KINEMATICS SECTION a)

PROJECTILES

An Italian scientist, Galileo (1564-1642), discovered the fact that two bodies of different mass released from the same height at the same time, reach the ground together. He deduced that this meant that the acceleration due to gravity is independent of the mass of the body on which it acts.

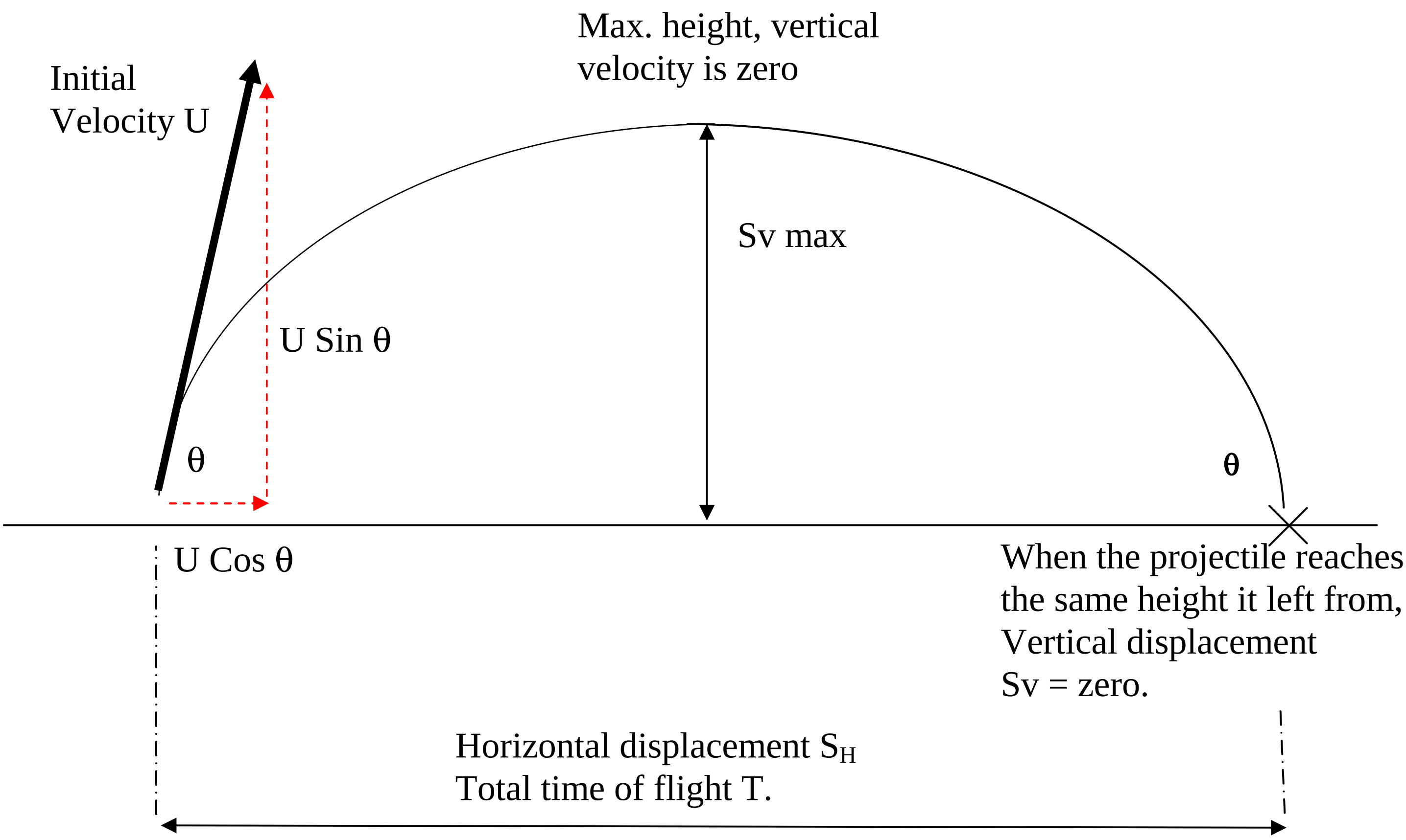
The acceleration due to gravity is a constant at any particular point on the earth's surface. Although it varies slightly from place to place and at different altitudes, it can usually be taken as 9.81 m/s^2 and is given the symbol 'g'. Note that we will never use 'g' for anything else here, since the SI unit of mass is the kilogram (kg). For the exact definition of units, a standard gravitational environment has been formulated for which g = 9.80665 m/s^2. This standard value is usually represented by "g" Newtons.

