both magnitude and direction** and both must be stated in order to specify that quantity completely.

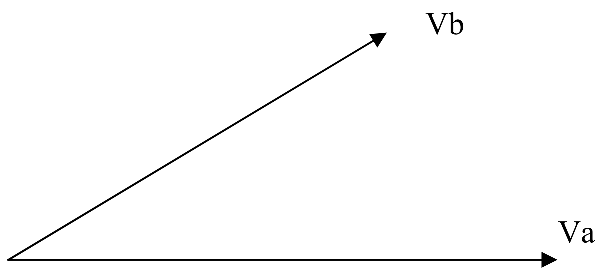
Examples of vector quantities are:-
The movement of a body is a specific direction
The force acting on a body with a definite line of action.

Vector quantities lend themselves to graphic representation, for instance a velocity of 10 m/s due north could be represented as a "vertical" line 10 cm long, the north being represented by the vertical and the length to a scale of 1 cm = 1 m/s.

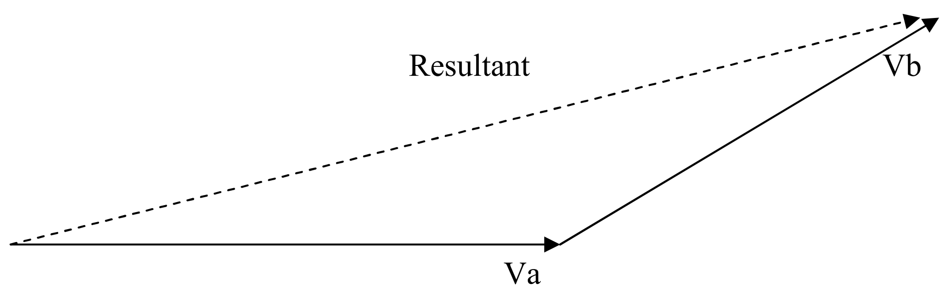


Resultant Velocity:-

Is the single velocity that can have the same effect as the velocities it replaces.
Resultant velocity can be obtained by **Vector Addition**. Since we will deal with the addition of more than two forces, the easiest graphical method of performing vector addition is to **draw the vectors nose-to-tail**. The **Resultant** Velocity is then the vector that **goes from the start of the diagram to the finish** of the diagram.
For a body subjected to the two velocities V_a and V_b.



Draw the vectors "nose-to-tail" (start each vector from the end of the one before), then the resultant goes from start to finish.

