HYDRODYNAMICS

The study of hydrodynamics is the study of fluids that have motion present. This motion will produce a velocity of fluid flow, which can produce forces and energy changes on the boundaries of the fluid, such as pipelines, and turbine vanes.

Before we can investigate this subject fully, we need to define a few fundamental points.

Volumetric flow rate, is the rate of fluid flow defined in m^3/sec, or other suitable units.

$$\dot{V} = A \times v$$

where V = volumetric flow rate in m^3/sec
A = cross section area of the pipe in m^2
v = fluid velocity in m/s

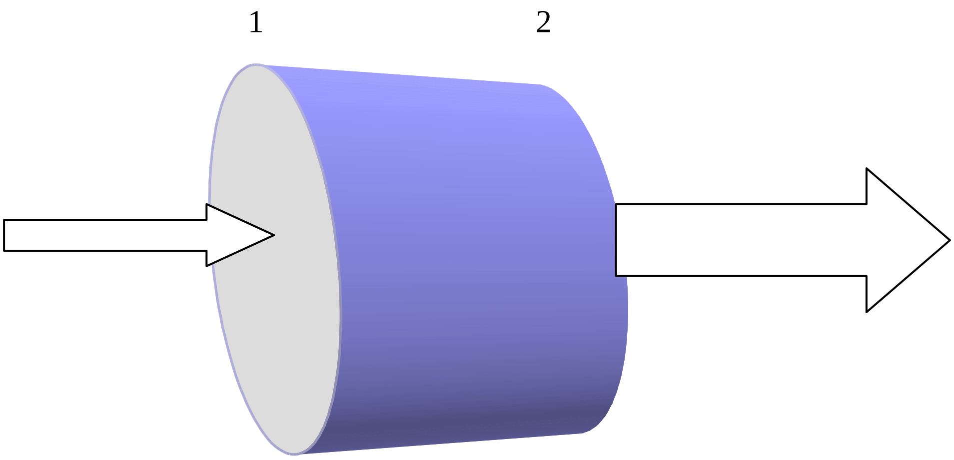
Mass flow rate, is the rate of fluid flow defined in kg/sec, or other suitable units.

$$\overset{o}{m} = \rho \times \dot{V} \text{ or } \rho A v$$

where ρ = fluid density

Continuity Equation

The continuity equation considers the preservation of mass entering and leaving a system. If the fluid is assumed incompressible, which we will do for this entire Class One course, then the mass entering and leaving must be equal. Assuming that the temperature of the fluid and hence the density will not change, then the volumetric flow into and out of the system must be constant.



Velocity	v_1	v_2
Diameter	d_1	d_2
Area	A_1	A_2

Flow rate at 1		=	Flow rate at 2	
	$A_1 v_1$	=	$A_2 v_2$	but $A = \pi d^2/4$,
so	$d_1^2 v_1$	=	$d_2^2 v_2$	

As well as considering the conservation of mass, we shall also consider the conservation of energy. Bernoulli identified three main elements of energy within a fluid flow, namely:

Pressure energy	$\dfrac{mp}{\rho}$
Kinetic energy	$\dfrac{mv^2}{2}$
Potential energy	mgZ

Bernoulli's original equation for steady, frictionless flow of an incompressible flow can be written in terms of the energy under consideration, but the equation is usually written in either head (Z) or pressure (p).

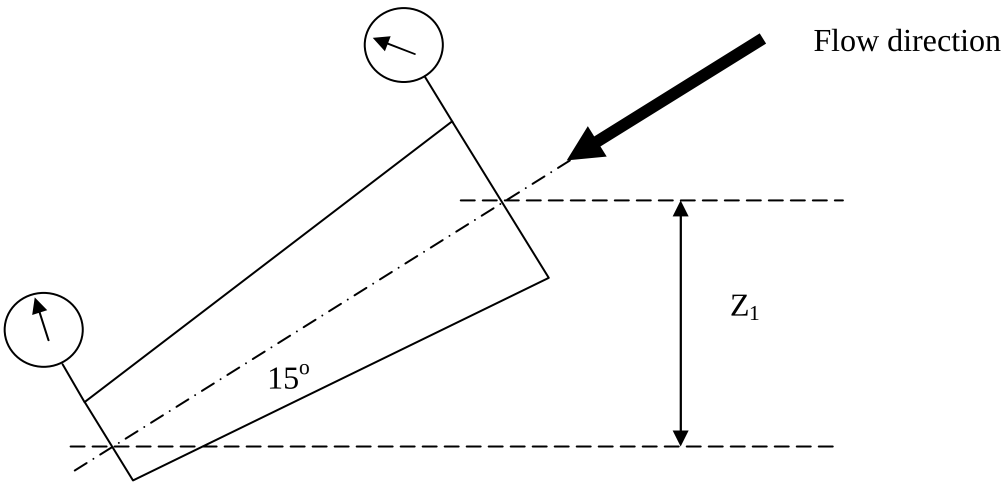
$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + Z_2$$ written in terms of the head (in metres), or

$$p_1 + \frac{\rho v_1^2}{2} + \rho g Z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g Z_2$$ written in terms of pressure (in N/m^2)

Either equation can be used, and their use may depend on the required unknown value.

<u>Example</u>

Consider a pipe of length 10m whose diameter tapers from 100mm to 60mm. The pipeline is fixed at an incline of 15^o as shown. A pressure gauge measures 2.3bar at the larger diameter. Calculate the pressure at the smaller diameter when a fresh water flow of 20 tonne/hour is present.



When using the Bernoulli equation we shall always assume that the pressure readings are from the middle of the pipe, so only the height change between the pipe centres is considered.

Assigning the subscript 1 to the inlet, and subscript 2 to the outlet.
Placing the height datum at the outlet (station 2), so any measurement above this will be positive (as it will have a positive potential energy)

Stating the known data:

p_1 = 2.3 bar or 230kN/m^2
Z_1 = $\sin 15^o$ x 10 = 2.59m
d_1 = 100mm

Z_2 = zero, as it is at the height datum
d_2 = 60mm

Knowing the flow rate allows the speed of fluid flow to be calculated.

From mass flow rate $\overset{o}{m}$ = ρAv,
where $\overset{o}{m}$ = 20 x 1000/3600 = 5.55kg/sec

then v_1 = 5.55/(1000 x $\pi 0.1^2$/4) = 0.71m/s

Also as the mass flow rate through the pipeline must be constant,
then v_2 = 5.55/(1000 x $\pi 0.06^2$/4) = 1.96m/s.

The relationship of the velocities could also be found from the continuity equation that is listed on page 1.

Now that the velocities are known, I shall use Bernoulli equation with the pressure equation to find the unknown pressure, p_2.

$$p_1 + \frac{\rho v_1^2}{2} + \rho g Z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g Z_2$$

$$230 \times 10^3 + \frac{1000 \times 0.71_1^2}{2} + 1000 \times g \times 2.59 = p_2 + \frac{1000 \times 1.96^2}{2} + 0$$

$230 \times 10^3 + 252.1 + 25.4 \times 10^3 \quad = \quad p_2 + 1.92 \times 10^3$

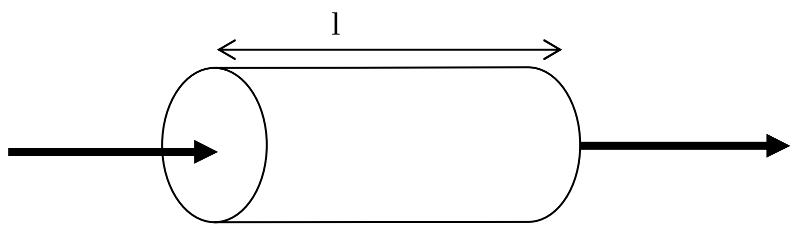
So p_2 = 253.73×10^3 N/m^2 or 2.54 bar.

SAQ #1

Consider a pipe of length 6m whose diameter tapers increases from 80mm to 120mm. The pipeline is fixed at an incline of 15^o with the inlet above the outlet. A pressure gauge measures 1.2bar at the inlet. Calculate the pressure at the outlet when a fresh water flow of 34 tonne/hour is present.
ANS on page 42

Pipeline friction

The use of Bernoulli's energy conservation equation is correct whenever the system conditions are constant and not changing. It can be applied over larger pipeline systems, although the various energy losses in a system should now be considered. The main cause of energy loss in fluid flow is due to friction between the fluid and the surrounding pipe wall. This energy loss can be seen as a pressure loss along the pipeline, and will reduce the ability of the fluid to transfer energy.



Consider a pipeline of uniform diameter of length "l" where a pressure drop occurs due to the pipeline friction.

Equating the forces at the inlet and outlet of the pipeline.

Force at inlet = Force at outlet + Force loss due to friction

Pressure_{IN} x area_{IN} = Pressure_{OUT} x area_{OUT} + Force loss due to friction

From experiments carried out by Froude the frictional resistance force can be shown to equal the friction coefficient x wetted area x velocity^2

Hence pipelines can be compared once their physical dimensions, fluid flow rate, and friction coefficient (or roughness) are known.

Hence force loss due to friction = $F\,\pi dl\,v^2$

So equating the forces gives

$$p_1\,\pi d^2/4 = p_2\,\pi d^2/4 + F\,\pi dl\,v^2$$

or $$p_1 - p_2 = 4\,F\,l\,v^2/d$$

Usually the figures for the pipeline friction (F) are not given in terms of pressure loss, but head loss. So replacing the term F with $\rho f/2$, and converting the pressure loss term with a head loss term z_f, where $p_1 - p_2 = \rho g z_f$

Then $$\rho g z_f = 4\,(\rho f/2)\,l\,v^2/d$$

or $$\mathbf{z_f} = \frac{4flv^2}{2gd}$$

This equation is known as Darcy's formula for head loss within pipelines.

The actual value of f could be given in the various questions (such as f = 0.006), or a relationship for the student to calculate f should be given such as
$f = 0.01\left(1 + \frac{1}{12d}\right)$.

<u>Class example</u>

Calculate the head lost due to friction within an 80mm bore pipeline of length 120m when the mass flow rate of the sea water is 24 tonne/hour. Assume the coefficient of friction for Darcy's equation to be 0.007 for this application.

From mass flow rate = ρAv

Then $24 \times 10^3 / 3600 = 1025 \times \pi 0.08^2/4 \times v$

Thus v = 1.3 m/s

From Darcy's equation $z_f = \frac{4flv^2}{2gd}$

$z_f = 4 \times 0.007 \times 120 \times 1.3^2 / 2 \times g \times 0.08 = 3.62\text{m}$

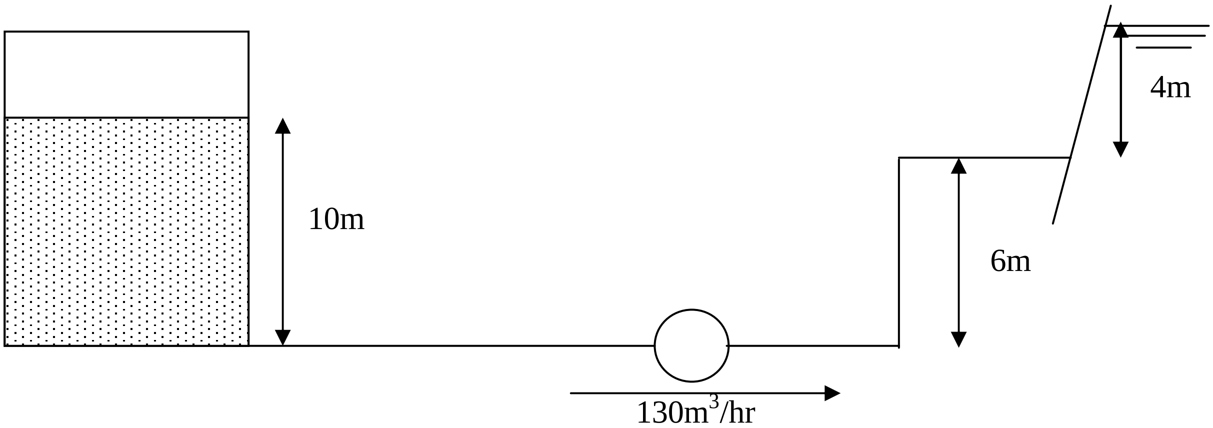
We can also calculate the pressure drop caused by this head loss from
$p_1 - p_2 = \rho g z_f$

So $p_1 - p_2 = 1025 \times g \times 3.62 = 36.4\ \text{kN/m}^2$ or 0.36 bar

SAQ #2
A pipe of 200mm diameter and 300m long is discharging fresh water with a velocity of 1.2 m/s. If f = 0.01, find the head loss, and hence pressure loss due to friction.
ANS on page 43

Class example

A ballast pipeline system is shown below.