The acceleration due to gravity acts vertically downwards, and is independent of the mass of the body on which it acts. When dealing with problems concerning the motion of bodies under the effects of gravity, it is important to adopt some convention concerning the sign to be given to distances, velocities and accelerations. Some direction must be assumed and stated to be the positive reference direction.

In our work on projectiles, ***the positive direction will be vertically upwards***. This means that a positive distance is one measured in an upwards direction from the starting point. Similarly, velocities and accelerations will be positive when they are in an upwards direction.
Note that since the ***acceleration due to gravity always acts vertically downwards***, for the sign convention adopted ***"g" will always be taken as a negative number in our work on projectiles***.

We can adopt a standard approach to any question on projectiles. The approach will be **to separate the motion of the projectile into its horizontal and vertical components**. The conditions that apply to each part of the motion are different. For the horizontal component, we can usually assume that the **horizontal velocity is constant**. In other words we neglect wind resistance etc. For the vertical component, we need to take into account the acceleration due to gravity. This, if we use our standard convention, means that the **vertical acceleration will be –9.81m/s^2**.

Since the acceleration is constant in both cases (constantly zero in the case of the horizontal motion), then the standard equations of linear motion can be applied. Remembering that we are dealing with vector quantities, and that the symbol ‘s’ is displacement, rather than distance travelled, we can make the following observations about projectiles.



There are a number of things we can deduce about the motion of the projectile.

Considering the vertical motion:

Initial vertical velocity = U Sin θ

Initial vertical acceleration = $-9.81 m/s^2$

At maximum height, the projectile is no longer moving upwards, but has not started moving downwards. Its’ **vertical velocity is zero**.

When the projectile **returns to the same height it left from**, the vertical displacement **Sv = ZERO**.

Using this information, it can easily be proved that:

The **time to reach maximum height is half the total time of flight ‘T’**.

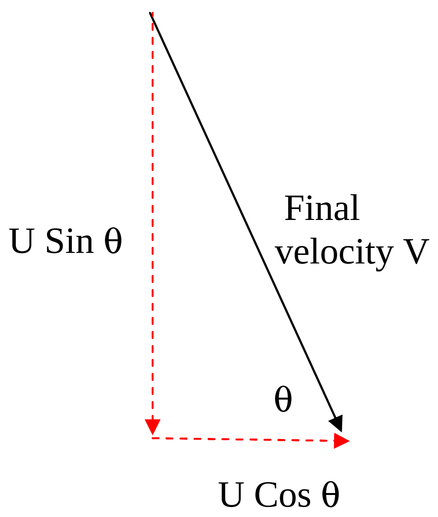
When the projectile **returns to the same height** it left from its **vertical velocity is equal in magnitude but opposite in direction to its' initial vertical velocity**.

Considering the horizontal motion:

Initial horizontal velocity = U Cos θ. This is constant and therefore also the final horizontal velocity.

The horizontal displacement is thus simply calculated by multiplying the constant horizontal velocity by the total time of flight, T.

We can combine the constant horizontal velocity with the final vertical velocity to give the final total velocity. Since their **magnitudes** do not change if the projectile returns to the same height, then under this condition, the magnitude of the final velocity is the same as that of the initial velocity, and the projectile returns to the ground at the same angle that it left from.



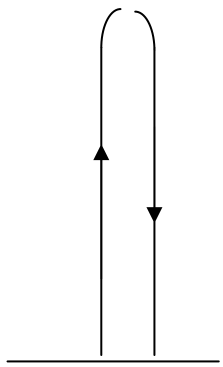
Now lets try a few examples.

Example 1:-

A stone is thrown vertically upwards with a velocity of 12 m/s. Find the greatest height reached, the time required to reach this height, and the total time before the stone reaches the ground again. How fast is the stone travelling when it hits the ground ?

Solution

$u = +12$ m/s, $v = 0$ (at highest point reached), $a = g = -9.81$ m/s^2.



$v^2 = u^2 + 2as$

$0 = 12^2 - 2 \times 9.81 \times s$

$19.62\, s = 144$

$s = \underline{7.33\ \text{m}}$

$v = u + at$

$0 = 12 - 9.81\, t$

$9.81\, t = 12$

$t = \underline{1.22 \text{ seconds}}$

For the total journey up and down:-

$s = 0$, $u = +12$ m/s, $a = g = -9.81$ m/s^2.

