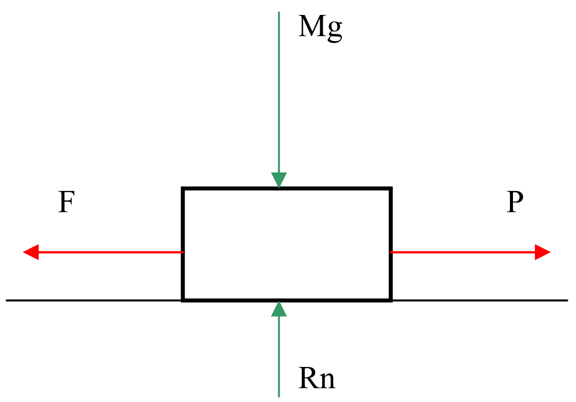
FRICTION

Friction is the resistance to relative motion which exists between two surfaces in contact. It may be undesirable, as in the case of shafts running in bearings, where it absorbs energy, and causes heat and wear, or it may be desirable, as in the case of belt drives or clutches, where it enables the transmission of power.

A body of mass M resting on a horizontal plane surface is subjected to gravitational force giving a downward load of Mg. This induces an upwards reaction R_n (equal to Mg), **normal** to the plane.

A gradually increasing force P applied to the body as shown will induce a frictional resistance F so that any instant F = P until the applied force P reaches a specific value, such that any further increase will cause motion. This specific value of F at this point is known as the LIMITING VALUE of "STATIC" friction.



This maximum or limiting value of friction is reached when motion is about to commence. When motion commences, the value of frictional resistance and hence the value of the coefficient of friction decrease very slightly, but in most cases this small change is ignored.

The Laws of Friction

The following conclusions are based on experimental work and usually referred to as the 'laws' of friction, but because of the difficulty in obtaining uniform surface texture, roughness and dryness they do not have the precision of scientific laws and apply only to moderate speeds and loads and even then are only approximate.

1. The frictional resistance is directly proportional to the forces at right angles to the rubbing surfaces. If the "normal" force is doubled then the frictional resistance is doubled. Thus:

   F is proportional to R_n
   $F = \text{Constant} \; x \; R_n$
   $F = \mu R_n$ (Where F is the maximum or limiting value of friction)

   This constant of proportionality μ is known as the "Coefficient of Friction"

2. The frictional resistance is dependent on the materials and on the degree of roughness of the contact surfaces.

3. The frictional resistance is **independent of the area of contact**. This means that two pairs of surfaces of the same materials, in the same condition and with the same force between them but having different areas will experience the same frictional force opposing motion. This may seem strange at first, but remember, if we have the same load distributed across a greater area we will reduce the contact pressure.

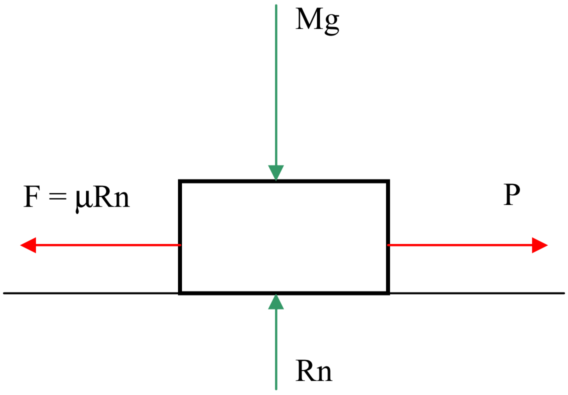
4. The frictional resistance is independent of the relative velocity of the surfaces.

Example1 :-

A horizontal force of 49 N is needed to slide a block over a floor at a steady rate. If the coefficient of friction between the block and floor is 0.5 find the mass of the block

$F = \mu R_n$

$49 = 0.5 \; x \; R_n$



$R_n = 98$

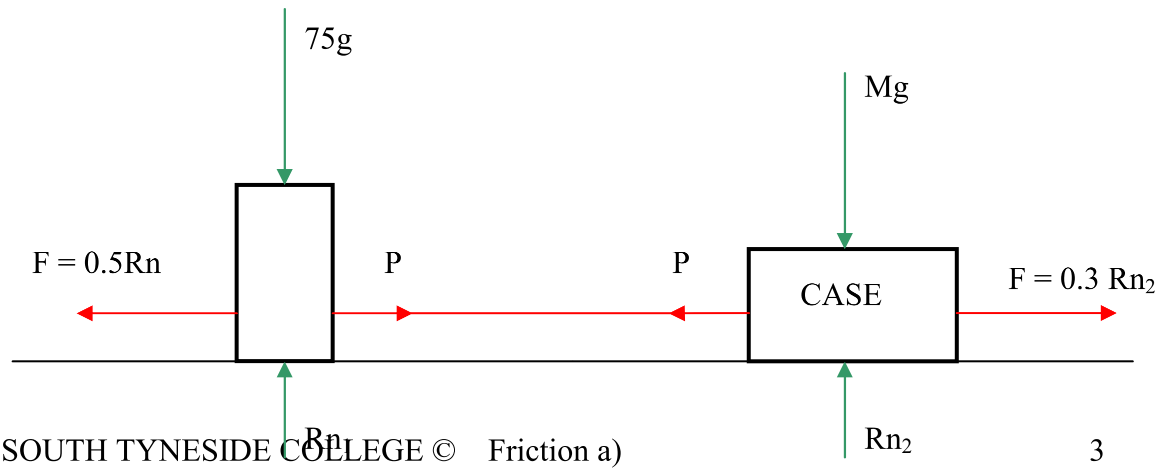
$R_n = Mg$

$$M = \frac{98}{9.81} = \underline{9.99\text{ kg}}$$

Example2 :-

A man has a mass of 75 kg, the coefficient of friction between the floor and his shoes is 0.5. A packing case stands on the same floor, the coefficient of friction between case and floor is 0.3.

What is the maximum mass of the packing case if the man can pull it along by means of a horizontal rope, without his shoes slipping on the floor.



$F = \mu R_{n1}$ and so for the man, without slip

$$F = 0.5 \times 75 \times 9.81 = 368 \text{ N} = P$$

This is the largest force that can be applied to the case:-

$$368 = \mu R_{n2} = 0.3 \times R_{n2}$$

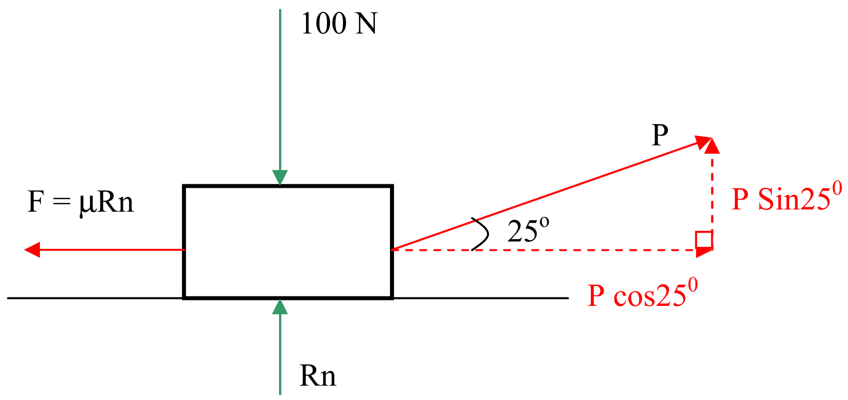
$$R_{n2} = Mg = \frac{368}{0.3} = 1227 \text{ N}$$

$$M = \frac{1227}{9.81} = 125 \text{ kg (max mass of packing case)}$$

So far so good, you should have met the laws of friction before. The situation becomes a little more complex though if either the applied force or the plane, or both, are inclined, since under these circumstances, **the normal reaction Rn is no longer equal to the weight of the body, Mg**.

Example 3

A body with a weight of 100 N lies on a flat surface, the coefficient of friction between the body and the surface is 0.3. Determine the force inclined at 25° above the horizontal, that will just cause motion.



First of all, resolve "P" into two mutually perpendicular components, normal and parallel to the plane. We can see from this that the normal component of “P” is upwards, and will therefore reduce the normal reaction, Rn. This in turn will decrease the frictional resistance.

Now we can apply the conditions of equilibrium, by summating the forces in two mutually perpendicular directions.

Summating the forces normal to the plane, upwards positive- which we can write shorthand as:-

$$\Sigma F_N \quad \uparrow +ve, \quad \Sigma = 0$$

$$R_n + P \text{ Sin } 25 - Mg = \text{zero}$$

$$R_n = 100 - 0.423P \qquad \text{(i)}$$

Summating the forces parallel to the plane, up the plane positive- which we can write shorthand as:-

$$\Sigma F_P \quad \rightarrow +ve, \quad \Sigma = 0$$

$$P \text{ Cos } 25 - \mu R_n = \text{zero}$$

$$0.906P = 0.3\, R_n \qquad \text{(ii)}$$

By substitution:-

$$0.906P = 0.3\,(100 - 0.423P)$$

$$= 30 - 0.127P$$

$$1.033P = 30$$

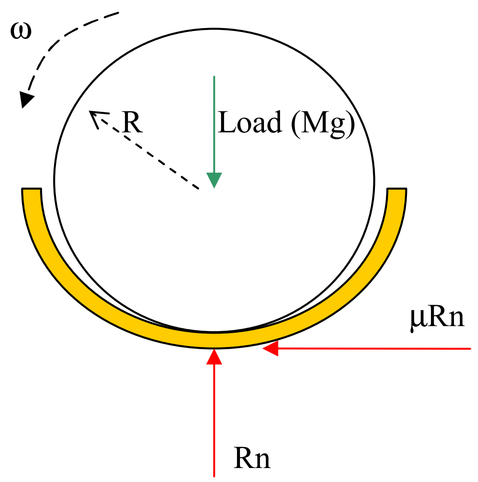
$$P = 29.05 \text{ N}$$

$$\longrightarrow$$

Note that if we had applied the force parallel to the plane we would have needed 30N, so by inclining our force **above** the plane, we have reduced the force necessary to move the body. This is because we have reduced the value of Rn. What do you think would happen if we pulled downwards, into the plane? Yes, the value of R_n would increase

Friction at bearings.