The total length of the suction pipeline is 45m, and the total length of the discharge pipeline is 12m. Assume the fluid is sea water with a density of 1025 kg/m^3, and that the friction coefficient is 0.06 for both the suction and discharge lines.

The pipeline size is 100mm for the suction, and 80mm for the diameter.

Calculate the pressure at the suction and discharge gauges of the pump when a flow rate of 130m^3/hr is present.

For this example I will use Bernoulli equation to calculate the energy on the suction side, and then the discharge side. Note that the equation CAN NOT be used to compare the suction and discharge sides as the energy of the pipeline will change due to the energy input of the pump.

I will use a modification of the Bernoulli's equation written on page 2 to include the loss of head due to friction, so

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + Z_f$$

Note that the energy or head loss will appear on the outlet, as this frictional loss will *reduce* some or all of the parameters at the outlet.

I will apply this equation to the suction pipeline first.

- The pressure at the tank outlet will be dependent upon the static head of the tank contents, so $p_1 = \rho gh = 1025g10 = 100.55\text{kN/m}^2$
- The pipeline has the same diameter along its length so $v_1 = v_2$
- The pipeline is horizontal so $Z_1 = Z_2$
- $Z_f = \dfrac{4flv^2}{2gd} = \dfrac{4 \times 0.006 \times 45 \times v^2}{2g \times 0.10} = 0.55v^2$

- The velocity of flow = Volumetric flow rate / Area = 130/3600 x $\pi 0.1^2$ x 0.25 = 4.6m/s

So removing those factors that equate to zero gives

$$\frac{p_1}{\rho g_1} = \frac{p_2}{\rho g} + Z_f$$

100.55 x 10^3/1025 x g	=	p_2/1025 x g	+	0.55 $(4.6)^2$
10	=	p_2/1025 x g	+	11.64
p_2	=	(10 – 11.64) 1025g		
	=	-16.47 kN/m^2 or –0.165bar		

What does the negative sign mean? This will indicate that the pressure is below atmospheric at the pump suction, so there is a slight suction or vacuum pressure present.

Applying the Bernoulli's equation to the discharge side

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + Z_f$$

where subscript 1 indicate the pump discharge, and the subscript 2 indicates the overboard end of the pipeline.

Points to note

- The discharge pipeline has the same diameter so $v_1 = v_2$
- The pressure p_2 will be the static head pressure due to the discharge line being 4m below the waterline. So $p_2 = \rho gh$ = 1025g4 = 40.22kN/m^2
- The datum will be placed at the pump discharge, so $Z_1 = 0$, and $Z_2 = 6$m
- $Z_f = \dfrac{4flv^2}{2gd} = \dfrac{4 \times 0.006 \times 12 \times v^2}{2g \times 0.08} = 0.183v^2$
- The velocity of flow = Volumetric flow rate / Area = 130/3600 x $\pi 0.0.08^2$ x 0.25 = 7.18m/s

Rewriting the Bernoulli's equation, omitting all zero values gives

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + Z_2 + Z_f$$

$p_1/\rho g$	=	40.22 x 10^3/1025 g	+	6	+	0.183 $(7.18)^2$
p_1	=	1025g(4 + 6 + 9.47)	=	195.8kN/m^2 or 1.96bar		

You may have noticed that rather that find the static pressure resulting from the static head, that I could have used the static head direct into the Bernoulli's equation as this has units of head. Always ensure you use the same units for all equations.

Student self test

SAQ

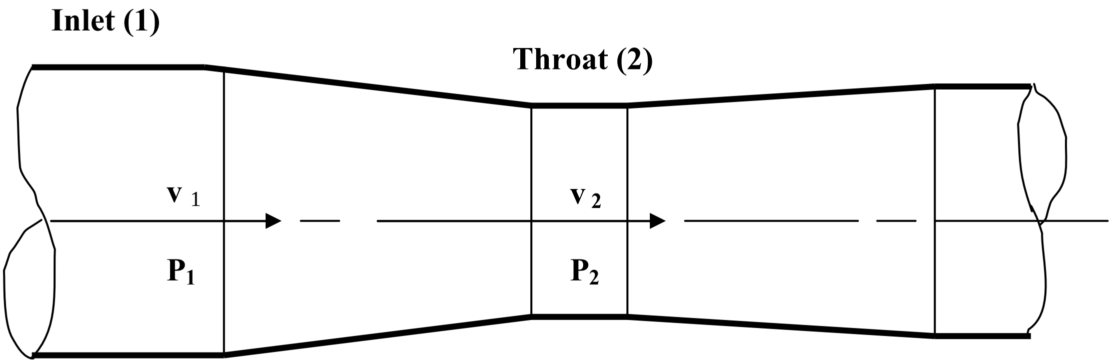
The diameter of a pipe changes gradually from 152 mm at a point A, 6.1 m above datum to 76 mm at B, 3.05 m above datum. The pressure at A is 103.5 kN/m^2 and velocity at A is 3.66 m/s. Neglecting losses, determine the pressure at B.
Ans: 32.6 kN/m^2

SAQ

Liquid of density 900 kg/m^2 is pumped upwards through a vertical tapered pipe of 100 mm diameter at the bottom and 175 mm diameter at the top and 0.75 m long. If the pressure difference across the two sections is 27.6 kN/m^2 determine the mass rate of flow in kg/s. Neglect losses.
Ans: 65.2 kg/s

Venturi meters

The Venturi Meter is a device used for the measurement of the flow rate of a fluid flowing through a pipeline. It comprises a pipe which is smoothly tapered to a reduced diameter known as the “Throat” then gradually increased to the original diameter, as illustrated below.



The Venturi Meter

From the continuity equation it follows that as the fluid flows from the inlet to the throat the velocity must increase; and from Bernoulli’s equation the pressure must therefore **reduce** to maintain the total energy of the flow constant – assuming no energy losses.

From the continuity equation:

$$v_2 = \left(\frac{D_1}{D_2}\right)^2 v_1 \quad \text{therefore} \quad v_2^2 = \left(\frac{D^1}{D_2}\right)^4 v_1^2$$

and from Bernoulli’s equation:

$$\frac{p_1}{pg} + \frac{v_1^2}{2g} + z_2 \text{ (neglecting losses)}$$

and if the device is horizontal, then $Z_1 = Z_2$, so

$$\frac{p_1}{pg} - \frac{p_2}{pg} = \frac{v_2^2 - v_1^2}{2g}$$

or since $p = \rho gh$

$$h_1 - h_2 = \frac{v_2^2 - v_1^2}{2g} = v_2^2\left(\frac{D_1}{D_2}\right)^4 - v_1^2$$

Meter Coefficient (C_d)

This is a factor which is introduced to account for the losses due to friction in the meter, which always results in the actual volume flow rate being less than the theoretical value: i.e.

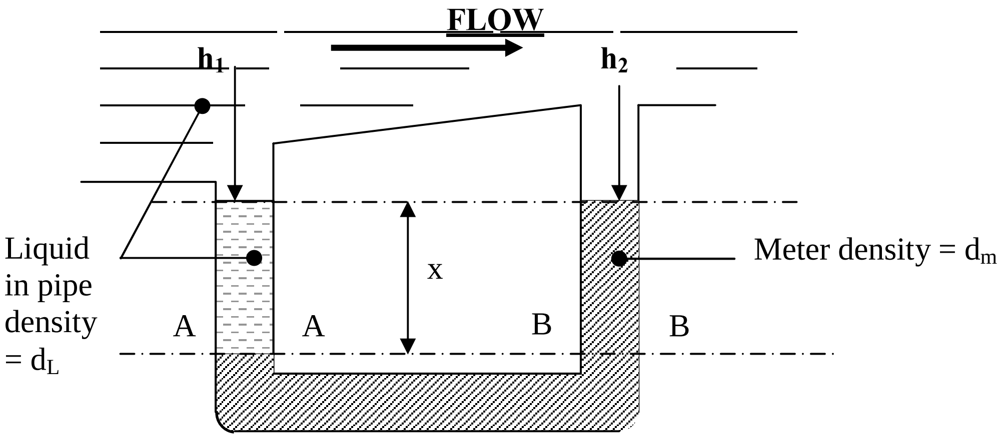
$\overset{o}{V}$ = cross section area (A) x theoretical velocity x C_d

Where the meter coefficient C_d is typically in the range of 0.85 to 0.95.

Differential Manometer

Differential manometers are commonly used on board as a device for measuring a relatively small pressure difference between two points in a pipeline, as for example between the inlet and throat of a venturi meter.

The principle of the device is illustrated below:



Differential Manometer

The pressure is the same at any uniform depth below the free surface in a continuous fluid,.

Hence; Pressure at AA = Pressure at BB

From pressure = ρgh, then $h_1 .\rho_L .g + x\rho_L .g = h_2.\rho_L .g + x.\rho_m .g$

Dividing by; $\rho_L.g$ $\boldsymbol{h_1 + x = h_2} + \left[x.\left(\frac{\rho_m}{p_L}\right)\right]$

Or $h_1 - h_2 = x\left[\left(\frac{\rho_m}{\rho_L}\right) - 1\right]$

Note! That in the case of where ρ_m is mercury of relative density 13.6, and ρ_L is fresh water of relative density 1.0, then the equation can be simplified to

$$h_1 - h_2 = x.\left[\frac{13.6}{1} - 1\right] = 12.6.x$$

Class example

In a test using a horizontal venturi meter, the measured flow of water over a period of 5 minutes was 7.29m^3. The inlet and throat areas were 0.0812m^2 and 0.0116m^2 respectively and the difference in pressure from a U-Tube manometer was 244mm of fresh water. Determine the meter coefficient, C_d.

The venturi is stated as being horizontal so that $Z_1 = Z_2$.

The pressure head difference $\frac{p_1}{\rho g} - \frac{p_2}{\rho g}$ is stated as 0.244m. p_2 must always be at a lower pressure than p_1 as the effect of the faster flow through the venturi meter will lower the pressure in the throat of the venturi.

Hence rewriting the Bernoulli equation in terms of head gives

$$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Remember that Bernoulli deals in theoretical velocities, the actual v_2 and v_1 can be found from the data given as these are actual velocities.
The meter coefficient will always cause the actual velocity ***to be lower than*** the theoretical velocity. The actual velocity must equate to the actual flow rate, whereas the theoretical flow rate can be found from the theory of energy conservation (i.e. Bernoulli's equation).

v_2 = Volumetric flow rate / area

$$= \frac{\frac{7.29}{5 \times 60}}{0.0116} = 2.1 \text{ m/s}$$

v_1 = Volumetric flow rate / area

$$= \frac{\frac{7.29}{5 \times 60}}{0.0812} = 0.75 \text{ m/s}$$

Hence rewriting the energy head equation as

$$0.244 = \frac{\left(v_2/C_d\right)^2}{2g} - \frac{\left(v_1/C_d\right)^2}{2g}$$

$0.244 \times 2g \quad = \quad \left(\dfrac{v_2^2 - v_1^2}{{C_d}^2} \right)$

$4.79 \quad = \quad \left(\dfrac{2.1^2 - 0.75^2}{{C_d}^2} \right)$

So $C_d \quad = \quad \sqrt{\left(\dfrac{2.1^2 - 0.75^2}{4.79} \right)}$

$= \quad 0.896$

<u>Class example</u>
A venturi meter measures the flow of water in a 75mm diameter pipe. The difference in head between the entrance and the throat of the meter is measured by a U-Tube manometer containing mercury, the space above the mercury on each side being filled with water. What should be the diameter of the throat in order that the difference of levels of the mercury shall be 250mm when the quantity of water flowing in the pipe is $0.64m^3$/min. Assume a discharge coefficient of 0.97.

The information contained in this question relies on our understanding the construction of the venturi, and the relationships of energy and flow conservation.