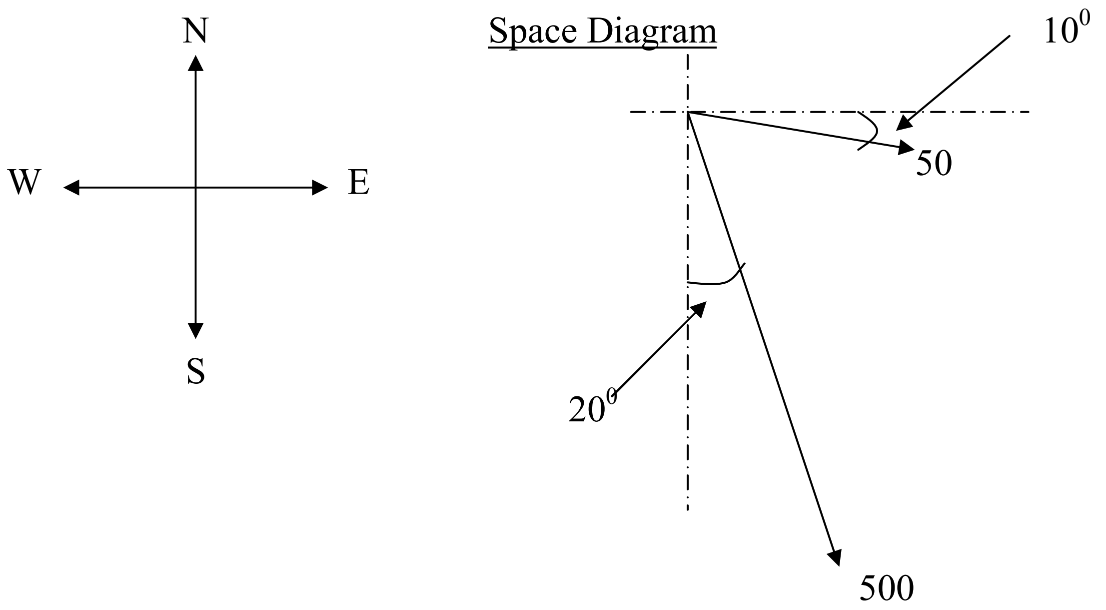


Example 5

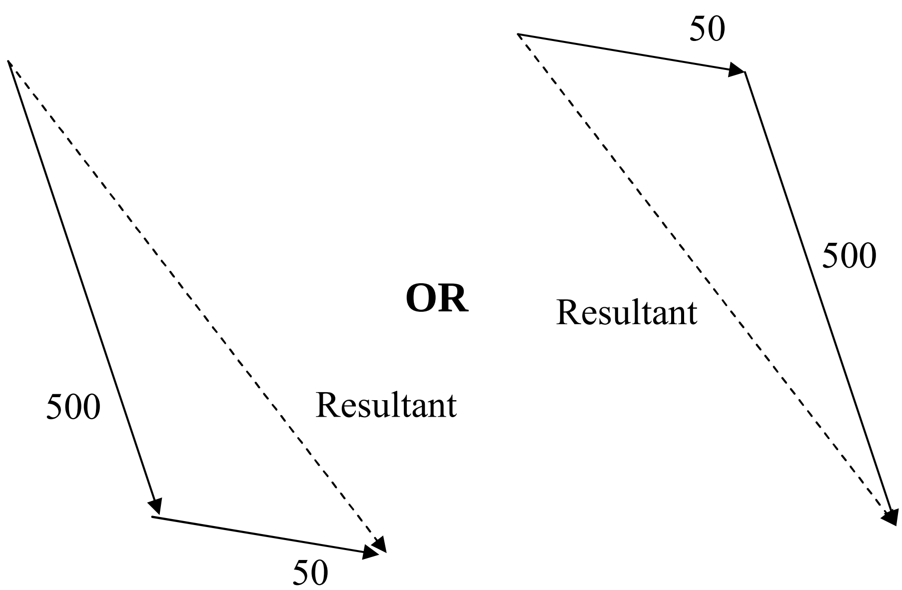
1. An aircraft flying at 500 knots in a direction of 20^o East of South encounters a gale force wind 50 knots blowing in a direction 10^o South of East.
Determine the resultant velocity of the aircraft.

Solution:

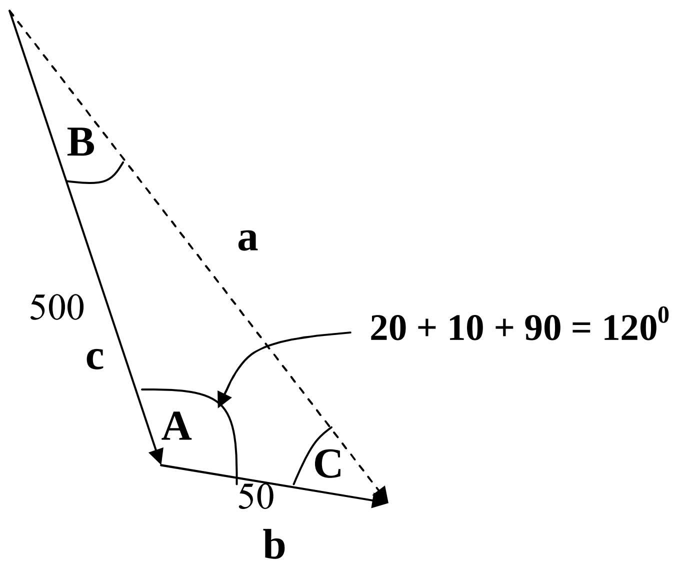
We want to find the resultant velocity, so we should draw the velocity vectors nose-to-tail. But first, we should draw a space diagram, representing the information given in the question. It is always a good idea with this type of question to sketch a compass so that we do not make any mistakes with the direction.



We can now draw the vector diagram, vectors nose-to-tail. Note that it does not matter which vector we choose to draw first, although the diagram will look different, the resultant velocity will be the same.