The illustration shows a shaft rotating in a plain bearing. The clearance between a shaft and its' bearing is very often only about 0.002 mm for each millimetre of shaft diameter. Hence the shaft diameter is very slightly less than the bearing diameter and it can be assumed that their centres coincide.



Let R = Shaft radius

M = Mass supported by the bearing

By summation of vertical forces :-

$$R_n = Mg$$

Friction Force $F = \mu R_n$

Friction Torque $Tf =$ Friction Force x Radius $= \mu R_n \, x \, R$

Power absorbed overcoming friction $= Tf \, x$ angular velocity $= \underline{\mathbf{Tf \times \omega}}$ [watts]

This is a simple but nevertheless effective approach to take. Let's tackle a few examples before we move on.

Example 4

The average load on each bearing of a 4-cylinder engine amounts to 10 kN. The coefficient of sliding friction is 0.01 and the engine runs at 2000 revs/min. Find the power lost in friction if there are five bearings each of diameter 50 mm.

Friction force at each bearing = μMg [approx]

= 0.01 x 10

= O.1 kN = 1OO N

Friction torque at each bearing = $\mu \, x \, Rn \;\; x \, R$

= 100 x 0.025

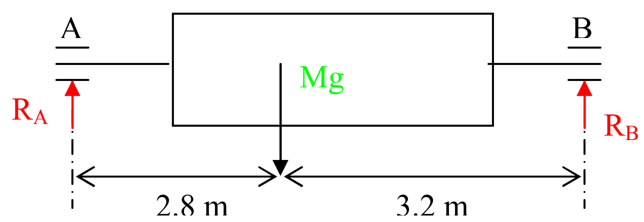
= 2.5 Nm

$$\text{Power lost in friction} = T\omega = \frac{(2 \, x \, \pi \, x \, 2000) \, x \, 2.5}{60} = 523 \text{ w}$$

Total friction loss = 5 x 0.523 = 2.615 kW

Example 5

A rotor of mass 3 tonne is 6 m long, the centre of gravity is 2.8 m from end A as shown. It is supported by bearings at each end. Calculate the power loss at the bearings given that the shaft diameter is 100 mm and the shaft speed is 500 rev/min. The coefficient of friction at A is 0.01 and at B is 0.015.



Solution.

In many questions, no matter how many bearings there are we will treat them as one bearing which takes all the load. This keeps things simple. However, we **cannot** do this here, because the load is not equally distributed, **and** the coefficient of friction is different at each bearing.

So first of all, we must find the reaction at each bearing.

Take moments about "B", clockwise positive.

$R_A x\ 6 - Mg\ x\ 3.2 = 0$. This gives $R_A = 15.7$ kN

Then since upward forces = downward forces, $R_B = 3000\ x\ 9.81 - R_A = 13.73$ kN

So, Power loss at A $= TF\ x\ \omega$

$= \mu R_A\ x\ R\ x\ \omega$

$= 0.01\ x\ 15.69x10^3\ x\ 0.05\ x\ (2\pi x500/60)$

$= \underline{410.9\ W}$

And Power loss at B $= TF\ x\ \omega$

$= \mu R_B\ x\ R\ x\ \omega$

$= 0.015\ x\ 13.73\ x10^3\ x\ 0.05\ x\ (2\pi x500/60)$

$= \underline{539.2\ W}$

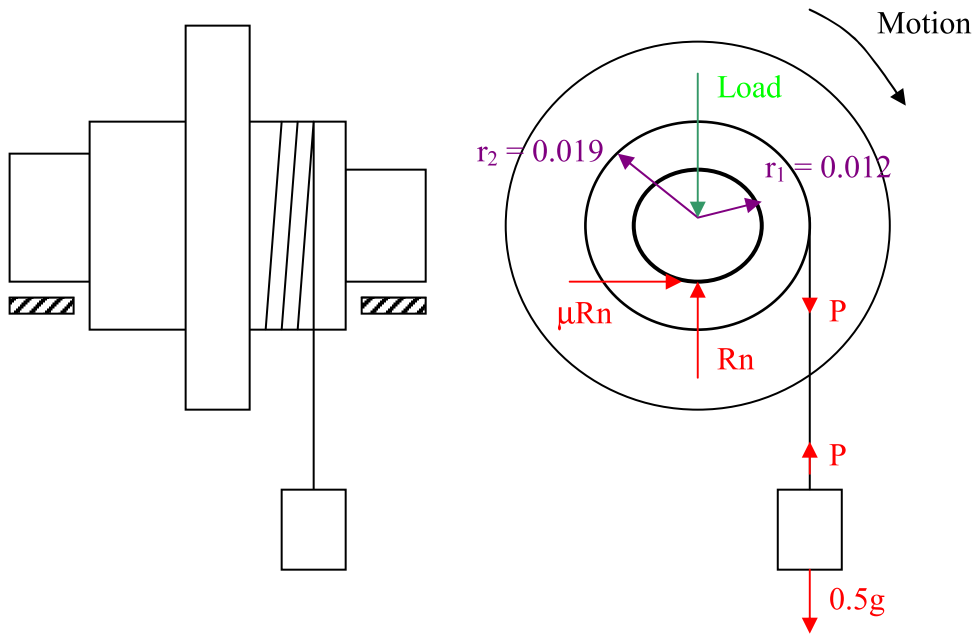
So total power loss $=$ 950.1 W.

Example 6

A flywheel mounted on a horizontal shaft 38 mm diameter runs in bearings 24 mm diameter. Flywheel and shaft can be turned at a slow uniform speed by a mass of 0.5 kg attached to a fine cord wound round the shaft. If the mass of the flywheel and shaft is 19 kg, what is the coefficient of friction for the bearings?

Solution.

As usual, first we should do a sketch. We might need more than one sketch here, just so we are sure we understand the problem.



Having drawn the sketch, we should put down the forces. Now consider the forces at the load. There are only two, the load itself, and the tension in the chord. In more complex problems we might need to summate the vertical forces, but here it is obvious that the tension in the chord is equal to 0.5g.

Now consider the forces at the bearing. There are two equal and opposite forces here, the load, and the normal reaction, Rn. We were told the weight of the shaft and flywheel is 19g, but **don't forget** that the shaft also supports the weight of the suspended 0.5kg mass.

So $\quad Rn \;=\; Load \;= (19 + 0.5)g = 191$ N

Hence friction torque = $\mu Rn \; x \; r_1$ (the friction torque acts at the **bearing** radius)

$$= \mu \; x \; 191 \; x \; 0.12$$

Since the speed is steady, we are in equilibrium, hence:

Sum of the Torques, clockwise positive = 0

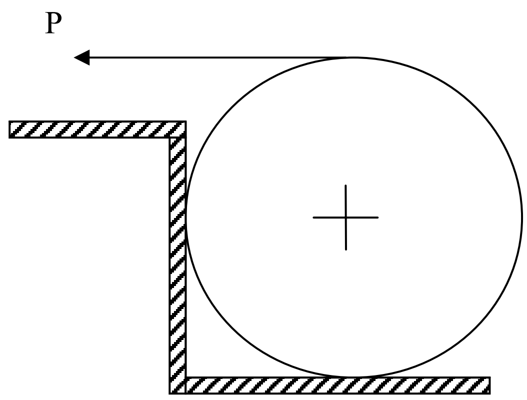
$P \times r_2 - \mu \times 191 \times 0.12 = 0$ (the torque due to the 0.5g acts at the **shaft** radius)

$0.5g \times 0.019 = \mu \times 191 \times 0.12$ Which gives $\mu = 0.04$.

One more example before we move on.

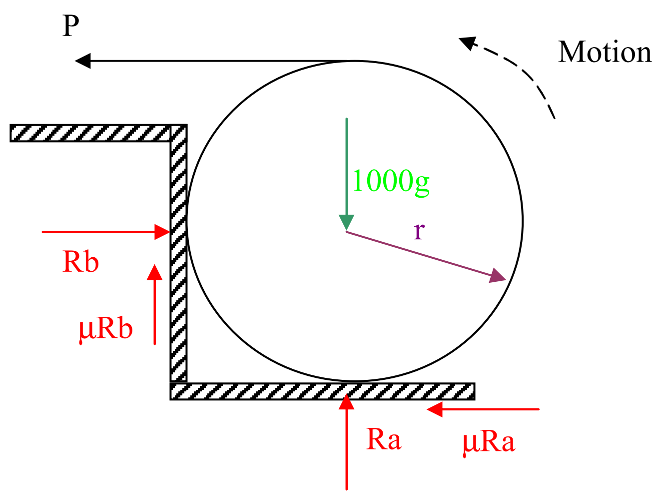
Example 7 (exam standard)

A shaft of mass 1 tonne is to be rotated by the application of a horizontal force P applied as shown. The coefficient of friction at all surfaces is 0.2. Calculate the minimum force required.



Solution

First, as usual, we should draw a sketch and put down the forces. Do **NOT** jump to the conclusion that Rn = Mg, as this is not the case. Take some time to consider the forces, and you should arrive at the diagram below.



Now we can see why Ra is not equal to mg, there will be an upwards frictional force at the vertical wall. It will pay you to put down the direction of motion, as shown, so you get the direction of friction correct. Remember, friction always opposes motion or the tendency to motion.

At first glance we have 4 unknowns here (Ra, Rb, P and r), but don't worry. Let us apply the conditions of equilibrium, and that will give us three equations.