From the previous relationship of venturi's, then the differential pressure at the inlet and throat can be found to be
$p_1 - p_2 \quad = \quad 0.25\,(13.6/1 - 1) \quad = \quad 3.15$m of water head

Also from $\quad h_1 - h_2 = \dfrac{v_1^2 \left(\dfrac{d_1}{d_2} \right)^4 - v_1^2}{2g}$

$v_1 \quad = \quad \dfrac{0.64/60}{\pi 0.075^2/4} \quad = \quad 2.41$m/s

So theoretical velocity = actual / C_d $= \dfrac{2.41}{0.97} = 2.48$ m/s

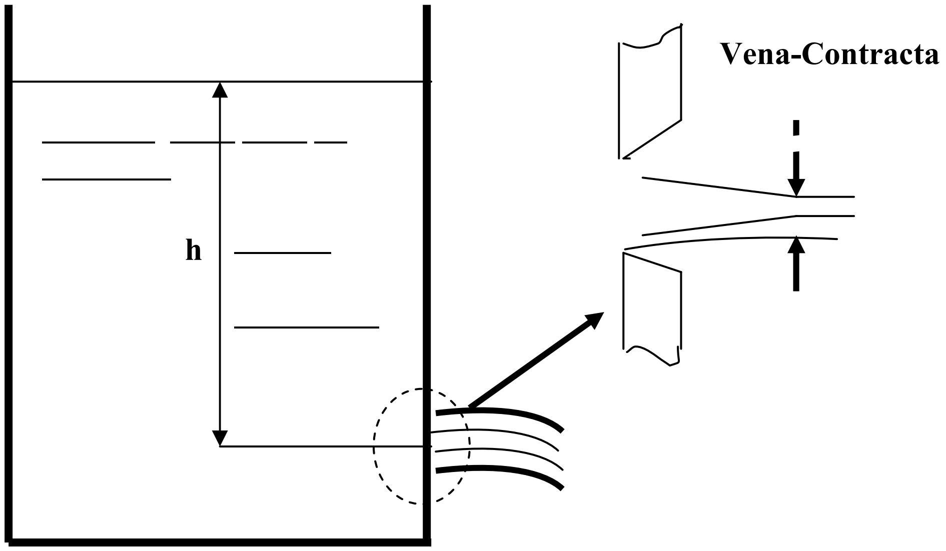
$3.15 \quad = \quad \dfrac{2.48^2 \left(\dfrac{0.075}{d_2} \right)^4 - 2.48^2}{2g}$

$61.8 + 6.15 \quad = \quad \dfrac{6.15\,(0.00003164)}{d_2^4}$

So $d_2 \quad = \quad 41.1$ mm

Flow through an Orifice

Consider a fluid flowing through an orifice due to potential head of '*h*' as shown below.



Flow through an Orifice

At the free surface the velocity is negligibly small and the pressure (gauge) is zero. Hence fluid energy is in the form of potential energy only.

In the fluid flow immediately outside the orifice the pressure energy and potential energy are zero (when the level of the orifice is taken as datum). Hence the fluid only has kinetic energy.

Assuming no energy loss:

KE (gained) = PE (lost)

i.e. $\frac{1}{2}mv^2 = mgh$

$v = \sqrt{2.g.h}$

Where v = theoretical velocity of the jet (m/s)
h = the static head of the liquid (m)

Coefficient of Velocity

Due to the friction losses at the orifice, the actual velocity of the jet is less than the theoretical velocity. The Coefficient of Velocity is the ration of the two; i.e.

$$\text{Coefficient of Velocity } (C_v) = \frac{Actual\ \ Velocity\ \ of\ \ Jet}{Theoretical\ \ Velocity\ \ of\ \ Jet}$$

Coefficient of Contraction (C_c)

Close examination of the jet shows that the fluid continues to converge after passing through the orifice; due to its inertia.
Thus the smallest cross section of the jet occurs outside the orifice and is called the ***'Vena Contracta'.*** The extent of the contraction is described by the;

$$\textbf{Coefficient of Contraction } (C_c) = \frac{Minimum\ \ cross\,\mathrm{sec}tion\ \ area\ \ of\ \ Jet}{cross\ \ \mathrm{sec}tion\ \ of\ \ Orifice}$$

Coefficient of Discharge (C_d)

This is the ration of the ***Actual Discharge Rate*** to the theoretical discharge rate; i.e

$$\textbf{Coefficient of Discharge } (C_d) = \frac{Actual\ \ Disch\,\mathrm{arg}\,e\ \ Rate}{Theoretical\ \ Disch\,\mathrm{arg}\,e\ \ Rate}$$

But the Actual Discharge Rate = Actual Velocity x Jet c.s.a.
and Theoretical Discharge Rate = Theoretical Velocity x Orifice c.s.a.

Therefore:

$$\textbf{Coefficient of Discharge } (C_d) = \frac{Actual\ \ Velocity \times c.s.a.Jet}{Theoretical\ \ Velocity \times c.s.a.Orifice}$$

So $$C_d = C_v \times C_c$$

Class example
An orifice 19 mm diameter discharges 0.0635 m^3 per minute. If the head of the water above the orifice is 1.83 m and the diameter of the vena-contracta is 15.2 mm determine:
a) C_c b) C_v c) C_d

from $v = \sqrt{2.g.h} = \sqrt{2x9.81x1.83}$, thus Theoretical Velocity, v = 6 m/s

a) Coefficient of Discharge = $\dfrac{Actual\ \ Discharge\ \ Rate}{Theoretical\ \ Discharge\ \ Rate}$

and Theoretical Discharge Rate = $\overset{o}{V}$ = $\pi/4 \text{ x } 0.019^2 \times 6 = 0.0017012\ m^3/s$

$C_d = \dfrac{Actual}{Theoretical} = \dfrac{0.0635}{60x0.0017012} = 0.62212$

b) Coefficient of Contraction = $\dfrac{c.s.a.ofJet}{c.s.a.ofOrifice}$

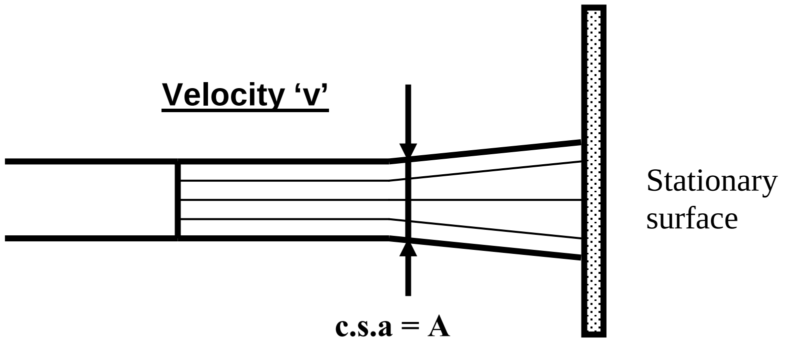
$C_c = \dfrac{\pi/4 \text{ x } 0.0152^2}{\pi/4 \text{ x } 0.019^2} = 0.64$

c) $C_d = C_v \text{ x } C_c$ therefore $C_v = \frac{C_d}{C_c} = \dfrac{0.662}{0.64} = 0.97$

Hydro-dynamically induced forces

Jet Impact on Stationary Surface

Consider a jet of liquid which strikes a stationary surface (i.e its velocity in the direction of the jet is zero) as shown, causing all the momentum in the jet to be destroyed.



Now; Force = Rate of Change of Momentum

Rate of change of momentum = mass flow rate $\times$ change in velocity

In the case of a stationary surface, the change in velocity of the jet is

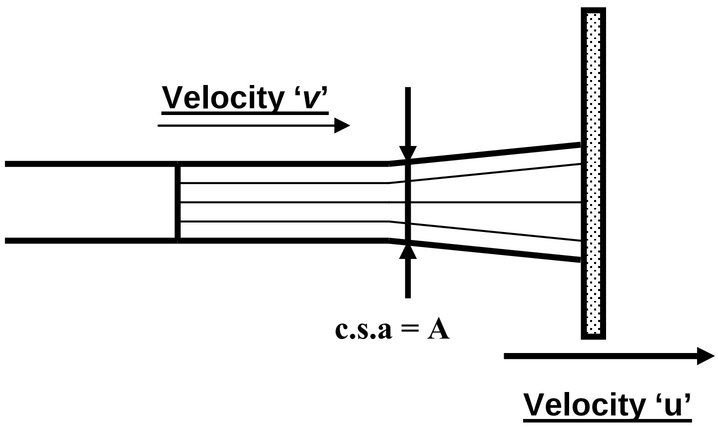
initial velocity – final velocity or (v – 0), since the final velocity of the jet is zero.

Thus Force $= \overset{o}{m} \times v$ and $\overset{o}{m} = \rho \times \overset{o}{V} = \rho \times c.s.a \times v$

Hence Force = $\rho A v^2$

Impact on a Moving Surface

If the surface is moving in the same direction of the jet with a velocity 'u' then the velocity of the jet relative to the surface is ($v - u$), i.e. the jet will overtake the surface at a rate of the velocity difference ($v - u$)



Thus the volume of liquid striking the surface in unit time is the area of the jet x difference in velocity $= A\,(v - u)$

Mass of liquid striking the surface in unit time is volume striking the surface x density , i.e. $= \rho \times A\,(v - u)$

As the change in velocity is ($v - u$), then as the force = mass flow rate x velocity change hence

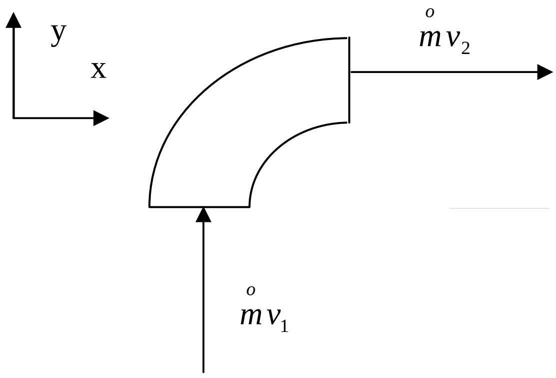
Force = $\rho\ A\,(v - u)^2$

Forces on pipe-bends

When the direction of a fluid flow is changed, as will occur within a pipe bend, then the vector direction of the fluid velocity will also change. As this will constitute a change in momentum, then a force must be present when this fluid flow change occurs.

In addition if the rate of water flow also changes, then there will be another force change present due to the change in pressure force on the bend.

<u>Momentum change</u>



Consider a 90° bend in a pipe line. We will consider the pipe section shown as the control volume. This control volume will allow us to simplify the forces present, and exists as the open ends and the surrounding metal surface.
If we assume that mass conservation is present, i.e. the mass flowing into the bend is the same as the mass flowing out of it, then the mass flow rate $\overset{o}{m}$ will be constant.

The momentum flowing *into* the control volume is $\overset{o}{m} v_1$

The momentum flowing *out* of the control volume is $\overset{o}{m} v_2$

Hence the summation of the these forces is $\sum F = \overset{o}{m} v_1 - \overset{o}{m} v_2$

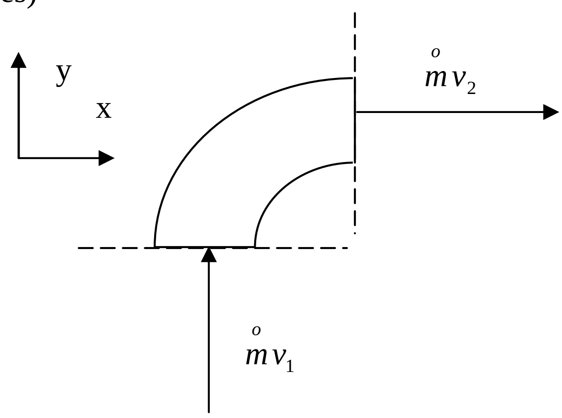
Note that the flow in is +ve and the flow out is –ve.
This total force will be the force of the water <u>*on*</u> the pipe.

If we wish to find the force required to hold the pipe in place, then this will be the equal and opposite of this force.

Class example

A pipeline with 2kg/sec of fresh water flowing within a 50mm bore line has a 90^0 bend. Find the force of the water on the bend.

Draw the control volume with the inlet and outlet clearly shown (I have used dotted lines)



Mass flow rate is given, so $\overset{o}{m}$ = 2kg/sec

$v_1 = v_2$ as the pipe line has a uniform bore or internal diameter

$$v_1 = \overset{o}{m} / \rho A \quad = \quad \frac{2}{\frac{1000 \times \pi 0.05}{4}} = \quad 1.02\text{m/s}$$

Consider the momentum change in the x direction

$$F_x \quad = \quad -\overset{o}{m} v_2 = \quad -2 \times 1.02 \quad = -2.04 \text{ N}$$

(Note the negative sign as it is flowing OUT of the control volume)

Consider the momentum change in the y direction

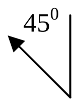
$$F_y \quad = \quad \overset{o}{m} v_1 = \quad 2 \times 1.02 \quad = 2.04 \text{ N}$$

(Note the positive sign as it is flowing INTO the control volume)

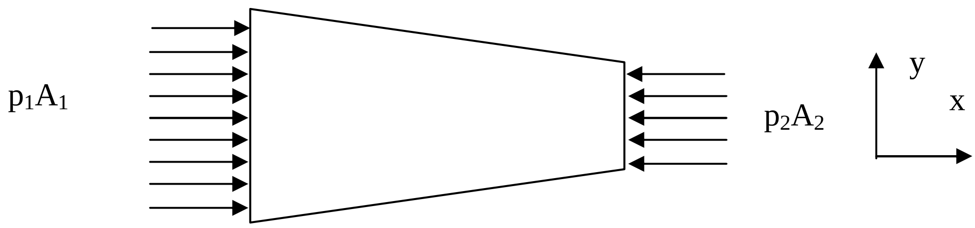
Hence force of water on pipe = $\sqrt{F_x^2 + F_y^2} = \sqrt{-2.04^2 + 2.04^2}$

$= 2.88\text{N}$

Direction of force = $\tan^{-1} F_y / F_x = 2.04/-2.04 = -45^0$ or



Pressure change



Consider a pipe line reducer as shown. We will now examine the force present due to a pressure change at the open ends of the control volume only, as these are the only forces entering (or exiting) the control volume.