We could find the resultant velocity simply by measurement of magnitude and direction if we drew the velocity diagram to scale. It is good practice to draw the diagram roughly to scale anyway as it will give us some confirmation that the answer looks right. If you are asked in a question to "Determine" then it means you have a choice of doing the problem by scale **or** calculation. Try this example yourself both ways. Here is how the answer would be calculated:



Using the Cosine Rule,

$a^2 = b^2 + c^2 - 2.b.c.\text{Cos A}$

$a^2 = 500^2 + 50^2 - 2 \times 500 \times 50 \times \text{Cos A}$

a = 527 knots (The resultant velocity, in magnitude only)

To get the direction, we could use the Cosine rule again, but we now have enough information to use the much simpler Sin Rule:

$$\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$$

$$\frac{50}{\text{Sin B}} = \frac{527}{\text{Sin 120}}$$

Sin B = 0.0822, so B = 4.72^0, which makes it in a direction 24.72^0 East of South.

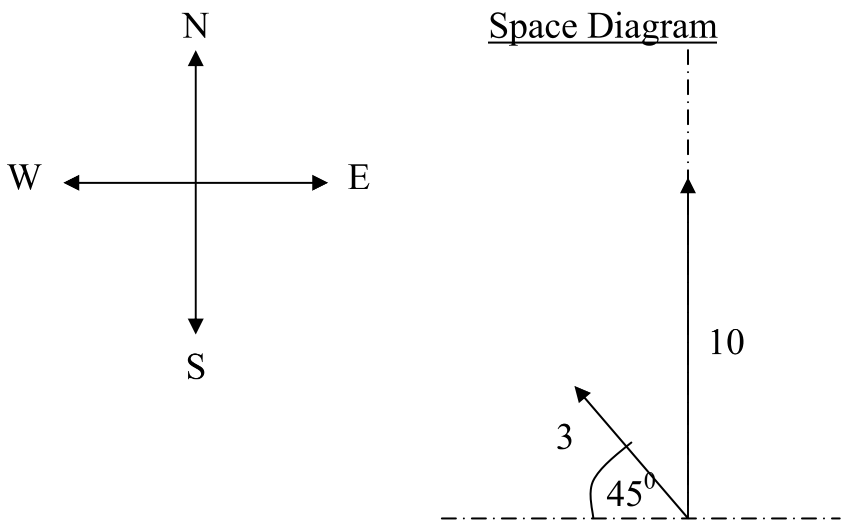
Example 6

A ship moving due north at 10 knots enters a 3 knot current running in a north west direction.

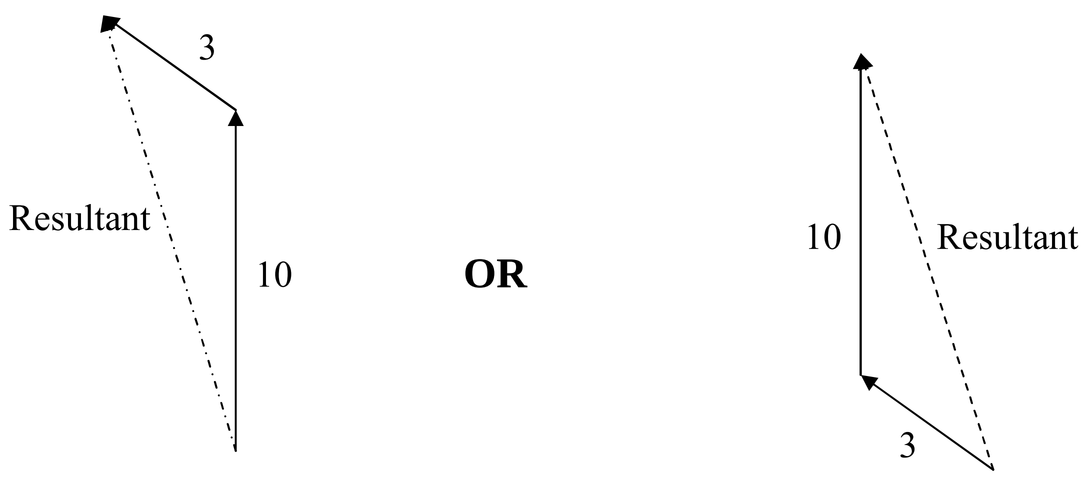
Determine its resultant velocity by calculation and by scale.