$s = ut + \frac{1}{2}at^2$

$0 = 12.t - \frac{1}{2} \times 9.81 \times t^2$

$4.905\, t^2 = 12t$

$t = \underline{2.44 \text{ seconds}}$

$v^2 = u^2 + 2as$

$v^2 = 12^2 - 2 \times 9.81 \times 0$

$v^2 = 144$

$v = -\underline{12\ \text{m/s}}$

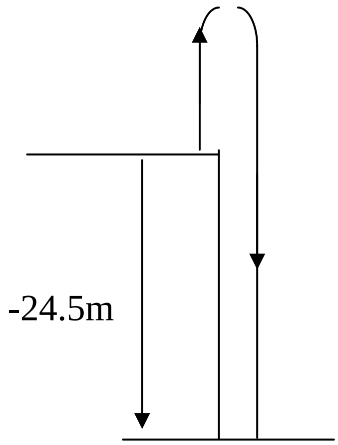
Note that we are used to just saying that the square root of 144 is +12. However, it is equally true that the square root of 144 is –12. Here, we choose the solution that v = -12 m/s because the projectile is travelling……..

That's right, downwards.

Example 2:-

A ball is thrown vertically up in the air from the edge of a cliff 24.5 m high, so that when it falls down again it just misses the edge and falls on to the base of the cliff. If the initial vertical velocity was 19.6 m/s, how long will it take the ball to reach the foot of the cliff, and what is its velocity when it does so ?

Solution



$u = +\ 19.6$ m/s, $a = g = -\ 9\text{-}81$ m/s^2,

$s = -\ 24.5$ m (ball ends up 24.5 m *below* starting point).

$$s = ut + \tfrac{1}{2}at^2$$

$$-24.5 = 19.6\,t - \tfrac{1}{2} \times 9.81 \times t^2$$

$$4.905t^2 - 19.6t - 24.5 = 0$$

$$t^2 - 4t - 5 = 0$$

$$(t - 5)\,(t + 1) = 0$$

$$t = 5 \text{ or } -1 \text{ seconds}$$

The negative answer is not applicable, (the ball cannot reach the ground before it has been thrown) so time, $t = 5$ seconds.

$$v = u + at$$

$$= 19.6 - 9.81 \times 5 \text{ m/s}$$

$$= 19.6 - 49.05 \text{ m/s}$$

$$= \underline{-\ 29.45 \text{ m/s}}$$

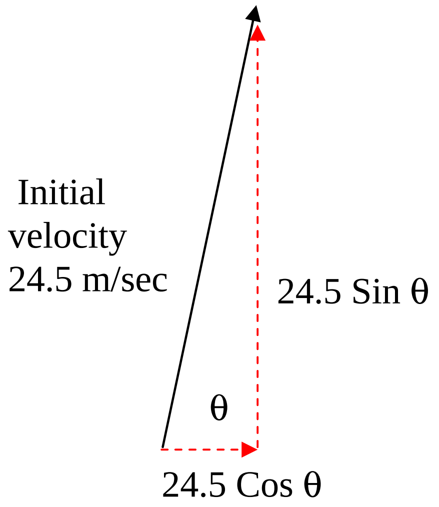
The negative value indicates a *downwards* velocity

Note that many students given this question divide the travel up into three or two parts. There is no need to do this! We can consider the total journey in one movement, remembering that the value of 's' in the formulae is that of **displacement**, which in this case is the height of the cliffs, 24,5m and negative because it is downwards.
Now lets try an example where we have both vertical and horizontal motion.

Example 3:-
A stone is thrown upwards at 24.5 m/s in a direction inclined at 53° to the horizontal. find;

(a) the time the stone is in the air
(b) the greatest height reached
(c) the horizontal distance covered
(d) the angle at which it hits the ground.

Solution.
This is a projectile problem, so the first thing we should do is separate the motion into horizontal and vertical components.



For the initial horizontal velocity,
horizontal component = (24.5 Cos 53° = 24.5 x 0.6)
= 14.7 m/s
Remember of course, the horizontal velocity remains constant, so this will also be the final horizontal velocity.

For the vertical velocity,
Initial vertical component = (24.5 Sin 53° = 24.5 x 0.8)
= 19.6 m/s
For the total time in the air, the vertical acceleration a = g = - 9.81 m/s^2
and the vertical **displacement** s = 0 (total distance above starting point when stone hits the ground again).

$s = ut + ½at^2$ hence:- $0 = 19.6t + ½ \times - 9.81\ t^2$

$4.905\ t^2 = 19.6\ t$

t = <u>4 seconds</u>

The greatest height reached is also determined by the vertical component of velocity and the vertical velocity v = 0 at maximum height.

$$v^2 = u^2 + 2as$$

$$0 = 19.6^2 + 2 \times -9.81 \times s$$

$$19.62\ s = (19.6)^2$$

$$s = \underline{19.6\ \text{m}}$$

The horizontal distance covered, will be given by the distance travelled at the constant value of the horizontal component of velocity (14.7 m/s), for the time the stone is in the air (4 seconds).