The pressure force acting *on* the control volume is p_1A_1 and p_2A_2

Hence the summation of the these forces is $\sum F_x = p_1A_1 - p_2A_2$

Note that in this case the orientation of the datum axis means the flow in is +ve and the flow out is –ve.

Again this total force will be the force of the water *on* the pipe.

Class example
A pipeline tapers from 75mm to 25mm in the horizontal plane. Calculate the forces that are present when a flow of 2kg/sec of fresh water is present. Assume the exit of the taper exhausts to atmosphere.

We have been told that the pressure p_2 exhausts to atmosphere so $p_2 = 0$, as we will work in gauge pressure for all these problems.

Using Bernoulli's equation to find the upstream pressure p_1, knowing that $Z_1 = Z_2$,

then $p_1 = \dfrac{\rho v_2^2}{2} - \dfrac{\rho v_1^2}{2}$

$v_1 = \overset{o}{m} / \rho A \quad = \quad 2 / 1000 \times \dfrac{\pi 0.075^2}{4} \quad = \quad 0.45\text{m/s}$

$v_2 = \overset{o}{m} / \rho A \quad = \quad 2 / 1000 \times \dfrac{\pi 0.025^2}{4} \quad = \quad 4.07\text{m/s}$

So $p_1 \quad = \quad 1000/2\,(4.07^2 – 0.45^2)$

$\quad = \quad 8.2\ \text{kN/m}^2$

Hence pressure force in the x direction $= p_1A_1 - p_2A_2$
$= 8200 \times \pi \times 0.075^2/4$
$= 36.2\text{N}$

This pressure force is the water force acting on the pipe, and is tending the pipe to move to the right. Note p_2 is atmospheric so we can ignore it on this occasion.

If you have ever held a pipeline with water ejecting out of a taper or nozzle, then you will realise that the tendency is for the nozzle to move away from the direction of fluid flow, not towards it as we have calculated. The reason for this discrepancy is that we must include the momentum forces as well, as the velocity of the water is changing as it leaves the nozzle.

Consider the momentum change in the x direction

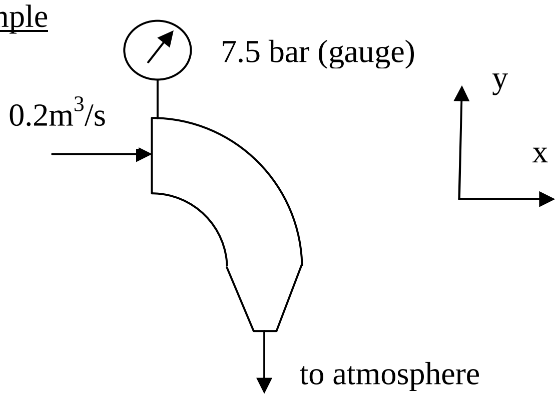
$$F_x \quad = \quad \overset{o}{m} v_1 - \overset{o}{m} v_2 = 2(0.45 - 4.07) \quad = \quad -7.24 \text{ N}$$

(Note the negative sign as it is flowing OUT of the control volume)

Hence the total force acting on the pipe taper is 36.2 – 7.24 = 28.96N.

So on this occasion the water is tending to push the pipe taper in the direction of fluid motion.

<u>Class example</u>



Oil of density 860 kg/m^3 flow through a nozzle discharging into an empty tank. The pipeline at inlet to the control volume is 150mm bore, and the nozzle bore is 80mm.

Calculate the force the pipe exerts on the oil flow

Resolving the forces in the x direction (these will also be the forces that are flowing into the control volume at inlet, examine the drawing to make sure you can see these forces – sketch them on the drawing)

$$\sum F_x = p_1 A_1 + \overset{o}{m} v_1$$

$$A_1 \quad = \pi\, 0.15^2 / 4 \quad = \quad 0.0177\text{m}^2$$

$v_1 \quad = \dfrac{\overset{0}{V}}{A_1} \quad = 0.2 / 0.0177 \quad = \quad 11.3$ m/s

$\overset{o}{m} \quad = \rho\overset{o}{V} \quad = 860 \times 0.2 \quad = \quad 172$ kg/s

So $\sum F_x = (750 \times 10^3 \times 0.0177) + (172 \times 11.3) \quad = \quad 15219$N

Resolving the forces in the y direction (these will also be the forces that are flowing out of the control volume at outlet, again sketch them on your drawing)

$\sum F_y = p_2A_2 + \overset{o}{m}v_2$

$A_2 \quad = \pi\, 0.08^2 / 4 \quad = \quad 0.005\text{m}^2$

$v_2 \quad = \dfrac{\overset{0}{V}}{A_2} \quad = 0.2 / 0.005 \quad = \quad 39.8$ m/s

$\overset{o}{m}$ will be the same for inlet and outlet $\quad = \quad 172$ kg/s

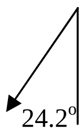
So $\sum F_y = (172 \times 39.8) \quad = \quad 6846$N

Remember that this force will be the +ve y direction, as the mass flow out is –ve, and that p_2 will be zero as the nozzle discharges to atmosphere.

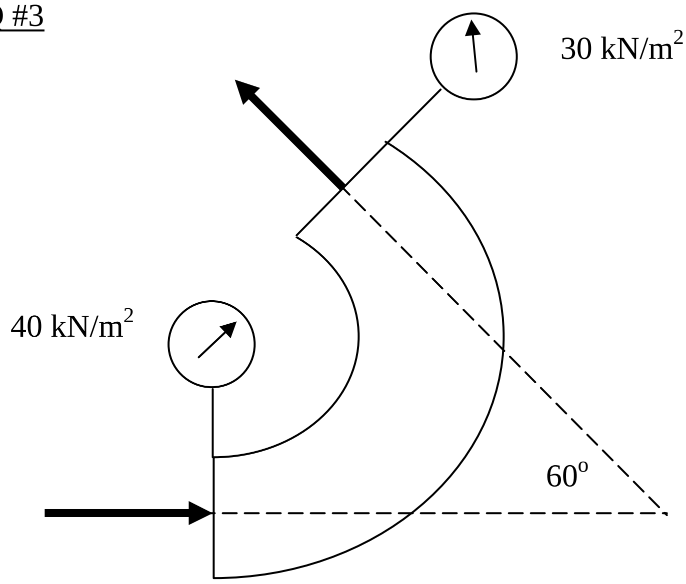
So total force is $\sqrt{F_x^2 + F_y^2} \quad = \sqrt{15219^2 + 6846^2} \quad = \quad 16.69$kN

Direction of force = $\tan^{-1} F_y / F_x = 6846 / 15219 \quad = 24.2^0$ or

However we have been asked for the force of the pipe on the fluid. This force will be equal and opposite to the force of the fluid on the pipe, so the force of the pipe acting on the fluid will be 16.69kN acting at



SAQ #3

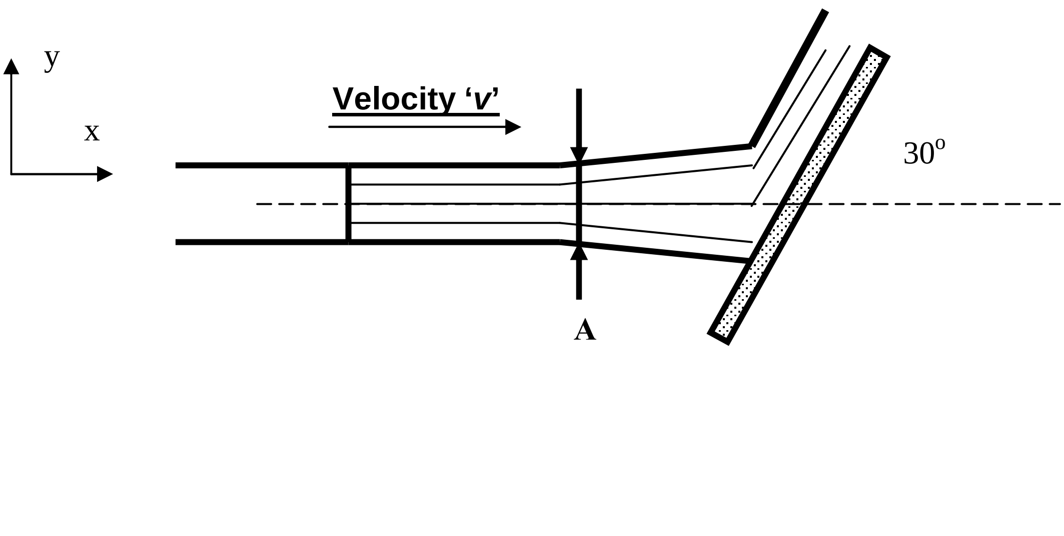


For the horizontal pipe bend shown above, calculate the force the pipe exerts on the oil.
The pipe bore is constant at 300mm, and the fluid has a velocity of 2 m/s, and density of 860 kg/m^3.
ANS on page 44

Class example
A jet of fresh water 50mm diameter with a velocity of 20 m/s strikes a flat plate inclined at 30° to the axis of the jet.
Determine the force exerted on the plate,
A) in the direction of the jet, and
B) normal to the plate

(a) when the plate is stationary
(b) when the plate is moving at 5 m/s in the direction of the jet.

This question combines the theory we have studied under the pipe bends and impact of jets onto plates. It is important that we ensure we have calculated the force in the direction required, as you will see in this following example.



We have already seen that the force exerted by a fluid flow is

Rate of change of momentum = mass flow rate $\times$ change in velocity

However as the plate is inclined, we must consider the change of velocity normal to the plate, as we know that the final water velocity in this direction must be zero.

So velocity of the fluid flow normal to the plate is $v \sin \phi$
The mass flow rate striking the plate will be $\rho A v$

Hence the force normal to the plate is $\rho A v^2 \sin \phi$

For the plate alignment given in the question then

$A = \pi\, 0.05^2 / 4 = 0.00196\text{m}^2$

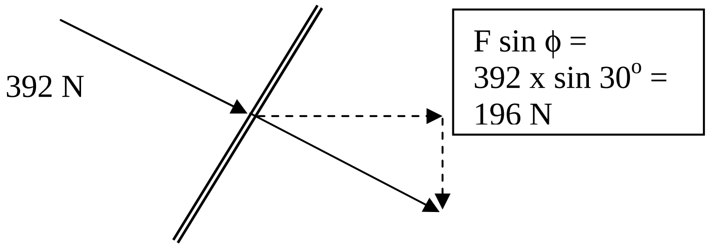
$v = 20$ m/s

Thus Force $= \rho A v^2 \sin \phi$

$= 1000 \times 0.00196 \times \sin 30^o \times 20^2$

$= 392\text{N}$

This force acts normal to the plate, so what is the force in the direction of the jet??



Well if we examine the force vector diagram, then we should see that this would be F sin ϕ, so the force in the direction of the jet will be $392 \times \sin 30^o = 196\text{N}$

When the plate is moving then the relative motion between the jet and plate should be used to assess the quantity of fluid hitting the plate. The mass flow rate striking the plate is $\rho A (v - u)$, and the change in velocity of the direction normal to the plate is $(v - u) \sin \phi$, as we have seen above the velocity normal to the plate after it strikes the plate must be zero.