Solution:

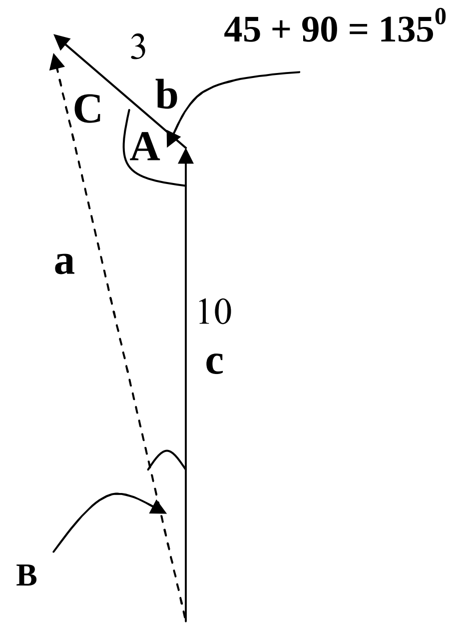
We want to find the resultant velocity, so we should draw the velocity vectors nose-to-tail. But first, we should draw a space diagram, representing the information given in the question



We can now draw the vector diagram, vectors nose-to-tail. Note that it does not matter which vector we choose to draw first, although the diagram will look different, the resultant velocity will be the same.



We can find the resultant velocity simply by measurement of magnitude and direction if we draw the velocity diagram to scale. Try this yourself. Here is how the answer would be calculated:



Using the Cosine Rule,

$a^2 = b^2 + c^2 - 2.b.c.\text{Cos } A$

$a^2 = 10^2 + 3^2 - 2 \times 10 \times 3 \times \text{Cos } 135^0$

a = 12.31 knots (The resultant velocity, in magnitude only)

Sin Rule:

$$\frac{3}{\text{Sin } B} = \frac{12.31}{\text{Sin } 135}$$

Sin B = 0.1723, so B = 9.92^0, which makes it in a direction 9.92^0 West of North.

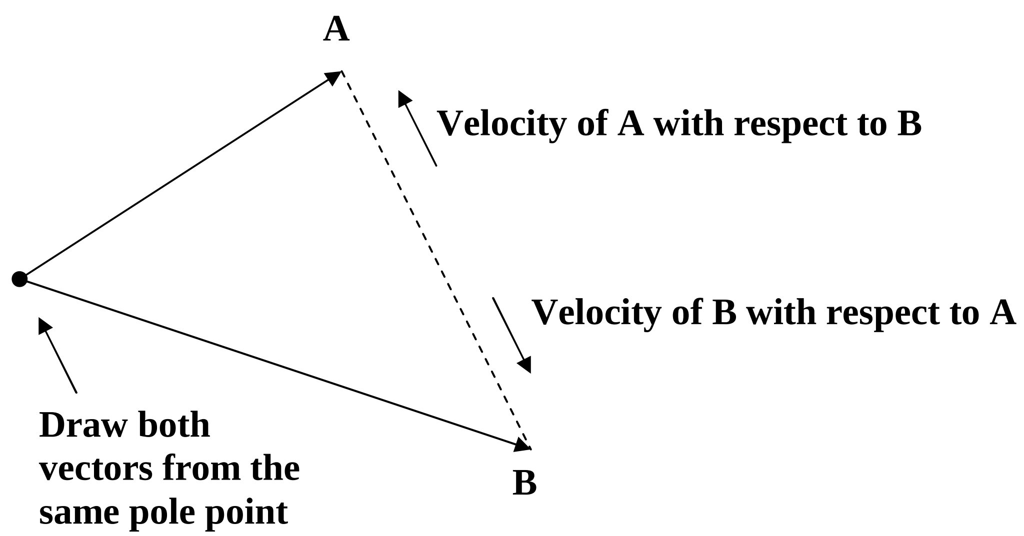
RELATIVE VELOCITY

The term RELATIVE VELOCITY is used for the velocity which a body **appears** to have when viewed from another body.

To find the velocity of A relative to B, that is, the velocity with which A appears to be moving when viewed from B, both absolute velocities are **drawn as vectors from the same "pole" or earth point.**
The relative velocity vector is that which joins the two outer ends of the absolute vectors. Effectively, we are finding the difference between the vectors, or subtracting them.

It may be helpful to consider that when finding the velocity of a body A relative to a body B, the observer is on B and looking towards the end of vector A.

For the velocity of B relative to A the observer is on A and looking towards the end of vector B.