Sum of the Torques, clockwise positive = 0

$$- P \times r + \mu \times Ra \times r + \mu \times Rb \times r = 0$$

The radius is common to all terms and will cancel, μ is 0.2, hence:

$$P = 0.2Ra + 0.2Rb \ldots\ldots.. 1$$

Horizontal Forces, $\Sigma F_H \rightarrow$ +ve, $\Sigma = 0$

$$- P - 0.2Ra + Rb = 0$$

Hence $\quad P = Rb - 0.2Ra \ldots\ldots\ldots\ldots 2$

Combining 1 and 2,

$$0.2Ra + 0.2Rb = Rb - 0.2Ra$$

$$0.4Ra = 0.8Rb, \qquad \text{So } Ra = 2\,Rb \ldots\ldots 3$$

Vertical Forces $\Sigma F_N \uparrow$ +ve, $\Sigma = 0$

$$Ra + 0.2Rb - 1000g = 0$$

Substituting for Ra from 3,

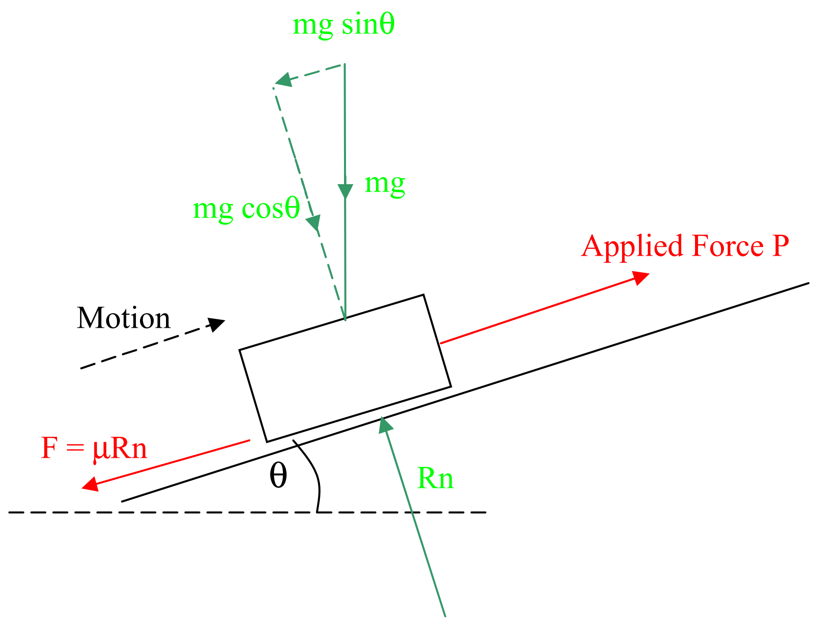
2.2Rb = 1000g, which gives Rb = 4459 N, and hence Ra = 8918 N.

Finally, putting these values in equation 2 gives;

$P = Rb - 0.2Ra = 4459 - 0.2 \times 8918 =$ **2675 N**

Friction on an inclined plane surface

The illustration shows a body moving with uniform velocity up an inclined plane under the action of a force "P". The line of action of the force is parallel to the plane. The expression for the value of "P" can be determined by equating forces "normal" and "parallel" to the plane. Note that this is easier than taking the forces horizontal and vertical because friction will always be parallel to the plane and the normal reaction will always be normal to the plane.



Sum forces "normal" to plane = zero, upwards +ve

R_n - Mg Cos θ = zero

R_n = Mg Cos θ (i)

Sum of forces parallel to the plane = zero, up the plane +ve

P - F - Mg Sin θ = zero

P = μR_n + Mg Sin θ ($F = \mu R_n$)

P = μ [Mg Cos θ] + Mg Sin θ [Substituting for R_n from (i)]

Summing the forces normal and parallel to the plane is one method we shall use to solve problems involving friction on an inclined plane.

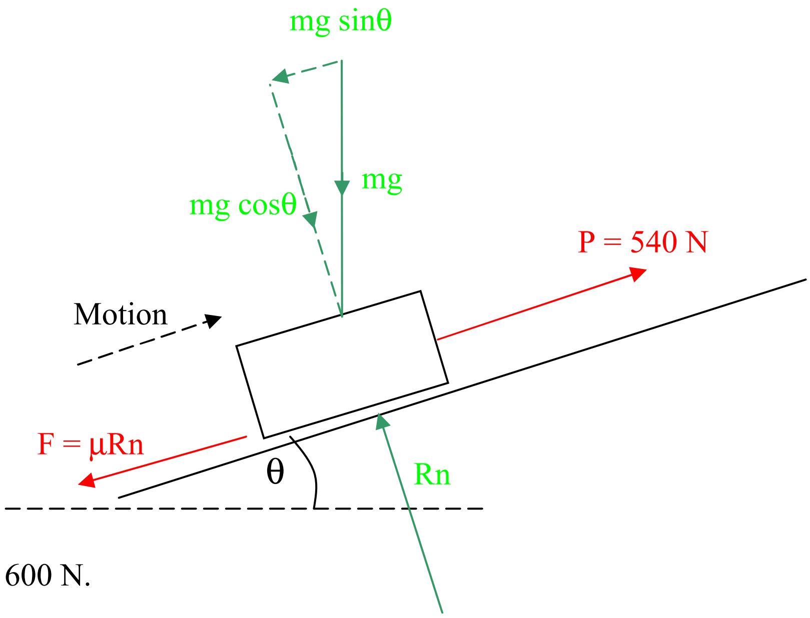
Example 8:-

A force of 540 N acting parallel to a plane inclined at 20° to the horizontal is required to just move a body of mass 613 kg up the plane. Find

(a) the coefficient of friction between the surfaces,

(b) the force parallel to the plane required to drag the body down the plane at a steady speed.

(c) If the surface of the plane can be lubricated to alter the value of the coefficient of friction, what should this value be for the body to be just on the point of moving down under its own weight?



(a) $Mg = 613 \times 9.81 = 600$ N.

$R_n = Mg \cos\theta = 600 \cos 20°$ N.

$F = \mu \, x \, 600 \cos 20°$ [acts down the plane]

Sum of forces parallel to the plane = zero:-

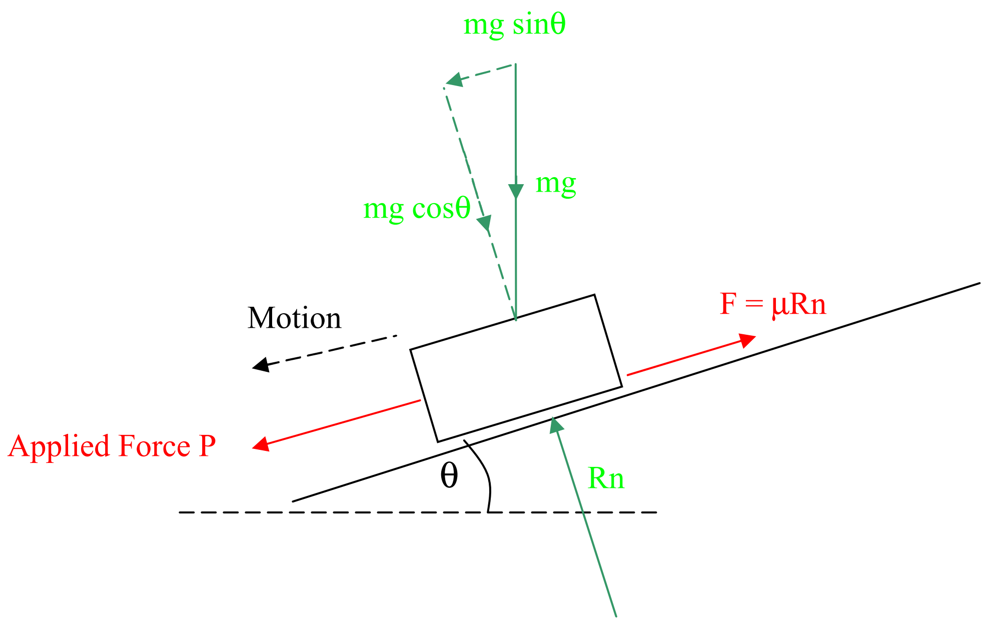
$540 - 600 \sin 20° - \mu \, x \, 600 \cos 20° = \text{zero}$

$540 = (600 \, x \, 0.342) + (\mu \, x \, 600 \, x \, 0.9397)$

$\underline{\mu = 0.593}$

(b)

As we are now moving the body down the plane, the friction force now acts up the plane.



Sum of forces parallel to the plane = zero (Note that Rn will be the same as before)

$$\mu \, x \;\; 600 \text{ Cos } 20^o - P - Mg \text{ Sin } 20^o \;\; = \text{zero}$$

$$P = 0\ 593 \, x \, (600 \, x \, 0.9397) - (600 \, x \, 0.342)$$

$$= 129.2 \text{ N}$$

(c) When the body is about to move down under its own weight, the component of the weight down the slope, just equals the friction force acting up the slope.

$$\mu \, x \, 600 \text{ Cos } 20^o = 600 \text{ Sin } 20^o$$

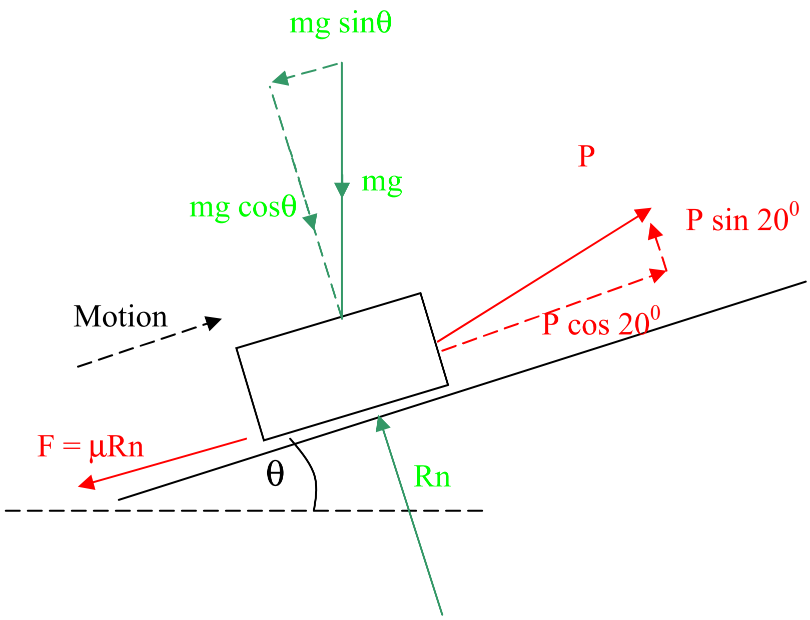
$$\mu \;\; = \;\; \frac{600 \text{ Sin } 20^o}{600 \text{ Cos } 20^o}$$

$$\mu = \tan 20 \qquad = \underline{0.364}$$

Example 9

A carriage of mass 1 tonne is to be pulled up a track inclined at 30° to the horizontal by a force P inclined at 20° to, and above, the track. Calculate the value of P if the effective coefficient of friction is 0.15.