Force $= \rho A (v - u)^2 \sin \phi$

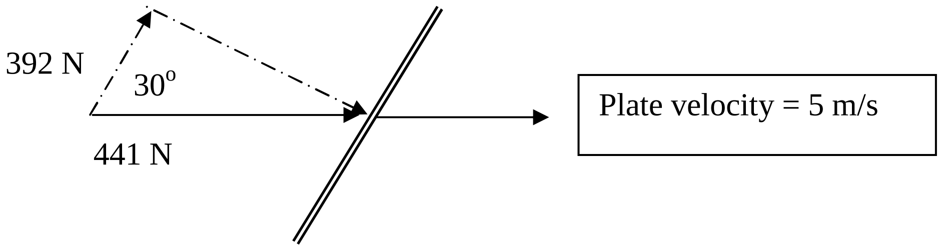
$= 1000 \times 0.00196 \times \sin 30^o \times (20 - 5)^2$

$= 220.5\text{N}$

Again the force in the direction of the jet will be F sin ϕ or 220.5 x sin 30^o = 110.25N

Ensure you are aware the direction of the force required in the question.

We will also use the analysis that was used on the pipe bends, to analyse this question. Note that as the jet is not confined within a pipe, then the pressure can be assumed constant at atmospheric pressure, so the pressure force can be neglected. The flow of the water will also be assumed not to separate on the plate, to make our analysis easier.



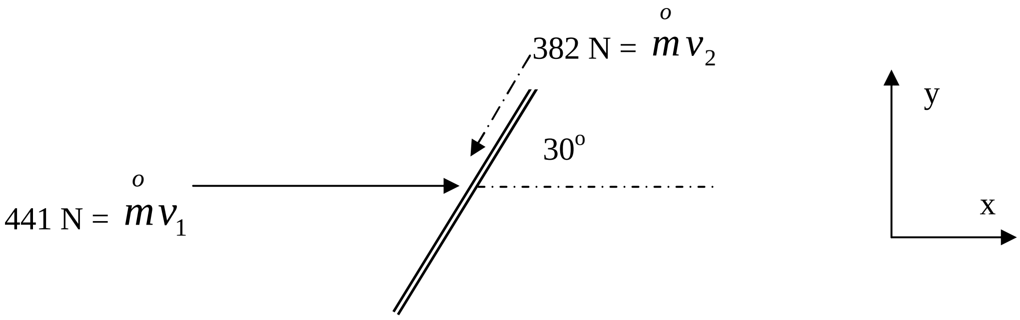
The momentum change entering the plate is $\rho\ A\,(v - u)^2$

A_1	=	A_2	=	$\pi\, 0.05^2 / 4$	=	$0.00196\ m^2$
v_1	=	$20 - 5$			=	15 m/s
$\dot{m}$	= $\rho A v$	= 1000 x 0.00196 x (20 – 5)			=	29.4 kg/s

So momentum change entering plate = 1000 x 0.00196 x $(15)^2$ = 441N

The vector of the momentum change parallel to the plate = $\rho\ A\,(v - u)^2 \cos\phi$ = 441 x cos 30^o = 381.9N

The vector of the momentum change normal to the plate = $\rho\ A\,(v - u)^2 \sin\phi$ = 441 x sin 30^o = 220.5N



Carry out our analysis in the x and y direction

$$\sum F_x = p_1A_1 + \dot{m}v_1 - \cos 30^o\,(p_2A_2 + \dot{m}v_2)$$
$$= 441 - \cos 30^o\,(381.9)$$
$$= 110.25\ \text{N}$$

This is the same answer as we achieved using the other analysis (in the **direction of the jet**)

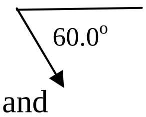
$\sum F_y = p_2A_2 + \overset{o}{m} v_2$

$= -\sin 30^o \times 381.9$
$= -190.95$ N

So total force is $\sqrt{F_x^2 + F_y^2}$ $= \sqrt{110.25^2 + 190.95^2}$ = 220.5 N

This is the same answer as we achieved previously when we calculated the force normal to the plate.

Direction of force = $\tan^{-1} F_y / F_x = 190.95 / 110.25$ $= 60^0$ or
Note the direction of this force is normal to the plate.



Carefully revise this example, as it contains many factors involving angles and relative motion that are useful through out the Mechanics module.

<u>Class example</u>
A jet of fresh water is discharged from an orifice 6.35 mm diameter under a head of 54.9 m and it impinges on a plate, which is normal to the jet. Given C_v is 0.97 and C_d is 0.62, find the force exerted by the jet on the plate.

Although the orifice size is 6.35mm, the vena-contracta will reduce the effective jet size, where $C_d = C_v \times C_c$
so the area of the jet will be $\frac{C_d}{C_v} \times \frac{\pi}{4} d^2 = \frac{0.62}{0.97} \times \frac{\pi}{4} 0.00635^2 = 20.24\text{mm}^2$

Assuming no energy loss: KE (gained) = PE (lost)

So $\frac{1}{2} mv^2 = mgh$
$v = \sqrt{2.g.h}$

Thus theoretical velocity will be $\sqrt{2g54.9}$ = 32.82 m/s

The actual velocity will be lower than this velocity as the figure of C_v has been given as 0.97, thus actual velocity = 0.97 x 32.82 = 31.84m/s

Thus Force $= \rho \, A v^2$

$= 1000 \times 20.24 \times 10^{-6} \times 31.84^2$

= 20.5N

SAQ #4

At two points A and B 3 m apart in a vertical pipe, the diameters are 25 mm and 50 mm respectively. End A is open to the atmosphere and fresh water flows upward from B to A, the pressure at B being 240 kN/m^2 (gauge). Just above A is a fixed flat plate with its surface normal to the axis of the pipe. Neglecting any losses, determine (a) the mass flow is kg/s

(b) the force exerted by the jet on a plate.

ANS on page 46

Typical examination question

An anchor wash pipe of 100mm diameter discharges sea water at the rate of 90 m^3/hour into the atmosphere through a nozzle. The nozzle converges the 100mm diameter pipe to 50mm, and deflects the jet horizontally through 40 degrees. Neglecting friction losses within the pipe and nozzle,

Calculate

a)	The exit velocity of the water from the nozzle	(3)
b)	The pressure at the 100mm section just upstream from the nozzle	(4)
c)	The resultant force on the nozzle	(9)

Take the density of seawater = 1025 kg/m^3

The volumetric rate of water discharge is given, and as the area of the pipeline is known, then the seawater velocity can be calculated.

Exit area A_2 = $\pi 0.05^2 / 4$ = 1.96×10^{-3} m^2

So exit velocity v_2 = $\dfrac{\overset{0}{V}}{A_2}$ = 90 / 3600x 0.00196 = 12.76 m/s

As the flow is constant we shall use Bernoulli's equation.

The inlet area A_1 is $\pi 0.1^2 / 4 = 0.00785$m^2,

so the inlet seawater velocity is 90/3600 x 0.00785 = 3.18m/s.

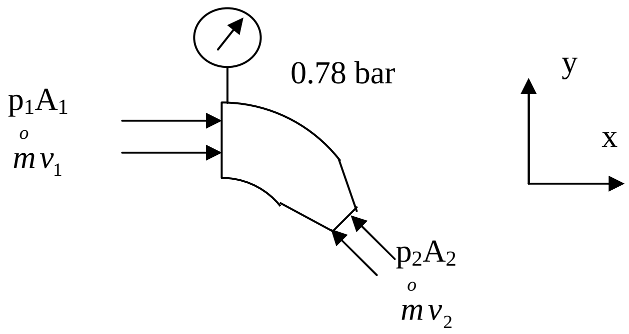
The question states that the nozzle is in the horizontal plane, so $Z_1 = Z_2$, hence

$$p_1 = p_2 + \frac{\rho v_2^2}{2} - \frac{\rho v_1^2}{2}$$

We shall assume that p_2 is at atmospheric pressure, so p_2 is 0 kN/m^2.

So $p_1 = \dfrac{1025 \times 12.76^2}{2} - \dfrac{1025 \times 3.18^2}{2}$ = 78.3 kN/m^2 or 0.78 bar

The mass flow through the nozzle is $\overset{o}{m} = \rho \overset{o}{V}$ = 1025 x 90/3600 = 25.63 kg/s



A sketch of the control volume (nozzle) indicates the direction of the four forces we need to equate. However we can omit the force p_2A_2 as the pressure p_2 is atmospheric.

p_1A_1	=	$78.3 \times 10^3 \times 0.00785$	=	614.66N
mv_1	=	25.63×3.18	=	81.5N
p_2A_2	=	0		
mv_2	=	25.63×12.76	=	327.0N

$$\sum F_x = (p_1A_1 + \dot{m}v_1) - \cos 40^0\ \dot{m}v_2$$
$$= (614.66 + 81.5) - 0.766 \times 327$$
$$= 445.68\text{N}$$

$$\sum F_y = \sin 40^0\ \dot{m}v_2$$
$$= 0.643 \times 327 = 210.2\text{N}$$

So total force is $\sqrt{F_x^2 + F_y^2} = \sqrt{445.68^2 + 210.2^2} = 492.76\text{N}$

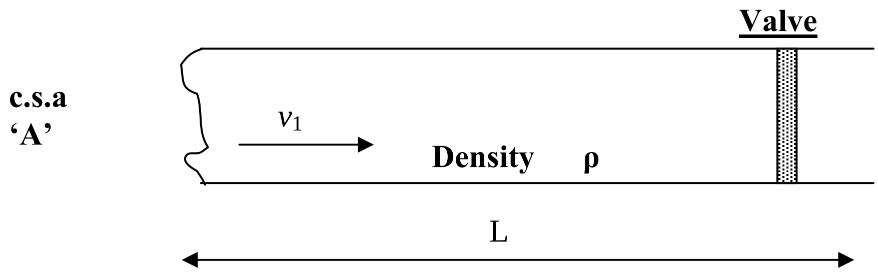
SAQ
A pipe 101.6 mm bore carries 1.132 m^3/min of fresh water. What is the force of impact at a right angled bend?
Ans: 62 N

SAQ
A jet of water 50.8 mm diameter moving at a velocity of 64 m/s strikes a flat plate in a direction normal to the surface. If the plate is fixed, find the force exerted on the plate by the jet. If the plate was moving at 15.23 m/s, determine the force exerted and the work done on the plate and the efficiency of the operation.
Ans: 8.3 kN 4.82 kJ 27.5%

Closure of valves within pipelines

The energy within a pipeline system would normally be calculated by using the various energy terms we have used on page 2 that make the Bernoulli equation. However to find the pressure rise within a pipeline when a valve closes requires the analysis of forces within the pipeline system as the energy will change and therefore Bernoulli equation is not valid for this application.

When a valve is closed against the flow of water, there will be a resultant pressure increase at the face of the valve, and also a pressure decrease downstream of the valve.



Slow or gradual valve closure

Consider a valve that is closed in a period of time 't'. This time is considered slow if the fluid that is slowed by the closing valve can partly escape through the valve as it is closing.

The force exerted on the equal to the rate of change of momentum of the fluid; i.e.

$$\text{Force} = \frac{\text{change of momentum}}{\text{time taken}}$$

As Change of Momentum = mass x change in velocity

Now the mass of fluid $= \rho \times A \times L$

and the change in velocity $= v_2 - v_1$ (but after valve closure the velocity $v_2 = 0$)

Hence Change in Momentum = ρ x A x L x v_1

Hence Force = Rate of change of Momentum = $\left(\frac{\rho.A.L.v_1}{t}\right)$

Hence Pressure = $\frac{Force}{Area} = \frac{(\rho.A.L.v_1)}{A.t} = \frac{\rho L v_1}{t}$

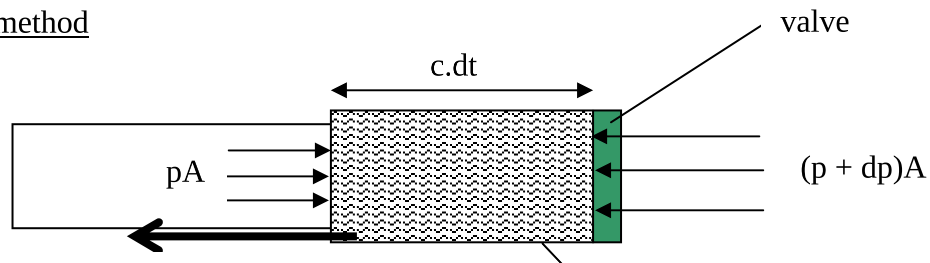
Rapid or sudden valve closure

When a valve on a long pipeline is shut instantaneously, the fluid upstream of the valve will impinge on the closed valve and the velocity energy will be converted to produce a pressure rise. The fluid layer behind the now stationary fluid mass will also be brought to rest, and the impact of this layer upon the first fluid mass will cause its pressure to rise as well as maintaining the pressure rise in the first fluid mass. Each layer of fluid will repeat this process as it strikes the stationary fluid. Eventually all the fluid in the pipe will be brought to rest, and the pressure within the whole pipe will be raised.