$s = ut + ½at^2$ (or we could just use s = v.t. since velocity is constant, and acceleration is zero)

$s = 14.7 \times 4 + 0$ m (horizontal acceleration = zero)

$s = \underline{58.8\ \text{m}}$

The vertical component of velocity on landing is given from

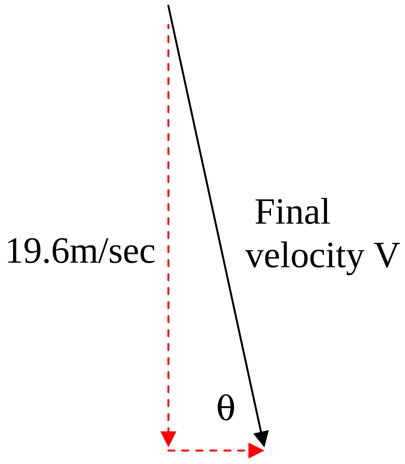
$$v = u + at$$

$$= 19.6 - 9.81 \times 4$$

$$= 19.6 - 39.2$$

$$= -19.6\ \text{m/s}$$

i.e. equal and opposite to the initial vertical velocity, The horizontal component will be constant throughout at 14.7 m/s, so the components of the velocity of the stone when it hits the ground are as shown.

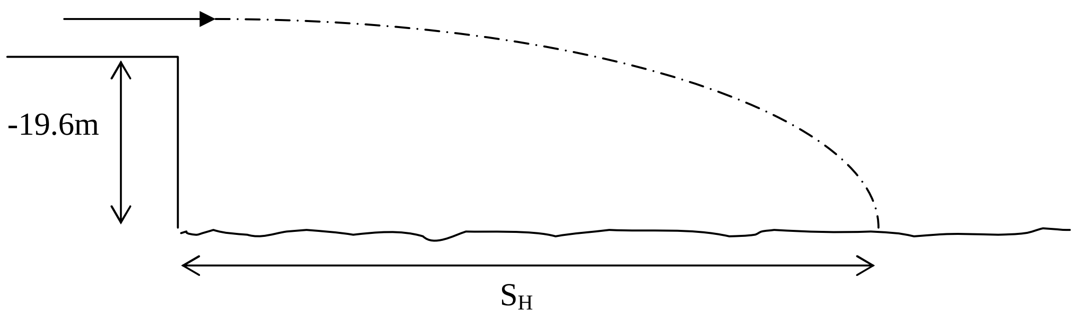


The resultant velocity is seen to be 24.5 m/s at an angle of 53° to the horizontal, directed towards the ground (a mirror image of the initial velocity).

Example 4

A shell is fired horizontally from the top of a cliff towards the sea, the surface is 19.6 m below the cliff top. If the muzzle velocity of the shell is 1200 m/s, how far from the base of the cliff will the shell strike the surface of the sea.

Solution.
First, draw a sketch.



Consider the vertical motion:-

$u = 0, \quad a = g = -9.81\ m/s^2, \quad s = -19.6\ m$

$s = ut + ½at^2$

$- 19.6 = 0 - \frac{1}{2} \times 9.81 \times t^2$

$4.905\, t^2 = 19.6$

$t^2 = 4$ $\quad$ $t = 2$ seconds

Consider the horizontal motion;-

The shell is in the air for 2 seconds:-

$s = ut + \frac{1}{2}at^2$

$s = 1200 \times 2 + 0$ m (horizontal acceleration is zero)

$s = \underline{2400 \text{ m}}$

Self Assessed Questions for you to try. (Answers given).

1 A body is projected vertically upwards at a velocity of 320 m/s. Find the height attained and the total time taken to return to its initial position.

Ans: 5219 m 65.24 s

2 A body is projected vertically upwards at 40 m/s whilst at the same time another body is allowed to fall freely from a height of 70 m. Determine the height above ground at which they meet and the time taken.

Ans: 55 m 1.75 s

3 A shell is fired vertically upwards with an initial velocity of 60 m/s. Later, another shell is fired with an initial velocity of 100 m/s and after 2 s in flight passes the first shell. Calculate the time of flight of the first shell.

Ans: 6.8 s or 5.3 s

4 A projectile is fired with an initial velocity of 100 m/s at an angle of 30^o to the horizontal. Determine its velocity, in magnitude and direction, after 2 s and 4 s respectively.

Ans: 92 m/s at 19.3^o to the horizontal
87.3 m/s at 7.1^o to the horizontal