Solution



Note that P has components parallel and normal to the track, and that it is no longer the case that Rn will equal mg cos θ.

Summating forces (in kN) parallel to the track, upwards positive,

P Cos 20 – μRn – Mg Sin θ = zero.

P x 0.9397 = 0.15 Rn + 9.81 x 0.5

P = 0.16 Rn + 5.22 ………………..1

Summating forces (in kN) normal to the plane = zero:-

Rn + P Sin 20 – Mg Cos θ = zero

Rn = 9.81 x cos 30 – P x Sin 20 = 8.5 – 0.342P…..

substitute for Rn in equation 1, P = 0.16 (8.5 – 0.342P) + 5.22

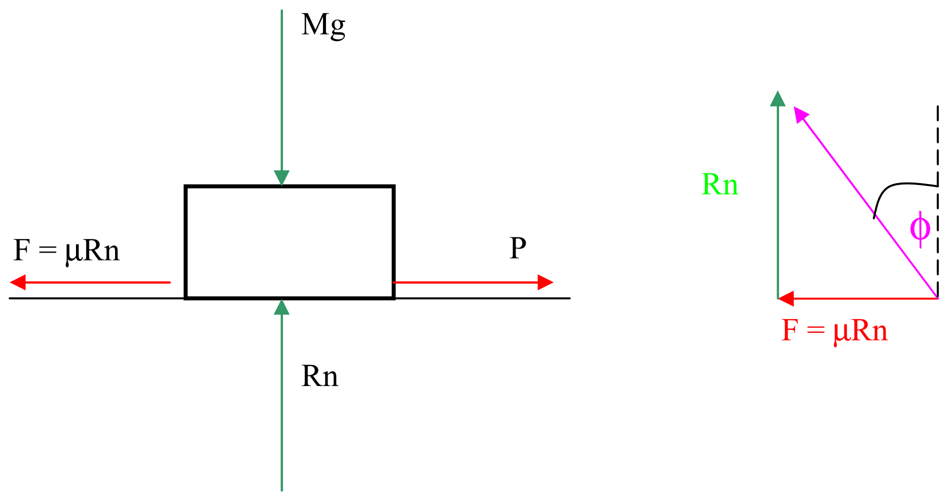
Which gives P = **<u>6.24 kN</u>**

THE LIMITING ANGLE OF FRICTION

Whilst summating the forces parallel and normal to the plane is a useful way of solving problems with friction on the inclined plane, there is another method which can sometimes be used and which is often quicker. This is the friction angle method.

A body of mass M resting on a horizontal plane surface, subjected to a gradually increasing force P applied to the body as shown induces a frictional resistance F. When the applied force P reaches a specific value such that any further increase will just cause motion, the frictional resistance F also attains a specific value at this point. This value is the maximum friction resistance.

The frictional resistance F, always opposes motion and will always be at right angles to the "normal" reaction R_n. If a resultant "R" of R_n and F is drawn it will always lean back from the direction of motion.
Let the angle of the resultant R = φ.



From:- $F = \mu R_n$ then $\mu = \dfrac{F}{Rn}$

But we can also see that Tan ϕ is also equal to $\dfrac{F}{Rn}$

So it follows that **Tan ϕ = μ**

Note! ϕ is known as THE ANGLE Of FRICTION and its value like that of μ depends on the nature of the contact surfaces.

The Friction Angle Method

In the problems considered so far, the body is in equilibrium under the action of **four** forces P, Mg, R_n and F. The friction angle method makes use of the resultant R of the friction force F and the normal reaction R_n, together with the angle φ between R and R_n. By combining two of the forces, Rn and μRn into one force R, we reduce the number of forces to **three**. We can therefore draw the vector diagram as **a three sided figure**, **a triangle of forces**, taking advantage of trigonometry to help us solve problems. This is a lot quicker than summing forces in two directions and producing two simultaneous equations.

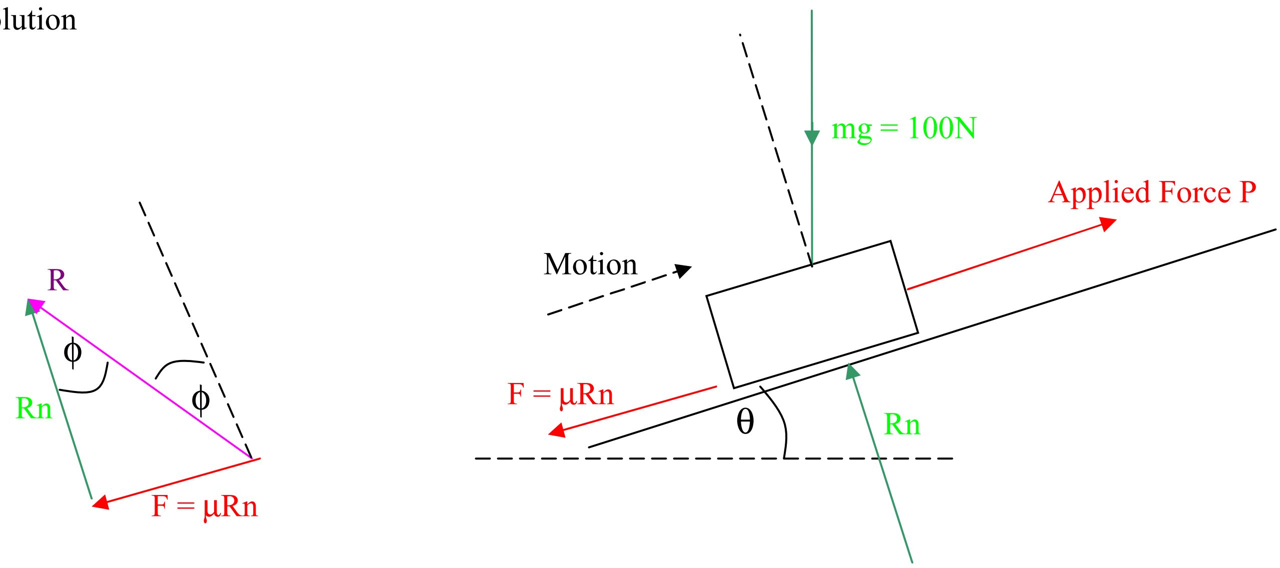
The friction angle method is limited in its application to systems where friction has reached its maximum value, and where there are no more than four forces. If we had a fifth force, (often by virtue of acceleration, there will be an inertia force, covered later in the Dynamics section), then the friction angle method is not so useful. Reducing five forces to four is not much help.

The friction angle method is best illustrated with an example.

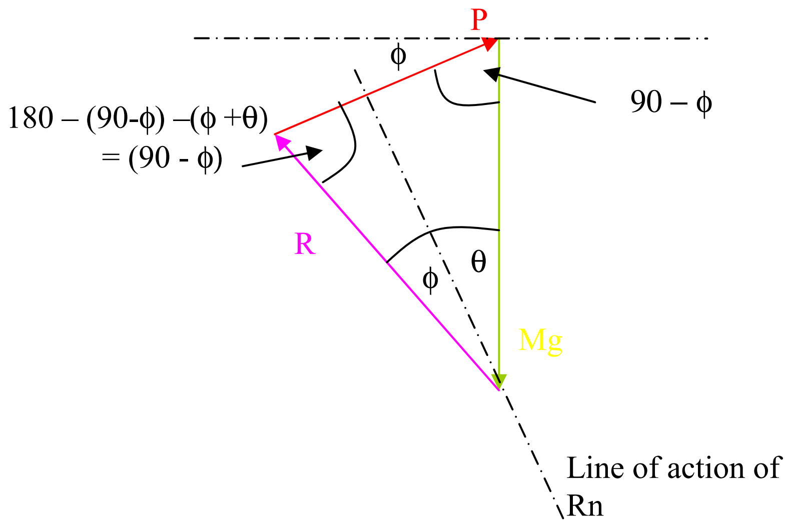
Example 10

Calculate the force necessary to pull a body of 100 N at a steady speed up a plane inclined at 30 degrees to the horizontal, if the coefficient of friction between the body and the plane is 0.2 and the force is applied parallel to the plane.
How much work is done in pulling it a distance of 5 metres up the plane?

Solution



The first step is to draw a sketch and put down the forces. Since we are using the friction angle method, we will combine the normal reaction Rn and the friction force μRn into one combined reaction R, which leans away from the direction at an angle of ϕ to the line of action of Rn. We can then draw our triangle of forces for Mg, Rn and P as shown below.