The increase in pressure will cause the fluid to be compressed, and the pipe itself to be distorted. The pressure increase or wave travels back up the pipeline as the fluid is stalled. **This wave will travel at the speed of sound for the fluid**. This pressure increase is much greater than that determined by the slow closure, and engineers should be aware of the damaging effects and magnitude of this pressure wave.

Two methods can be used to calculate the pressure increase, force and energy.

<u>Force method</u>



When the valve is closed rapidly, a pressure wave travels back up the pipe with a speed c.

Within a short period of time dt, a length of fluid c.dt is brought to rest.

The force against the valve face is increased from the original pressure “p” to (p + dp)

From Force. dt $=$ Momentum. dv

$(pA - (p + dp)A)dt = m.dt.dv$

$dp.A.dt = \rho Ac.dt.dv$

So $dp = \rho.c.dv$

As the valve is fully shut then the change in velocity dv of the fluid will be the original fluid flow speed, v,

Hence increase in pressure **dp** $=$ $\boldsymbol{\rho c v}$

Energy method

For this method we must consider the real properties of a fluid, in that it can be compressed, as well as the real properties of the pipeline, in that it will expand when the pressure inside the pipe increases.

The compressibility of a fluid is known as the Bulk Modulus, as is similar to the Modulus of elasticity of a solid.

The Bulk Modulus,κ, is defined as change in volume for change in pressure and equals

$$\kappa = \frac{V}{dV}dp$$

The bulk modulus for water at 1 bar and 20°C is 2130MN/m^2, whereas the volume modulus of steel is 170,000 MN/m^2, hence water is about 80 times more compressible than steel.

Consider a fluid flow of v m/s travelling along a pipeline, when it is stopped suddenly by a rapid valve closure. The kinetic energy of this fluid is $\rho v^2/2$.

This energy will be converted into strain energy of the pipeline as the pressure increase distorts the pipeline.

The strain energy per unit volume = $dp^2 / 2\kappa$

But the velocity of sound in a fluid c = $\sqrt{(\kappa/\rho)}$

So strain energy = $dp^2 / 2c^2\rho$

Equating the two energies gives

$$dp^2 / 2c^2\rho = \rho v^2/2$$

$$dp^2 = \rho^2 c^2 v^2$$

So increase in pressure **dp** **=** $\boldsymbol{\rho c v}$

Class example

Fresh water is flowing at 2kg/sec within a 75mm bore pipeline of length 20m. A valve at the end of the pipeline is closed

a) Slowly within 1.2 seconds

b) Rapidly

Assume the speed of sound within the fluid is 1280m/s

For a slow closure we can use the given relationship of $\dfrac{\rho L v_1}{t}$

Fluid velocity v = $2 / \pi 0.075^2 \times 1000/4$ = 0.45m/s

Pressure rise = 1000 x 20 x 0.45 / 1.2 = 7.5kN/m^2 or 0.075 bar

For a rapid closure we can use the relationship of $\rho c v$

Pressure rise = 1000 x 1280 x 0.45 = 576kN/m^2 or 5.76 bar

At this point we should discuss what is the distinction between a slow and rapid closure. The critical time period can be shown to equal 2L/c, which is the time it takes for the pressure wave to travel along the whole pipeline and be reflected back. If, by the time the initial pressure wave (from the closing valve) travels the length of the pipeline twice and the valve is still not closed, then there will be some leakage of pressure and the valve closure is termed slow. However if the valve is closed when the reflected pressure wave returns, then the valve closure is termed fast or rapid.

For the example the critical time or pipe period as it is known will be 2 x 20 / 1280 or 0.03125 seconds or 31.25 ms. As most valves can not be closed within these very small times, the high pressure surges from rapid valve closure is unlikely, but not unknown.

Student test

Compare the rise in pressure caused by the sudden closure of a valve at the end of a 60m pipeline when the fresh water is flowing at 3 m/s, with the rise in pressure when the valve is closed in 0.5 seconds. Calculate the pipe period time.

Assume c = 1440 m/s

Ans: 43.2 bar, 3.6 bar, 0.083 seconds

Centrifugal Pumps

This type of pump is the most versatile and popular pump unit used onboard.

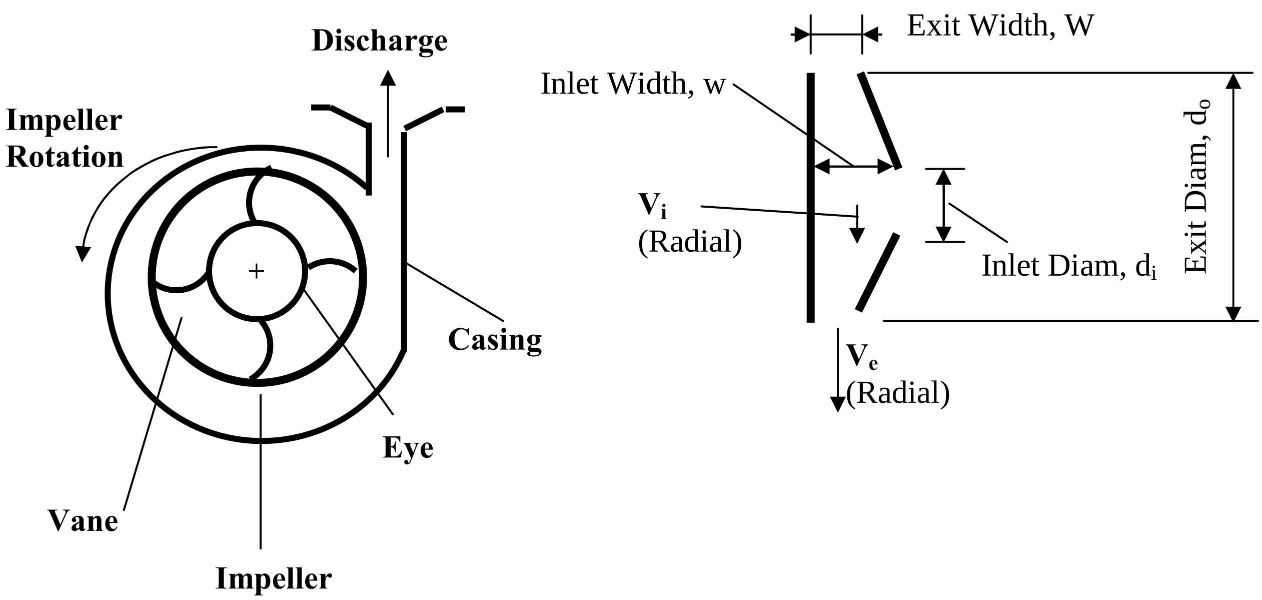
The main component is the impeller that rotates inside a casing. This impeller imparts a force on the fluid within it, which causes the fluid to flow out of the impeller as it rotates. This fluid flow produces a vacuum to be formed in the pump suction, which will draw other fluid into the impeller suction.
Thus fluid flow will occur from the suction to discharge due to the rotating impeller; and from tank suction to the pump due to the tank pressure being higher than the pump impeller suction pressure.

The kinetic energy produced by the impeller (fluid flow) is then converted into pressure energy at the volute or diffuser. Both of these systems operate by increasing the discharge area and slowing the fluid down.

We shall examine the operation of the pump impeller in more detail. Fluid enters the eye of the impeller radially (normal to the direction of rotation), and is accelerated tangentially (parallel to the direction of rotation) by the impeller, and moves to the outside of the casing. It is normally assumed that the fluid inlet velocity is entirely radial, and that *it keeps the radial velocity constant* as it moves across the impeller.

The Continuity equation implies that for constant radial velocity, the cross sectional area of flow must be constant.
Since the cross sectional area of flow is equal to the impeller circumference times its' width, this means that as the impeller diameter increases, its' width must decrease accordingly. This gives the impeller its characteristic shape.

CENTRIFUGAL PUMP IMPELLER



At the ***Inlet to the Impeller***

$\overset{o}{V}$ = inlet width "w" x circumference at inlet "πd_i" x inlet velocity (v_1)
= $w.\pi d_i.v_1$

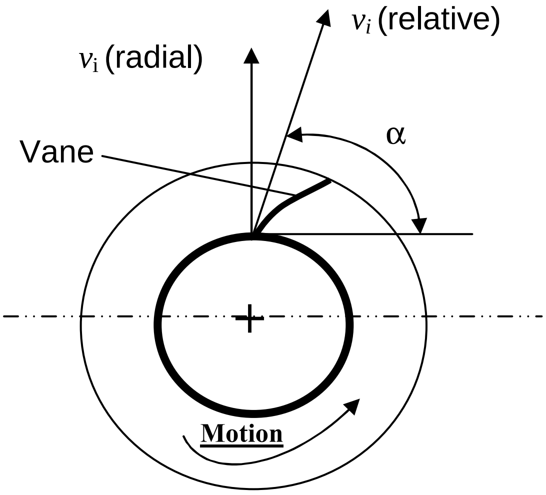
At the ***Exit from the Impeller***

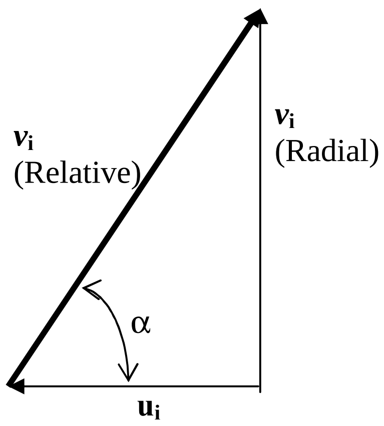
$\overset{o}{V}$ = exit width "W" x circumference at exit "πd_e" x radial exit velocity "v_e"
= $W.\pi d_e.v_e$

for continuity Flow rate at inlet = Flow rate at outlet
So $w.d_i.v_I = W.d_e.v_e$

As stated earlier we would normally assume (as no friction is present) that radial components of velocity through the impeller are equal; i.e $v_{e(rad)} = v_1$

Centrifugal Pump Velocity Diagrams
To gain a greater understanding of the actual velocity vectors that exist at the inlet and outlet of the impeller we shall construct velocity diagrams.

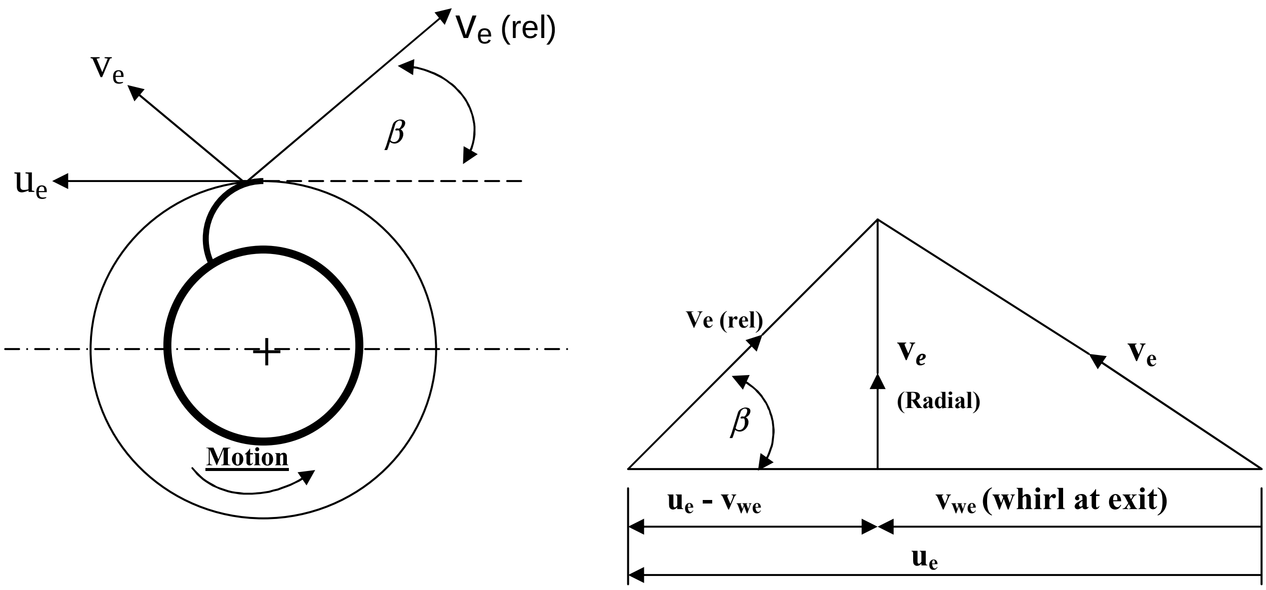




Consider a vane within the impeller rotating at N rev/sec. The inside tip of the vane will have an absolute speed of πNd, where d is the inside diameter. We will give this tangential impeller speed the symbol u_i, with the subscript "i" indicating the inlet.

where u_i = Tangential Velocity of Impeller Vane Root
v_i = Absolute velocity of liquid at Inlet (radial only)
v_i = Relative velocity of liquid at Inlet
α = Angle of impeller at inlet for shockless entry

It is the relative velocity of the fluid flow that is important for the required angle α of the impeller vane, as it is relative to the impeller motion.