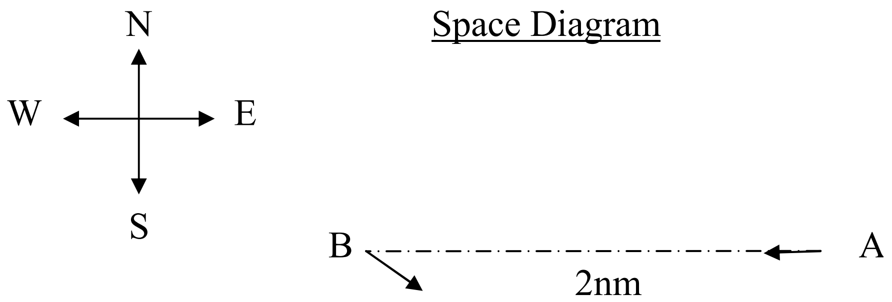


Example 7

A ship "A" is steaming due west at 5 knots. Ship "B" is dead ahead at a distance of 2 nautical miles. Ship "B" is steaming at 30^0 South of East at 10 knots. **Calculate** their nearest distance of approach.

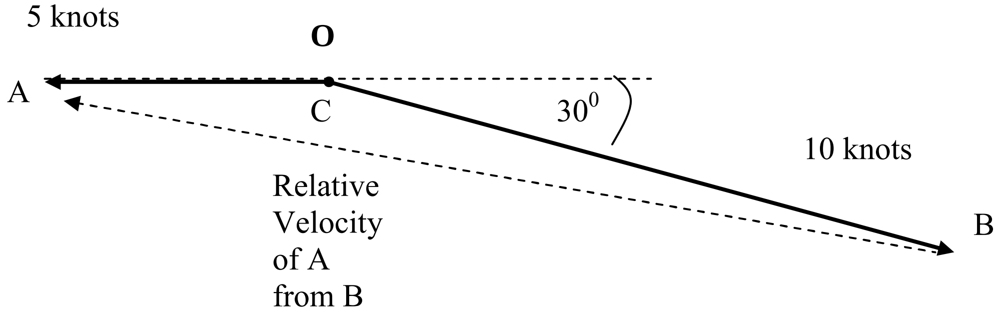
Solution

This type of question is best solved by finding the relative velocity of the two ships. First, we should draw a rough sketch of the space diagram to put down the information in the question. We will draw a more accurate one later.



Note that space diagrams are for "spatial" information and vector diagrams are for vector, in this case velocity, information, and to avoid confusion, the two should not be mixed into the same diagram. On a rough sketch of the space diagram, it is usual to give some indication of the heading of the vessels as shown.

Now we need to find the relative velocity of the two vessels, and this is done by drawing them from the **SAME POLE POINT**.



To calculate the relative velocity requires the use of the cosine rule and the sine rule. From the Cosine Rule,

$$(\text{Relative velocity})^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \text{Cos}150^0$$

$$= 211.6 \quad \text{i.e.} \quad AB = \underline{14.55 \text{ knots}}$$

Knowing this value, we can now use the Sin Rule,

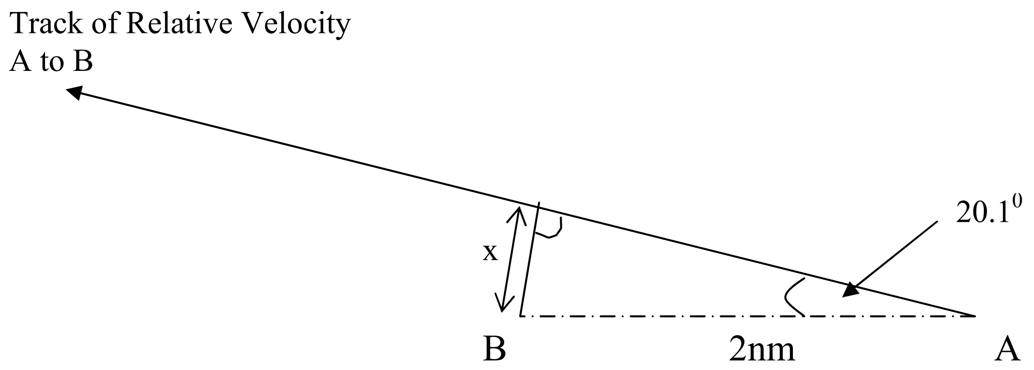
$$\frac{10}{\text{Sin A}} = \frac{14.55}{\text{Sin } 30^0}$$

$$A = 20.1^o$$

We can use this information to find the nearest distance of approach. What we need to do is:

1. Draw the Space Diagram
2. Consider one vessel, (in this case B) stationary
3. Draw a line **of no fixed length** that represents the course or track of A relative to B. Note that the velocity "A relative to B" is the velocity of A considering B as stationary.
4. From the diagram, it should be obvious that the nearest distance of approach occurs when the **track** of A relative to B is perpendicular to the object considered stationary.

Space Diagram



From this diagram, the distance of nearest approach, "x" is $2\text{Sin}20.1^0 = 0.688$ nm

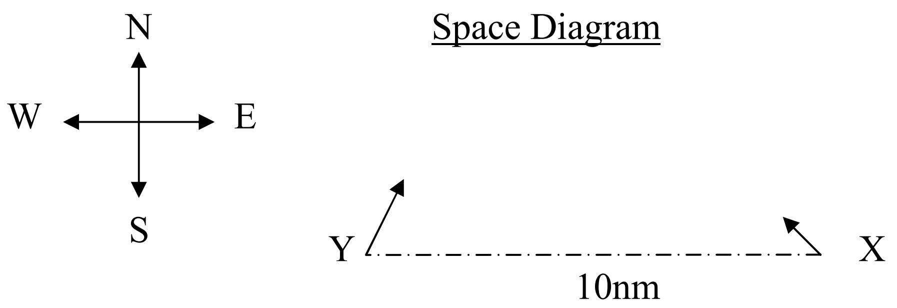
Relative velocity is thus a useful method of being able to predict how close objects will pass, or what change of course might be necessary to meet one another. By using relative velocity, one object is considered stationary whilst the other moves at the relative velocity.

Example 8

Ship, X is 10 nautical miles due east of a ship Y. Ship X makes 12 knots on a bearing of 315^0 and ship Y makes 15 knots on a bearing of 30°.
Find:
(a) the relative velocity of X to Y and its direction from the vertical (i.e, north-south line),
(b) the minimum distance eventually separating the ships.
(c) The time to reach the point of nearest approach
(d) If ship X wanted to contact ship Y at what speed should she proceed on her original course?

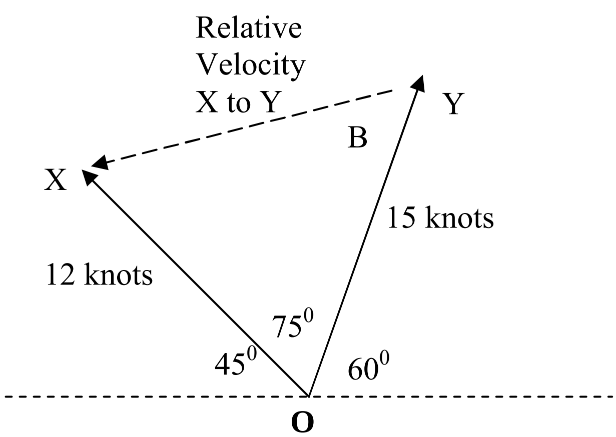
Solution

First, we should draw a rough sketch of the space diagram to put down the information in the question. We will draw a more accurate one later.



Remember, space diagrams are for "spatial" information and vector diagrams are for vector, in this case velocity, information, and the two should not be mixed into the same diagram. We have though given some indication of the heading of the vessels as shown.

Now we need to find the relative velocity of the two vessels, and this is done by drawing them from the **SAME POLE POINT**.



From a "pole point" O the absolute velocities are represented by their respective vectors. o-y and o-x, the relative velocity is shown on the velocity diagram.