We know the value of ϕ, since $\mu = \tan\phi$. So $\phi = \tan^{-1} 0.2 = 11.31^0$

We can now use the sine rule to find P.

$$\frac{P}{\sin(\phi + \theta)} = \frac{mg}{\sin(90 - \phi)}$$

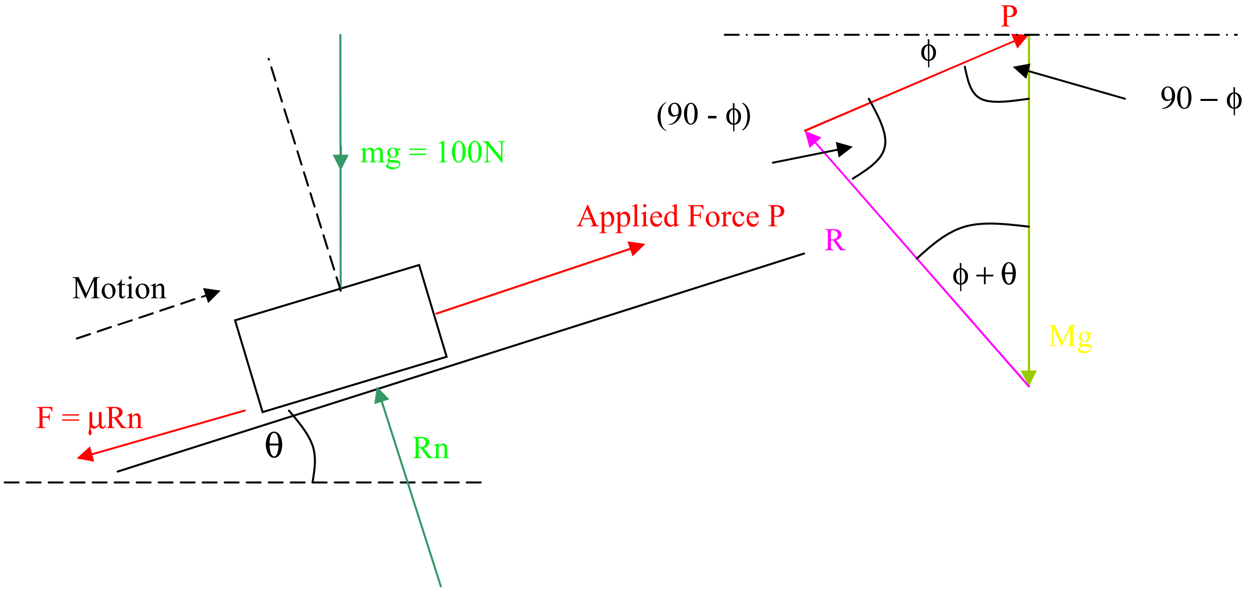
Which gives $P = \dfrac{100 \times \sin 41.31^0}{\sin 78.69^0}$

So P = 67.32 N

Then Work Done = Force *x* Distance moved = 67.321 *x* 5 = **<u>336.6 J</u>**

Example 11

A force of 336 6 N applied parallel to the plane, is required to move a body of 500 N up the plane. If the coefficient of friction is 0.2 find the angle of the incline.



So, do the sketch, draw the triangle of forces, and then apply the sin rule as before. Try it for yourself, you should get an answer of $\theta = 30^0$.

Example 12

A load of 224 N rests on a plane inclined at 15 degrees to the horizontal, the coefficient of friction between load and plane being 0.24. Find

(a) the magnitude *and direction* of the **minimum** force that will pull the load up the incline,

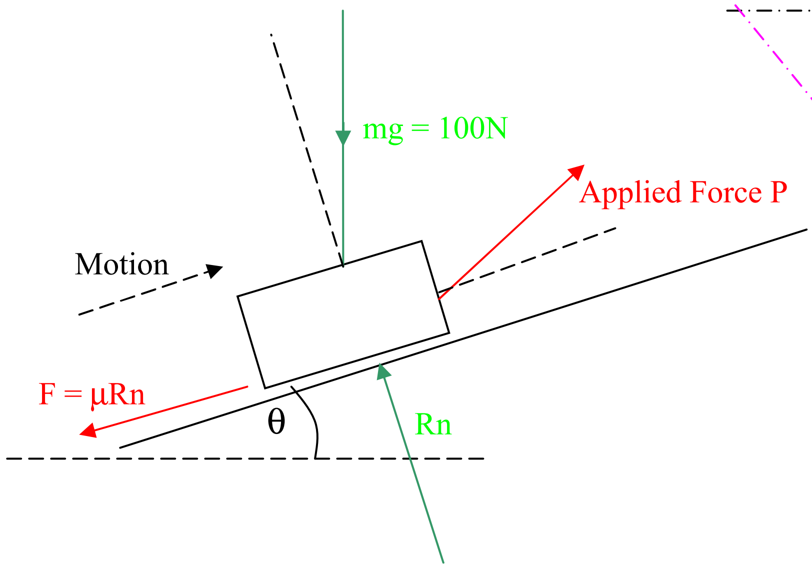
(b) the magnitude of the force required to pull the load up if it is applied horizontally.

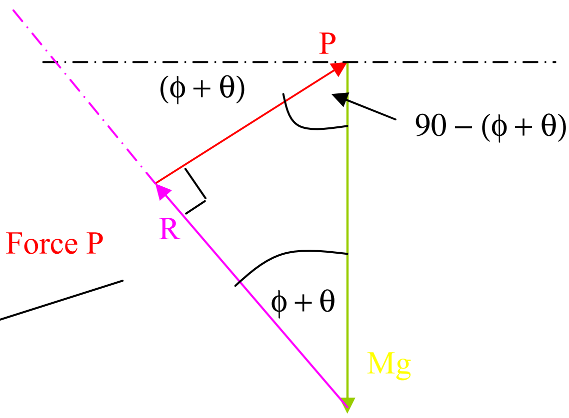
(c) If the plane is now inclined at 10 degrees to the horizontal, calculate the force to be applied parallel to the plane to pull the body down the plane.

Solution.

What we need to do here is draw the sketch and the vector diagram, and consider what direction of applied force "P" will give us a minimum value. We know the value of ϕ, since $\mu = \tan \phi$.

So $\phi = \tan^{-1} 0.24 = 13.469^0$





Close inspection of the vector diagram shows that P will be at a minimum when it is at 90 degrees to "R". Try moving P to any other angle and you will see that this is the case. Since the angle at the bottom of the triangle is $(\phi + \theta)$, this means our remaining internal angle is $(90 - (\phi + \theta))$. This in turn means that P must make an angle of $(\phi + \theta)$ above the horizontal, that is to say ϕ above the plane. We have just shown then, that **to move a body with minimum force we should apply that force at an angle equal to the friction angle ϕ, above the plane**.

To calculate this minimum force P is relatively straightforward, since we have a right angle triangle.

$P = 224 \sin(15 + 13.5) = 106$ N

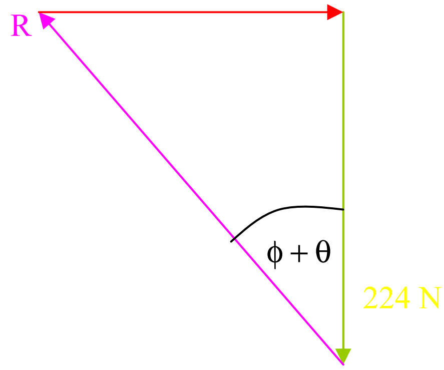
b) If the force P were applied horizontally, then our vector diagram would be as below;

P

$P = 224 \tan(12 + 13.5)$

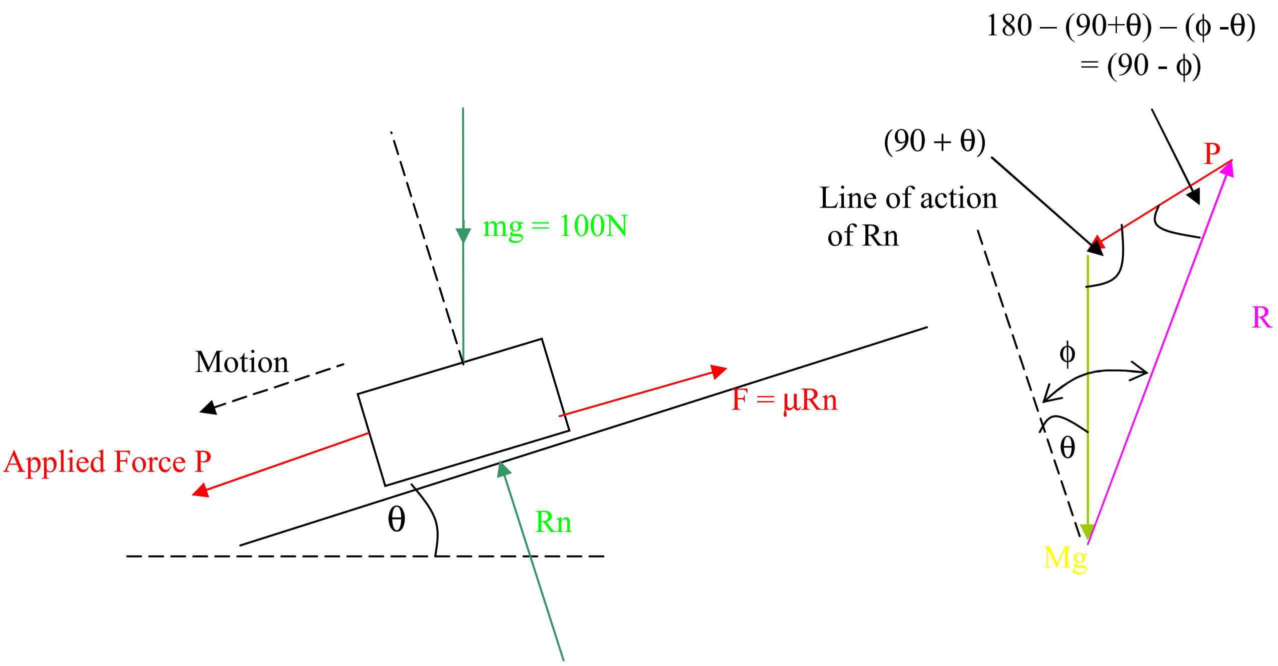
So P = **121.6 N**

As expected, this is greater than our minimum force



If we pull the body down the plane, now inclined at 10 degrees, then the direction of "R" is altered.

In this instance the resultant R has a component up the plane and is directed backwards to the direction of motion at an angle ϕ to the normal to the plane Rn as shown.