Exit Velocity Diagram

In the exit diagram, the flow from the impeller in the radial direction v_e (radial) is assumed to equal the inlet radial velocity if we ignore the friction effects of the impeller / fluid interface.

As with the inlet velocity diagram, the tip of the impeller *at exit*, u_e, will have a velocity of πND, where D is the external diameter of the impeller.

The absolute velocity of the fluid flow from the impeller v_e has two components, the radial component we have already seen is present at the inlet, and the whirl (meaning it is a tangential direction) velocity v_{we}. This whirl velocity is important, as it is this velocity that is gained when the fluid passes through the impeller, and hence is a measure of the work done on the fluid by the impeller.

To ensure there is shockless exit of the fluid leaving the impeller, then the relative flow v_e should leave with the correct angle β

The work done on the fluid per second = Force x velocity

$$= \overset{o}{m} v_{we} u_e$$

Another term used within pumps is the head of a pump. The head can be defined as the vertical lift that a pump is capable of achieving, thus if a vertical pipe was fitted to the outlet of the pump, then the height of the fluid column would be the head of the pump.

Work done in terms of potential energy $= \overset{o}{m} g h_t$

Thus the theoretical head, h_t, can be defined by equating the potential energy and work done terms as $h_t = \frac{v_{we}u_e}{g}$

Also the pump head can be related to the discharge and suction pressures of the pump, as the head is the difference between these two pressure, i.e. a measure of the energy that has been transferred to the fluid

$$h_t = \frac{pressure_{disch} - pressure_{suct}}{\rho . g}$$

Class example

A centrifugal pump operates at 390 rev/min and discharges fresh water at 0.12m^3/s. The impeller is 500mm diameter and 85mm wide at outlet. The blade outlet angle is 28^o. Radial flow can be assumed at inlet, and is constant through the impeller. Calculate the theoretical head, power required by the pump, and the pressure rise across the impeller.

Impeller speed at exit, u_e = πND = π 390/60 0.5 = 10.21m/s

Volumetric flow rate $\overset{o}{V}$ = 0.12 = W.$\pi d_e.v_e$ = 0.085π0.5v_e so v_e = 0.9m/s

Once the radial velocity is known together with the impeller angle, then the dimension ($u_e - v_{we}$) can be found from the trigonometric relationship.

So $\tan\beta = \frac{v_e}{(u_e - v_{we})}$, thus ($u_e - v_{we}$) = v_e / tan 28^o = 0.9 / tan 28^o = 1.69m/s

We can now find the whirl velocity at exit v_{we},
from $u_e - (u_e - v_{we})$ = 10.21 – 1.69 = 8.52m/s

Thus head $h_t = \frac{v_{we}u_e}{g} = \frac{8.52 \times 10.21}{9.81} = 8.87\text{m}$

Power = $\overset{o}{m}\,gh = \rho \overset{o}{V}\,gh$ = 1000 x 0.12 x g x 8.87 = 10.44kW

Note this is the mass flow rate, not the mass. Using mass flow rate will give the units in J/s or W

The pressure rise can be equated to the static head minus the velocity head, as these are the two components of energy at the outlet of the pump.

Hence p = (ρgh - ρv_{we} / 2)

Static pressure = 1000 x g x 8.87 = 87kN/m^2

Velocity "pressure" = 1000 x 8.52^2 / 2 = 36.3kN/m^2

Hence pressure rise across the pump = 87 – 36.3 = 50.7kN/m^2

<u>Class example</u>
A centrifugal fan running at 750 rev/min, has impeller diameters at inlet and outlet of 53.4 cm and 76.2 cm respectively. The velocity of whirl at exit is 22.85 m/s and the blade outlet angle 70°. The width of the impeller at outlet is 101.6 mm.
If the specific volume of the air is 0.795 m^3/kg;
Find the mass of air delivered/min, the blade inlet angle and the impeller width at inlet. Assume radial flow velocity is constant.

The density of the air = 1/specific volume = 1/0.795 = 1.258 kg/m^3

As we know the impeller exit angle, we can construct part of the outlet velocity triangle.
$u_e = \pi ND = \pi \times 750/60 \times 0.762 = 29.92$ m/s

Thus $(u_e - v_{we}) = 29.92 - 22.85 = 7.07$ m/s

As $\tan\beta = \dfrac{v_e}{(u_e - v_{we})}$, then $\tan 70^o = v_e / 7.07$, then $v_e = 19.43$ m/s

Using the inlet $\overset{o}{m} = \rho.W.\pi d_e.v_e = 1.258 \times 0.1016 \times \pi \times 0.762 \times 19.43$
$= 356.7$ kg/min

Note the need to place all the units into metres from the original quoted figures of cm and mm.

The inlet velocity diagram will yield the inlet angle from $\tan\alpha = \dfrac{v_1}{u_i}$

$u_i = \pi Nd = \pi \times 750/60 \times 0.534 = 20.97$ m/s

As v_1 is considered the same as v_e, then v_1 is 19.43m/s
Thus $\tan\alpha = 19.43 / 20.97$, thus $\alpha = 42.82^o$

From the mass or volume continuity equation of fluid flow, then wd = WD

So inlet width w, = WD / d = 0.1016 x 0.762 / 0.534 = 0.145m or 145mm

Class example

A pump is to draw water from a depth of 1.83 m below its centre line and discharge it to a height of 6.1 m above its centre line through a pipe 76.4 m long and 101.6 mm diameter. If the rate of discharge is 226 m^3/h and the pump efficiency is 60%, find the input power to the pump.

Loss of head due to friction in discharge pipe is given by $\frac{4\ f\ l\ v^2}{2\ g\ d}$ where f = 0.0048.

In this example, the total head delivered by the pump will consist of three parts:

1. Suction head lift or head (1.83m)
2. Static head (6.1m)
3. Discharge pipe friction head (Darcy equation)

From volumetric flow rate $\overset{o}{V} = uA$, thus fluid velocity, v;

v = 226/(3600 x π0.1016^2/4) = 7.74 m/s

Friction head lost in the discharge pipe = $\frac{4 \times 0.0048 \times 76.4 \times (7.74)^2}{2 \times g \times 0.1016}$ = 44.1m

Hence total lift of the pump = 1.83 + 6.1 + 44.1 = 52.03m

Mass flow rate of the pump = $\rho\overset{o}{V}$ = 1000 x 226/3600 = 62.78 kg/s

Thus outlet power of the pump = $\overset{o}{m}gh$ = 62.78 x g x 52.03 = 32.04kW

Hence inlet power of the pump = outlet / η_{pump} = 32.04 / 0.6 = 53.4kW

Typical examination question

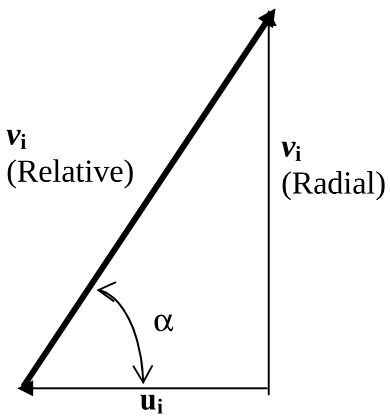
The impeller of a sea water circulating pump rotates at 720 rev/min and discharges 400 tonne/hour. The impeller dimensions are inlet diameter 180mm, and exit diameter 420mm. The exit vane angle is 70°. At entry to the impeller the water flow has a velocity of 2 metres/s in the radial direction only.

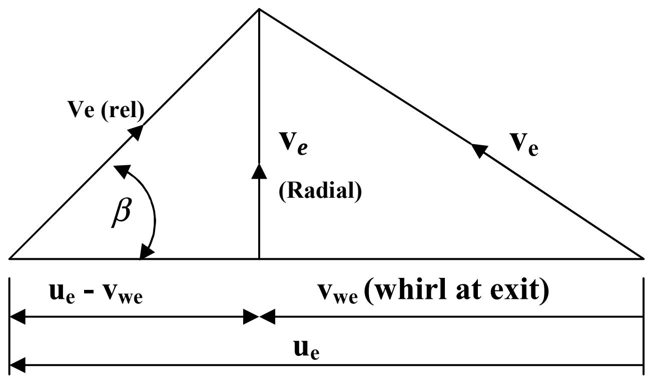
Calculate EACH of the following:

a)	The inlet vane angle for shock-less entry	(4)
b)	The width of the impeller at inlet and exit	(5)
c)	The required power of the pump	(7)

The density of the sea water = 1025 kg/m^3

If we construct the two velocity diagrams for the pump, then the data given within the question will assist you in deciding how and what is required to answer the question. It is always good practice to make a sketch of any mechanics problems, as an aid to understanding.





Data given in question:

$v_1 = v_e$ = 2m/s (radial)

$\beta = 70^o$

Calculated information:

u_i	=	πNd	= π 720/60 x 0.18	= 6.79m/s
u_e	=	πND	= π 720/60 x 0.42	= 15.83m/s

From the study of the inlet velocity diagram we can see that $\alpha = \tan^{-1} v_1 / u_1 = 16.4^o$

From the relationship for volumetric flow rate $\overset{o}{m} = \rho.w.\pi d_i.v_I$,
then 400 x 10^3 /3600 = 1025 w π 0.18 x 2, so w = 95.85mm

Note the flow through the impeller is dependant upon the ***radial flow rate*** NOT the impeller speed induced velocity of v_e

Similarly the exit impeller width can be found from $\overset{o}{m} = \rho.W.\pi D.v_e$,
then 400 x 10^3 /3600 = 1025 W π 0.42 x 2, so W = 41.08mm

To calculate the power required, we need to find the head produced by the pump, and hence the whirl velocities at the exit of the impeller, as it is this velocity flow energy that will be converted in the pump exit volute to produced the discharge head.

As $\tan\beta = \dfrac{v_e}{(u_e - v_{we})}$, then $\tan 70^o = 2/(u_e - v_{we})$, then $(u_e - v_{we}) = 0.73$m/s

Thus $v_{we} = u_e - (u_e - v_{we}) = 15.83 - 0.73 = 15.1$m/s

Thus head $h_t = \dfrac{v_{we}u_e}{g} = \dfrac{15.1 \times 15.83}{9.81} = 24.37\text{m}$

Previously we have seen that Power = $\overset{o}{m}gh$, so

Power = 400 x 10^3 / 3600 x g x 24.37 = 26.56kW

Typical examination question

A centrifugal pump fills a diesel oil header tank through a 75mm bore line at 14 tonne/hour. The service tank used for the oil supply applies a constant 2 metre positive head on the suction side of the pump, whilst the discharge side requires a 12 metre lift. The total pipe length from pump to header tank is 15 metres.

The pipeline friction can be found from the Darcy relationship, where

$f = 0.005\left(1+\frac{1}{40d}\right)$ where d is the pipeline diameter in metres

Calculate EACH of the following

a)	Pump suction pressure	(2)
b)	Head lost to friction between the pump and header tank	(5)
c)	The discharge pressure of the pump	(4)
d)	The pump power required	(5)

The density of the diesel oil = 890 kg/m^3

The tank suction pressure will be positive as the suction tank is above the pump. From p = ρgh, then suction pressure = 890 x g x 2 = 17kN/m^2 = 0.17bar

The Darcy equation gives the head lost due to friction, but the friction co-efficient needs to be calculated from the relationship $f = 0.005\left(1+\frac{1}{40d}\right)$.

Hence $f = 0.005\left(1+\frac{1}{40\times 0.075}\right)$ = 0.0067

From $\overset{o}{m} = \rho.u.A$, so u = 14000/(3600 x 890 x π0.075^2/4) = 0.99m/s

Thus from the Darcy equation, head lost between the pump and the header tank will be $\frac{4flv^2}{2gd}$ = $\frac{4\times 0.0067\times 15\times 0.99^2}{2\times g\times 0.075}$ = 0.266m

The discharge pressure of the pump will be the resistance due to the static head plus the resistance or head from the friction.