Either by drawing to scale, or by calculation, the value of the relative velocity vector is determined as 16.607 knots, at an angle of 74^o to the north south line.
The calculation involves the use of the cosine rule and the sine rule.

$$\text{Relative velocity}^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \times \text{Cos}75$$

$$= 275.8 \quad \text{i.e.} \quad AB = \underline{16.607 \text{ knots}}$$

$$\frac{12}{\text{Sin B}} = \frac{16.607}{\text{Sin } 75}$$

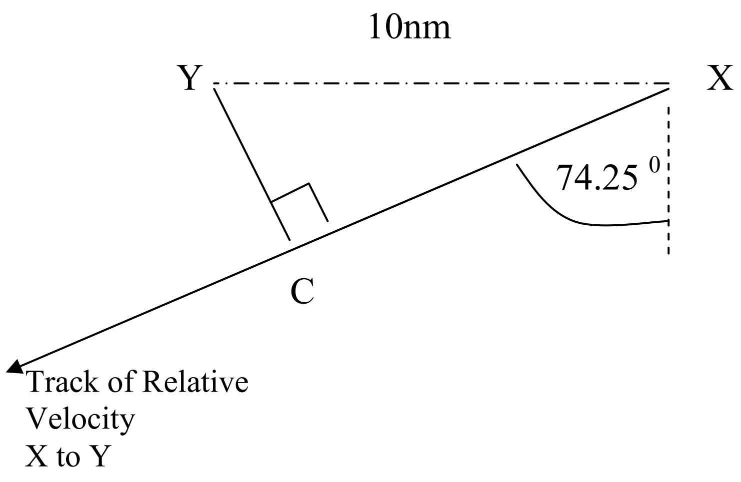
$$B = 44.258^o$$

Angle to North - South Line = $\underline{74.25^0}$ (as drawn)

As the ships proceed, this relative velocity is maintained and X will eventually pass astern of Y achieving its closest position at point C, on the space diagram, where angle YCX is 90°. YC is the minimum distance apart.

YC = 10 sin 15.75° or <u>2.713 nautical miles</u>.

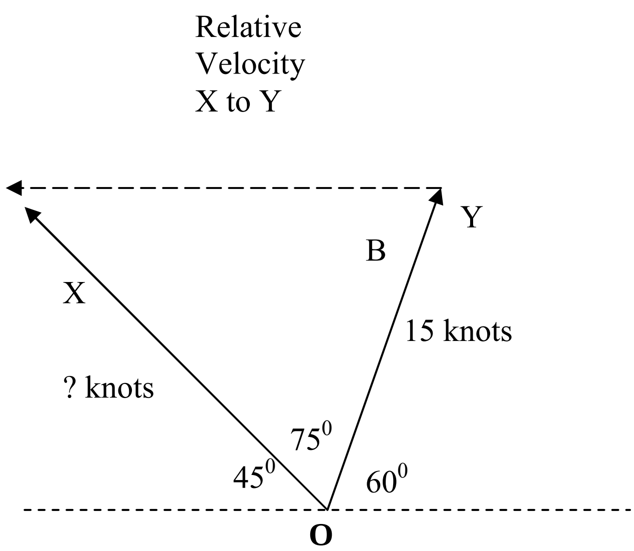
<u>Space Diagram</u>



Distance to run to point of nearest approach, "XC" = $10\text{Cos}15.75^0$ = 9.625nm

Time to nearest approach $= \dfrac{\text{Relative Distance}}{\text{Relative Velocity}} = \dfrac{9.625}{16.61} = 0.58$ hours (35mins) $\longrightarrow$

If ship X is to contact ship Y, the velocity of X relative to Y must be due west. The new velocity diagram is shown.



The direction (course) of ship X and the speed and direction (course) of ship Y are to remain unchanged.
The new direction of the relative velocity 'Vr' must be due west, the new speed of X (on the original course) can be determined by scale or calculation (Cosine rule).

The required speed would be <u>18.37 knots</u>.

Self Assessed Questions for you to try. (Answers given).

1 A body is attached to a string which is wound round a shaft of 80 mm diameter. If the body falls 2.5 m in 3 s, find the angular acceleration of the shaft in rad/s^2 and its speed in rev/min at the end of the fall.

Ans: 13.9 rad/s^2 398 rev/min

2 A vessel steaming due west at 5 knots sights another vessel dead ahead at a distance of two nautical miles. If the second vessel is steaming 30^o south of east at 10 knots find their nearest distance of approach and the time taken to reach this position.

Ans: 0.687 nautical miles 7 min 44 s

3 A ship steaming due west at 16 knots runs into a 3 knot current running south west. Find the resultant velocity of the ship and the distance travelled after 1.5 hours.

Ans: 18.24 knots at 6.7^o south of west
27.36 nautical miles