We can now use the sine rule to find P.

$$\frac{P}{\sin(\phi - \theta)} = \frac{224}{\sin(90 - \phi)}$$

Which gives $P = \underline{224 \times \sin 3.5^0}$

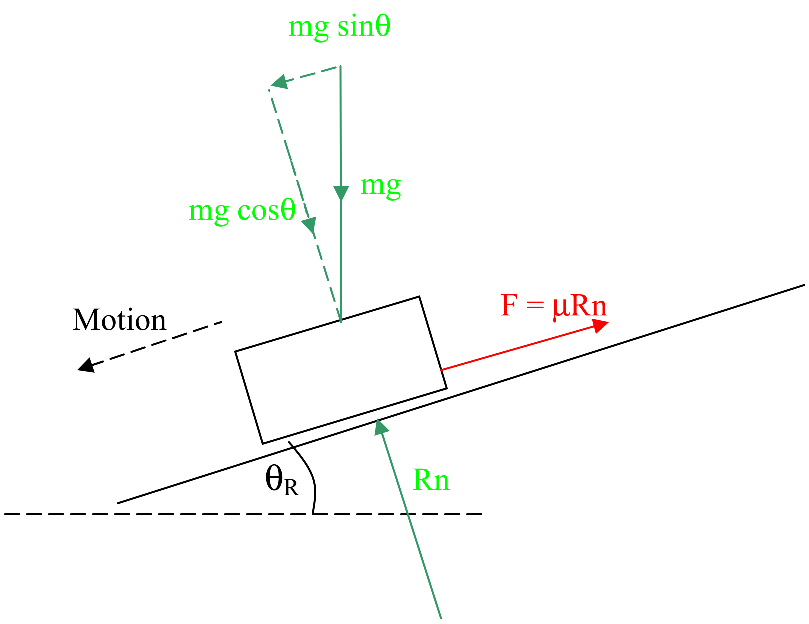
$\text{Sin } 76.5^0$

So <u>P = 14.1 N</u>

We will come back to the friction angle method again when considering friction at screw threads. Note that we have seen that the friction angle is quicker than the summation of forces method, particularly so if the vector diagram turns out to be a right angle triangle, for instance when the applied force is horizontal. Just one more observation about inclined planes before we move on to screw threads.

The Angle of Repose

If the angle of the plane, θ is gradually increased, there comes a point when the component of its weight acting down the plane, mg sin θ , is enough to overcome friction, and the body is just about to slide down the plane due to its own weight. This angle is known as the angle of repose.



Sum forces "normal" to plane = zero, upwards +ve

$$R_n - Mg \text{ Cos } \theta_R = \text{zero}$$

$$R_n = Mg \text{ Cos } \theta_R \qquad \text{(i)}$$

Sum of forces parallel to the plane = zero, up the plane +ve

μR_n - Mg Sin θ_R = zero

$$\mu Mg \cos\theta_R = Mg \sin\theta_R \qquad (Rn = Mg \cos\theta_R)$$

Hence $\mu = \dfrac{MgSin\theta_R}{MgCos\theta_R} = \text{Tan}\,\theta_R$

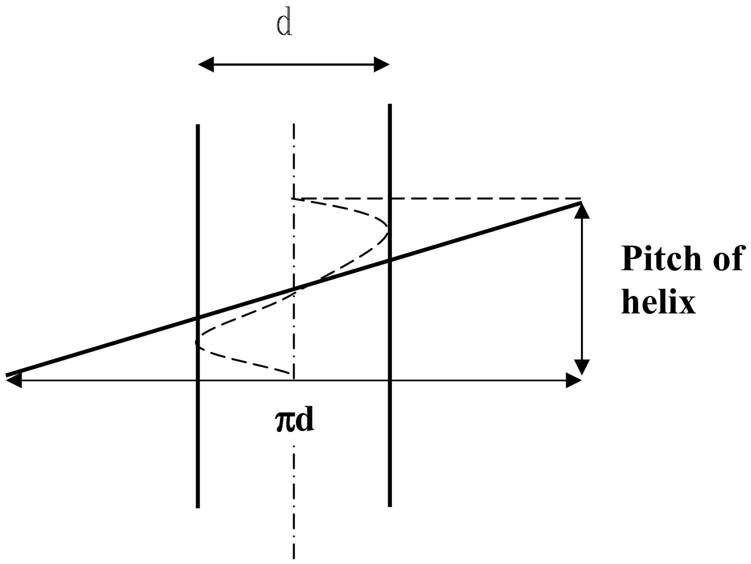
But we know from our previous work that $\mu = \tan\phi$

Hence $\mu = \text{Tan}\,\theta_R = \tan\phi$

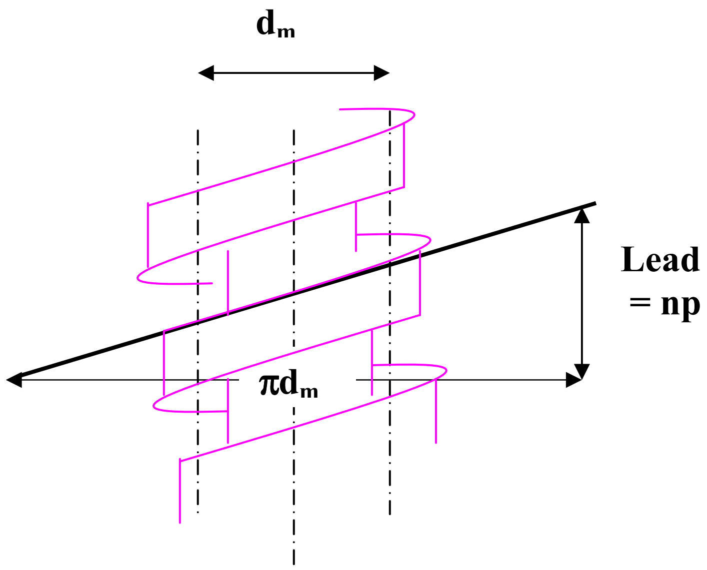
Hence **the angle of repose is the same angle as the friction angle**.

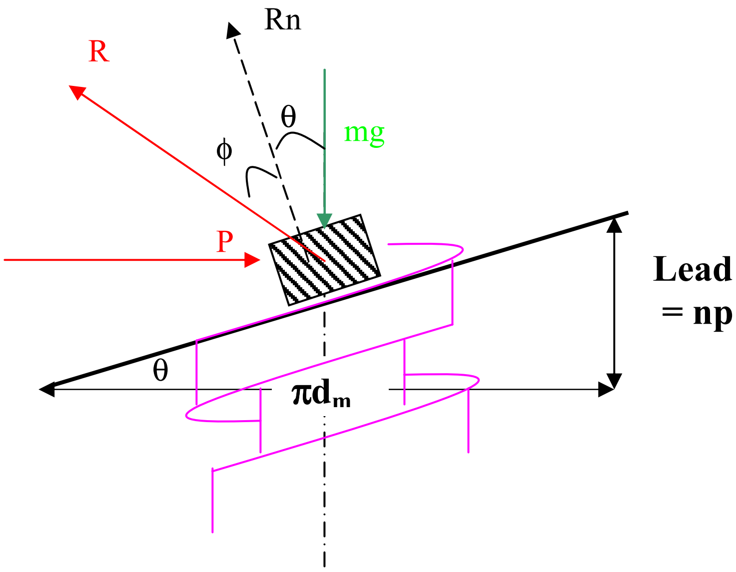
THE SCREW THREAD AS AN INCLINED PLANE

A helix of diameter d and pitch p can be developed as a straight line having an inclination of $\tan^{-1} p/\pi d$.



The square screw thread can be regarded as such a helix and can be developed as an inclined plane. The inclined plane formed will have a base of length πd where d is the mean diameter of the thread. The height of the plane is the lead of the thread, which is the number of starts "n" times the pitch.





The small shaded block represents a portion of the nut on the sliding thread.

Consider the nut to be forced along the incline of the screw thread, against an axial load Mg by an applied torque. This torque will effectively cause a force P to be applied horizontally at the mean radius of the thread.

We can therefore consider the screw thread as an inclined plane with the force to cause motion applied horizontally. Because the applied force is horizontal, the friction angle method of solution is particularly suited to the screw thread because we will obtain a right-angle triangle.

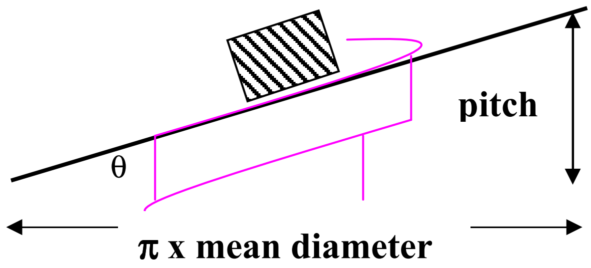
Example 12

A single-start square thread screw of 56 mm mean diameter and 12 mm pitch is required to raise a load of mass 910 kg by an effort applied at the end of an arm 610 mm effective radius. If the coefficient of friction is 0 2, determine the minimum force required.

Solution

The screw is single-start so, lead = pitch.

$$\tan\theta = \frac{p}{\pi d} = \frac{12}{56\pi} = 0.0682$$

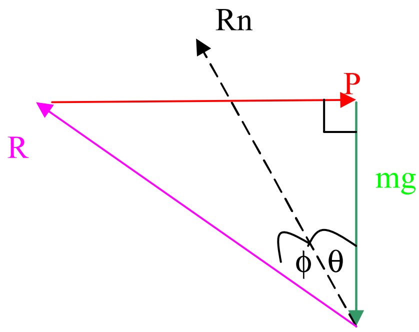


$\theta = 3.9^o$

$Mg = 910 \times 9.81 = 8.924$ kN.

From the triangle of forces

$$\tan(\theta + \phi) = \frac{P}{mg}$$



Tan $\phi = \mu = 0.2$. So $\phi = 11.3^0$

So $P = mg \tan 15.2^0 = 8{,}924 \times 0.2717$

So P = 2,425 N

Note that **this is the force required at the mean radius of the thread**.

Torque Applied at the end of the arm = Torque required at mean radius of thread.