Hence total discharge head = 12 + 0.266 = 12.266m

p = ρgh = 890 x g x 12.266 = 107.1 kN/m^2 or 1.07 bar

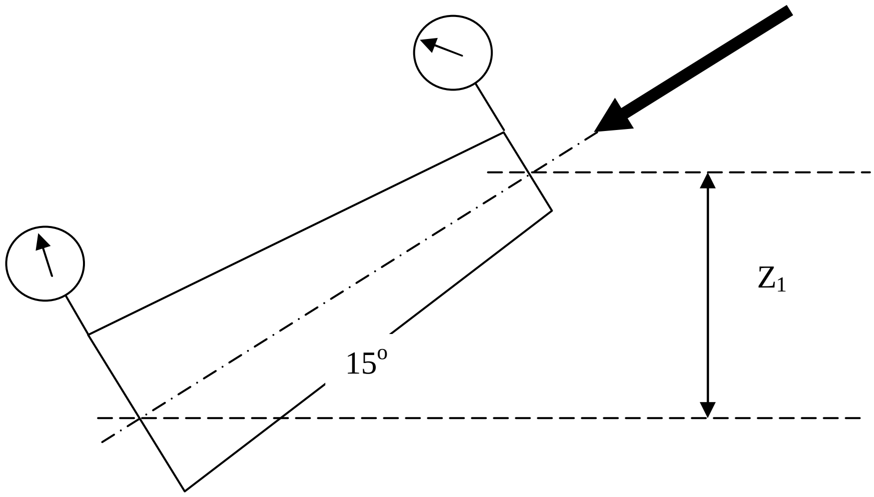
The power required is the total head from the suction to the discharge. Previously we added the suction head to the discharge head, but *as the suction is positive* we need to **remove** this assistance from the total discharge head.

Pump head = total discharge head – suction head = 12.266 – 2 = 10.266m

Power = $\overset{o}{m}gh$ = 14000/3600 x g x 10.266 = 391.65W

SAQ #1

Consider a pipe of length 6m whose diameter tapers increases from 80mm to 120mm. The pipeline is fixed at an incline of 15^o with the inlet above the outlet. A pressure gauge measures 1.2bar at the inlet. Calculate the pressure at the outlet when a fresh water flow of 34 tonne/hour is present.



Assigning the subscript 1 to the inlet, and subscript 2 to the outlet.

Placing the height datum at the outlet (station 2), so any measurement above this will be positive

Stating the known data:

p_1	=	1.2 bar or 120kN/m^2
Z_1	=	$\sin 15^o \times 6 = 1.55$m
d_1	=	80mm
Z_2	=	zero, as it is at the height datum
d_2	=	120mm

As before, knowing the flow rate allows the speed of fluid flow to be calculated.

From mass flow rate $\overset{o}{m} = \rho Av$,

where $\overset{o}{m} = 34 \times 1000/3600 = 9.44$kg/sec

then $v_1 = 9.44/(1000 \times \pi 0.08^2/4) = 1.88$m/s

Also as the mass flow rate through the pipeline must be constant,

then $v_2 = 9.44/(1000 \times \pi 0.12^2/4) = 4.23$m/s.

Now that the velocities are known, I shall use Bernoulli equation with the pressure equation to find the unknown pressure, p_2.

$$p_1 + \frac{\rho v_1^2}{2} + \rho g Z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g Z_2$$

$120 \times 10^3 + 1000 \times 1.88^2/2 + 1000g\ 1.55 = p_2 + 1000 \times 4.23^2/2 + 0$

$120 \times 10^3 + 1.77 \times 10^3 + 15.2 \times 10^3 = p_2 + 8.95 \times 10^3$

So $p_2 = 128 \times 10^3$ N/m^2 or 1.28 bar.

SAQ #2

A pipe of 200mm diameter and 300m long is discharging fresh water with a velocity of 1.2 m/s. If f = 0.01, find the head loss, and hence pressure loss due to friction.

From Darcy's equation $z_f = \frac{4flv^2}{2gd}$

So $z_f = \frac{4 \times 0.01 \times 300 \times 1.2^2}{2 \times g \times 0.2}$

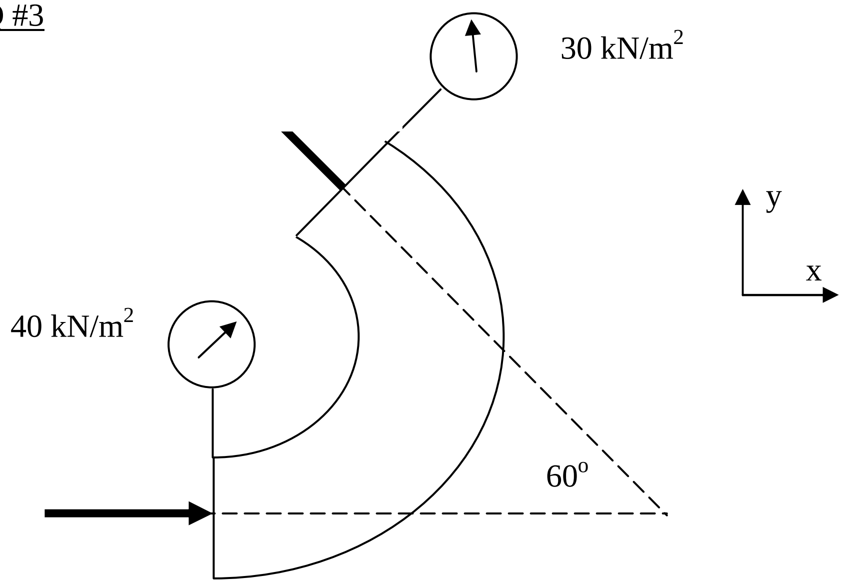
$= 4.4$m

Pressure loss $= \rho g z_f$

$= 1000g4.4$

$= 43.2$kN/m^2 or 0.43bar

SAQ #3



For the horizontal pipe bend shown above, calculate the force the pipe exerts on the oil. Fluid density is 860 kg/m^3
The pipe bore is constant at 300mm, and the fluid velocity is 2 m/s.

I have placed the x and y datum next to this sketch.

Resolving the forces in the x direction

$\sum F_x = (p_1A_1 + \overset{o}{m}v_1) + \cos 60^0 (p_2A_2 + \overset{o}{m}v_2)$

Study your sketch to ensure you can see that these forces are added.

$A_1 \quad = \pi\, 0.3^2 / 4 \quad = \quad 0.071\text{m}^2$

$\overset{o}{m} \quad = \rho A v_1 \quad = \quad 860 \times 0.071 \times 2 = \quad 121.6 \text{ kg/s}$

$A_2 \quad = A_1 \quad = \quad 0.071\text{m}^2$

$v_2 \quad = v_1 \quad = \quad 2 \text{ m/s}$

$\overset{o}{m} \quad = \quad 172 \text{ kg/s}$

So $\sum F_x = ((40 \times 10^3 \times 0.071) + (121.6 \times 2)) +$
$\cos 60^o ((30 \times 10^3 \times 0.071) + (121.6 \times 2)) \quad = \quad 4270\text{N}$

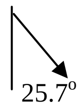
Resolving the forces in the y direction

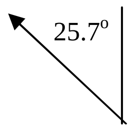
$\sum F_y = $ -sin 60^o (p_2A_2 + $\overset{o}{m} v_2$)

Note the negative sign as the forces are acting into the outlet of the pipe.

So $\sum F_y = $ - (sin 60^o (30 x 10^3 x 0.071) + (121.6 x 2)) = - 2055N

So total force is $\sqrt{F_x^2 + F_y^2}$ $= \sqrt{4270^2 + 2055^2}$ = 4739N

Direction of force = tan^{-1} F_y / F_x = -2055 / 4270 = -25.7^0 or 

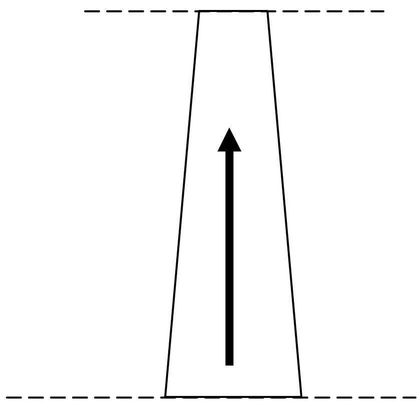
However we have again been asked for the force of the pipe on the fluid. This force will be equal and opposite to the force of the fluid on the pipe, so the force of the pipe acting on the fluid will be 4.74kN acting at 

SAQ #4

At two points A and B 3 m apart in a vertical pipe, the diameters are 25 mm and 50 mm respectively. End A is open to the atmosphere and fresh water flows upwards from B to A, the pressure at B being 240 kN/m^2(gauge). Just above A is a fixed flat plate with its surface normal to the axis of the pipe. Neglecting any losses,
determine

(a) the mass flow is kg/s

(b) the force exerted by the jet on a plate.



Using the Bernoulli equation between the inlet (50mm) and outlet (25mm), with the height datum at the inlet B, or subscript 2.

$$p_1 + \frac{\rho v_1^2}{2} + \rho g Z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g Z_2$$

where p_1 = zero (atmospheric)
Z_1 = 3m
p_2 = 240 kN/m^2
Z_2 = zero (datum)

From the continuity equation $v_1 = v_2\ A_2 / A_1 = v_2\ d_2^2 / d_1^2 = v_2\ 50^2 / 25^2 = 4v_2$

Thus $$\frac{1000(4v_2)^2}{2} + 1000g \times 3 = 240 \times 10^3 + \frac{1000v_2^2}{2}$$

$$\frac{1000 \times 16{v_2}^2}{2} + 1000g \times 3 = 240 \times 10^3 + \frac{1000v_2^2}{2}$$

So $7500v_2^2 = 240 \times 10^3 - 29.43 \times 10^3$

Hence v_2 = 5.3 m/s

$\overset{o}{m} = \rho Av$, so $\overset{o}{m} = 1000 \times \pi/4 \times 0.050^2 \times 5.3$ = 10.4 kg/sec

For a stationary plate the Force = $\rho\ A v^2$ or $\overset{o}{m} v_1$

Thus Force = 10.4 x (4 x 5.3) = 220.5N

SAQ
A centrifugal fan running at 750 rev/min, has impeller diameters at inlet and outlet of 53.4 cm and 76.2 cm respectively. The velocity of whirl at axis is 22.85 m/s and the blade outlet angle 70°. The width of the impeller at outlet is 101.6 mm. If the specific volume of the air is 0.795 m^3/kg, find the mass of air delivered/min, the blade inlet angle and the impell4er width at inlet. (Radial flow velocity constant).
Ans: 350 kg/min 42°33′ 14.53 cm

SAQ
A centrifugal pump impeller has an eye diameter of 38 cm and an outside diameter of 76 cm. If the width of the impeller passage at the eye is 15.24 cm and the radial velocity of flow of the water through the passage is constant, what is the width of the passage at outlet from the impeller?
If the speed of the pump is 100 rev/min and the radial velocity of flow is 1.525 m/s what must be the inlet angle of the vanes relative to the tangent to the eye such that the water enters the impeller without shock?
Ans: 76.2 mm 37°23′

SAQ
A pump draws water from a tank 3.048 m below its own level and discharges into another tank 76.2 m above its own level. The suction pipe diameter is 15.24 cm and the velocity through this pipe is 1.525 m/s with a friction head loss of 0.61 m. On the delivery side the pipe diameter is 12.7 cm and the friction head loss is 13.72 m. If the efficiency of the pump is 65%, calculate the power required to drive the pump.
Ans: 39.4 kW

SAQ
A hose carrying sea water is held over the side of a ship with a nozzle at the end horizontally placed, and 9.15 m above water level. If the jet strikes the water at 18.3 m from the ships side, calculate the pressure in the hose (neglect friction losses in hose) give C_v is 0.9. Determine also the power required to drive the pump if it is placed at water level and its efficiency is 75%. The nozzle diameter is 12.7 mm.
Ans: At nozzle 113.5 kN/m^2 At pump 205.5 kN/m^2 0.466 kW

SAQ
A pump fills a tank with fresh water at the rate of 0.906 m^3/min through a pipe 50.8 mm diameter and 45.75 m long. The suction lift is 1.22 m and the discharge head to the bottom of the tank is 9.75 m. The friction head is given by $\frac{4flv^2}{2gd}$,
where f = 0.01
What is the power exerted by the pump at the instant when there is 1.22 m of water in the tank?
Ans: 17 kW

YOU HAVE NOW COMPLETED THE HYDRODYNAMICS SECTION, AND SHOULD BE ABLE TO ATTEMPT THE MODULE ASSESSMENT QUESTIONS FOR THIS SECTION.