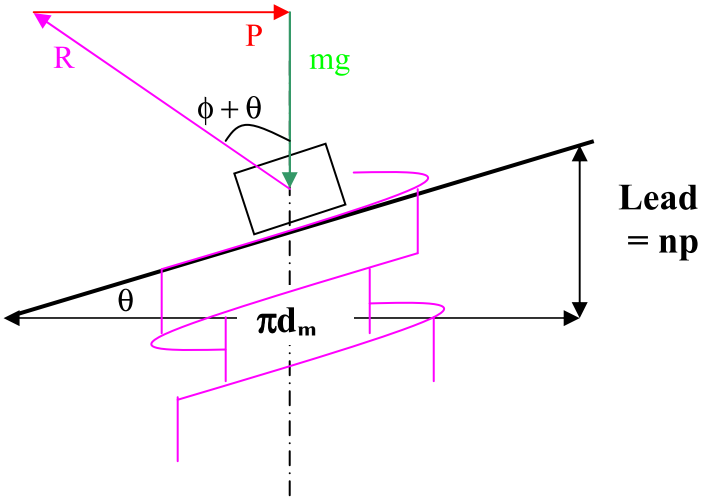
So $P_{ARM} \; x \; R_{ARM} = P_{THREAD} \; x \; R_{THREAD}$

$$\text{Effort required on arm} = \frac{2425 \; x \; 0.028}{0.61} = \underline{\mathbf{111\ N}}$$

Note that we could have done this question by summating forces parallel and normal to the plane, but that ***the friction angle method is particularly useful*** because since the applied force at the thread comes from a torque, it is always applied at right angles to the axis of the thread, so ***we always get a right angle triangle***.

EFFICIENCY OF A SCREW THREAD

From previous consideration of a horizontal force "P" acting up an inclined plane:-



$$P = Mg \tan(\phi + \theta)$$

This force is considered as being applied at the mean radius of the thread. This means that the distance moved by the force P to turn the nut one revolution is the mean circumference.
i.e. $2\pi \times R_{mean}$

For one revolution:-
Work Done = Force x Distance = $Mg \tan(\phi + \theta) \times 2\pi \times R_{mean}$

Work Achieved = Load x Distance Raised = $Mg \times$ Pitch $= Mg \times 2\pi \times R_{mean} \times \tan\theta$

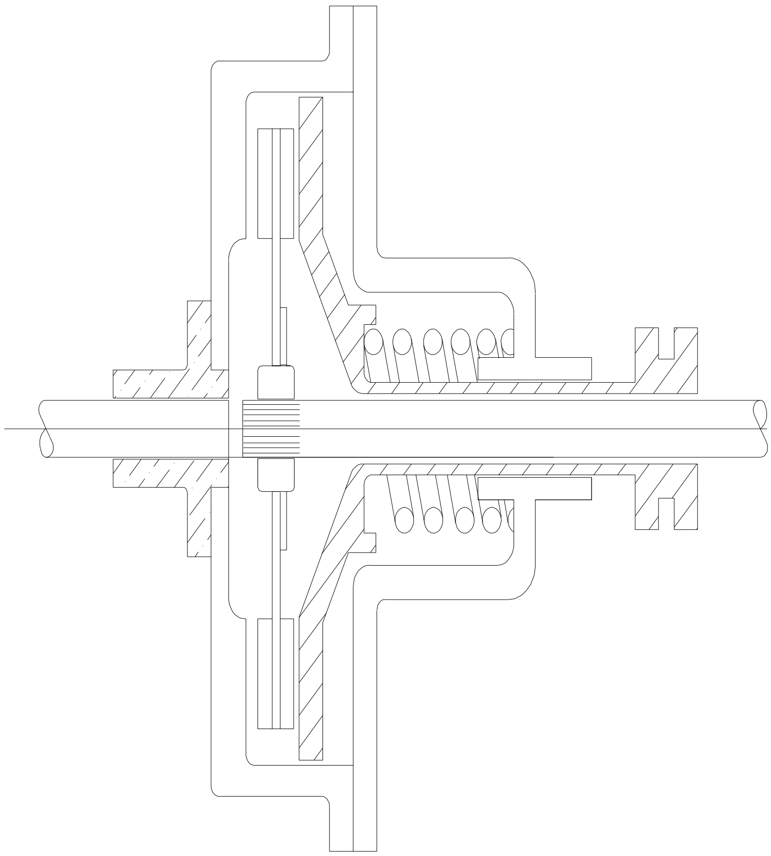
$$\text{Efficiency} = \frac{Mg \times 2\pi \times R_{mean} \times \tan\theta}{Mg \tan(\phi + \theta) \times 2\pi \times R_{mean}}$$

$$= \frac{\tan\theta}{\tan(\phi + \theta)}$$

The above expression shows that for maximum efficiency θ should be much bigger than φ. There are a number of practical points about this though. The larger θ, the greater the effort required to turn the nut through one revolution. Also, if θ is greater than φ, then from our work on the angle of repose, the load would slip back down if the effort is removed:- not much good for a car jack!!

Plate Clutches

Plate clutches are used where it is necessary to repeatedly couple and uncouple two co-axial shafts. The simple clutch consists of two flat circular plates each coupled to one section of shaft, the plates may be pushed together or parted according to whether or not power is to be transmitted.



Use is made of expendable friction material (Compound Asbestos Fibre being common). The force holding the plates together enables a friction torque to be transmitted from one shaft section to the other. Use of such a clutch gives a smooth take up of power and sudden torque surges are eliminated. Additionally, such a clutch can be used as a torque limiting device.

Assumptions Made

If the surfaces are coaxial and the force between them is normal to their plane, then the pressure on them will be uniform when they are new. However, since points at different radius have different velocity there will be some variation (very small) in the value of μ. This will apply particularly in the case of a flat pivot where the velocity at the center is zero. Thus the initial wear will not be uniform but will increase with radius and be greatest at the outside. This variation in the rate of wear alters the pressure distribution, increasing the intensity towards the center. This in turn alters the rate of wear so that the variations in wear and pressure are continuous. Ultimately the pressure may become uniform again though the surfaces will not necessarily regain their original profile. In view of the foregoing it is usual to assume **either**

(1) that the pressure is uniform, or

(2) that the rate of wear is uniform.

For **Uniform Pressure**, it can be shown that:

Torque = $\frac{2}{3}\mu W\left(\frac{r_1^{\,3}-r_2^{\,3}}{r_1^{\,2}-r_2^{\,2}}\right)$ Per working surface

For **Uniform Wear**, it can be shown that:

Torque = $\mu W\left(\frac{r_1+r_2}{2}\right)$ Per working surface

Where:

T = Torque (Nm)

μ = coefficient of friction

W = Axial Load (N)

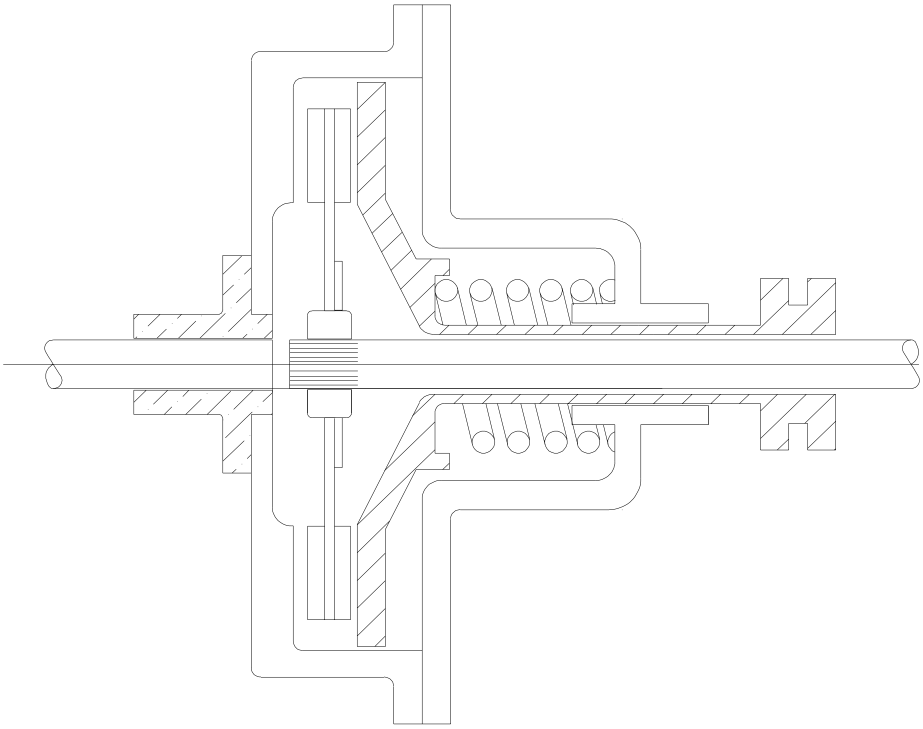
R_1 = Outer radius of friction surfaces

R_2 = Outer radius of friction surfaces

Both of these formulas will be given for the Class One Mechanics Exam. The first assumption gives a slightly higher value of frictional torque and may be used in the estimation of power loss in a bearing. Clutch design is usually based on the second assumption.

Multi Plate clutches and Multi Collar Bearings

The friction torque transmitted by a clutch is dependent on the inner and outer diameters. If it is necessary to transmit greater friction torque without making having too large a diameter, then a multi collar clutch is used. A similar approach is used with collar bearings to reduce the specific load of the transmitted thrust. The friction torque transmitted by a **five** plate clutch would be **four** times that transmitted by a simple clutch.



Three Plate Clutch (Single plate clutch, both sides effective)

Note that if there are three plates then there are two working surfaces, and for five plates there are four working surfaces, and in general **terms if there are n plates there are (n-l) working surfaces**. A three plate clutch is sometimes referred to as a "single plate clutch, both sides effective".

It is the coefficient of static friction (stiction) that is most appropriate until slipping occurs, then the coefficient of sliding friction applies.

Example 13

A plate clutch has 3 discs on the driving shaft, and 2 discs on the driven shaft, providing 4 pairs of working surfaces, each of 240mm OD and 120mm ID. Assuming uniform pressure, find the total spring loads pressing the plates together to transmit 25kW at 1575r.p.m. Take the coefficient of friction, $\mu = 0.3$.

Solution

$$\text{Power} = 2\pi \text{ N T}$$

$$\text{Friction Torque} = \frac{\text{Power x } 60}{2\pi \text{ N}} = \frac{25 \text{ x } 10^3 \text{ x } 60}{2\pi \text{ x } 1575} = 151.6 \text{ Nm}$$

For 1 working face:-

$$\text{Friction Torque} = \frac{2}{3}\mu \text{ w} \frac{[Ro^3 - Ri^3]}{[Ro^2 - Ri^2]}$$

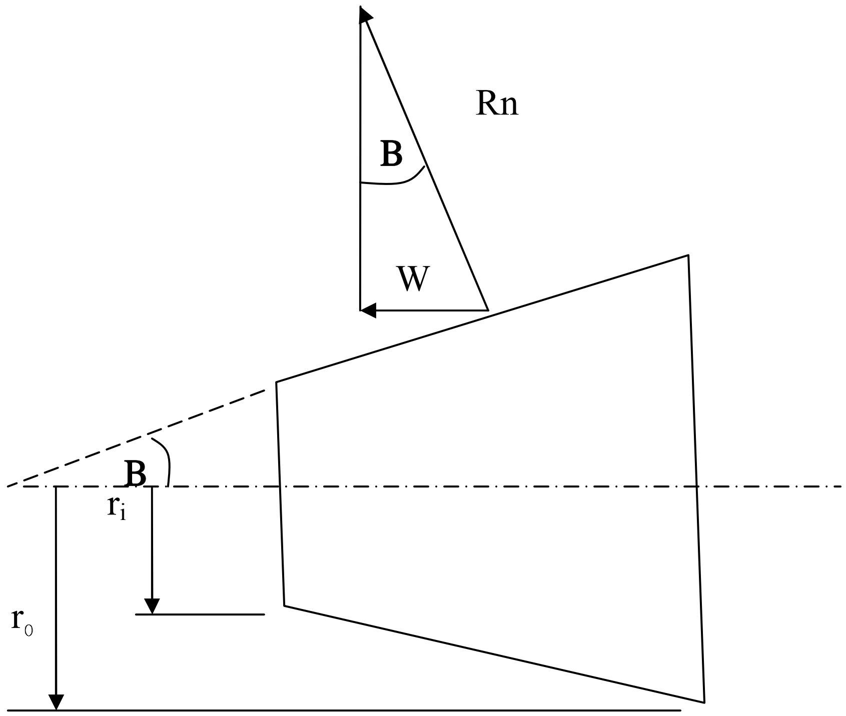
For 4 working faces

$$151.6 = 4 \text{ x } \frac{2}{3} \text{ x } 0.3 \text{ x W x } \frac{[0.12^3 - 0.06^3]}{[0.12^2 - 0.06^2]}$$

Which gives W = <u>**1354 N**</u>

Conical Clutches and Bearings

On this type of device the contact faces are inclined at an angle to the shaft axis, giving a wedge effect. This results in a higher than normal force at the contact faces for the same axial load.



From the above,

$$Rn = \frac{W}{\text{Sin } B}$$ (Which means Rn is in excess of the axial load 'W')

So Torque = μRn x Mean Radius = $\dfrac{\mu Rn\,(r_0 + r_i)}{2}$

Substituting for Rn gives,

$$\text{Torque} = \frac{\mu W\,(r_o + r_I)}{2\sin B}$$

Note that this assumes **Constant Pressure** across the clutch surface

Example 14.

In a conical friction clutch the effective diameter of the contact surfaces is 80mm, the semi-apex angle of the cone is 150 m is 0.3 and the axial force transmitted is 180N. Calculate the Power that can be transmitted at 1000r.p.m.

Solution

$$\text{Torque} = \frac{\mu W (r_o + r_I)}{2\sin B} \qquad \text{Here, } \frac{(r_o + r_I)}{2} = \text{effective } \textbf{radius} = 0.04\text{m}$$

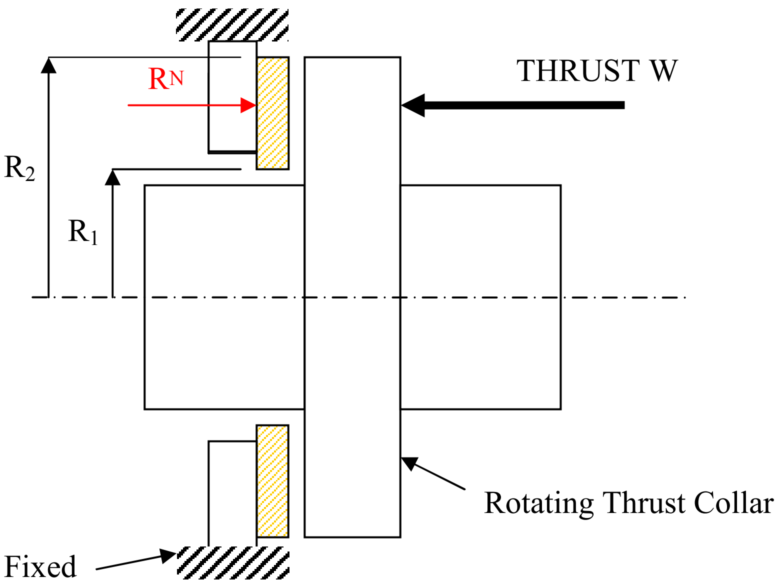
$$\text{Torque} = \frac{0.3 \text{ x } 180 \text{ x } 0.04}{\text{Sin } 15^0} = 8.345 \text{ Nm}$$

$$\text{Power} = T\omega = 8.345 \text{ x } \frac{1000\text{x } 2\pi}{60} = \underline{\mathbf{873.9\ W}}$$

Friction at a Single Collar Thrust Bearing – Simple Approach

In taking a "simple" approach to deriving an expression for the friction torque developed at a single collar thrust bearing, we assume that:-

1. Pressure is uniform across the thrust faces
2. The thrust acts at the mean radius



Friction at Bearing Surfaces $= \mu R n = \mu W$

Friction Torque $= \mu R n \times$ Mean Radius $= \mu W \dfrac{(R_1 + R_2)}{2}$

The syllabus states that you should be able to derive this equation. You will notice that it is the same expression as that for the Torque transmitted by a clutch assuming constant pressure.

Self Assessed Questions for you to try. (Answers given)

1. A body with a mass of 50 kg requires a force of 250 N to pull it up a certain incline, whilst to pull it down the same incline, the force required is 60 N. Find the angle of the incline and the coefficient of friction.

   Ans: 11.2^{o} and 0.323

2 A body of 100 kg mass is pulled up an incline of 0.7 radian by a rope running parallel to the incline and passing over a drum of 457 mm diameter. The coefficient of friction between the contact surfaces is 0.25 and an effort of 450 N applied at the end of a crank handle on the drum shaft just causes uniform motion up plane without acceleration. Calculate the effective radius of the crank handle.

   Ans: 416.2 mm

3 A body with a mass of 200 kg rests on a rough horizontal surface and μ between body and plane is 0.3. Calculate the value of a force P, which if inclined upward at 10^{o} to the plane, would just cause motion at uniform velocity.

   Ans: 566 N

4 To the crest of a plane inclined at 30^{o} to the horizontal is connected another plane which is horizontal. A body with a mass of 25 kg rests on the horizontal plane and on the inclined plane a body mass 75 kg. The bodies are connected by a cord parallel to both planes and passing over a light frictionless pulley at the junction of the two planes. Given μ in both cases is 0.25, determine
(a) the acceleration of the system when the body on the incline is released
(b) the tension in the cord when motion commences.

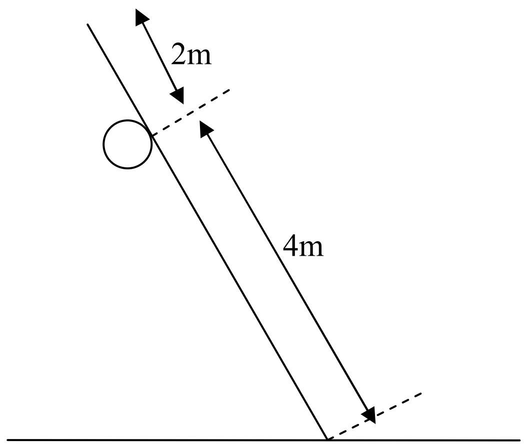
   Ans: (a) 1.472 m/s^2 (b) 98.1 N

5 A ladder inclined at 60^{o} to the horizontal is resting against a smooth vertical wall. The length of the ladder is 8 m and its mass 50 kg with its centre of gravity 3 m from the lower end. A man with a mass of 70 kg is standing on the ladder at a distance of 1.5 m from the top. What is the least coefficient of friction between ladder and ground to prevent the ladder from slipping.

   Ans: 0.365

6 A regular wooden beam with a mass of 100 kg and 6 m long is placed with its base on the ground and resting against a pipe 2 m from the top of the beam as shown. A piece of machinery with a mass of 300 kg is to be slung from it. The beam makes an angle of 30^{o} to the vertical and the coefficient of friction is 0.2 at the base and 0.3 at the pipe. Calculate the highest point along the beam that the machinery can be hung without the beam slipping.

Ans: 1.458 m from base



7 A single start square thread has a mean diameter of 50 mm and a pitch of 12.5 mm. The coefficient of friction between the thread and the nut is 0.15. Find the efficiency of the thread when lifting a load of 4.5 kN and the torque required.

Ans: 34.2% 26.15 Nm

8 A watertight door of mass 300 kg is lifted by a single start square thread having a core diameter of 20 mm and a pitch of 20 mm. If the torque required to raise the door is 15 Nm find the coefficient of friction between screw thread and nut.

Ans: 0.1195

9 A single plate clutch, both sides effective, has to transmit 30kW at 3000 r.p.m. The coefficient of friction at the working surfaces is 0.3. Working to a pressure limit of $83kN/m^2$, find:-

(a) The dimensions of the thrust faces if the O.D. = 1.5 I.D.

(b) The value of the axial thrust.

Ans: Ri = 78.75mm, Ro = 118.1mm, W = 1.617kN