CONSERVATION OF ENERGY AND CONSERVATION OF MOMENTUM

You should already be familiar with the **principle of conservation of energy**. This says:

***Energy cannot be created or destroyed but can only be changed from one form to another.***

For example when a moving body is brought to rest by the application of a brake, its kinetic energy is transformed to heat energy at the brake. Note that conservation of energy does not mean that "useful" forms of energy are conserved. For instance, not all a body's potential energy is always converted to kinetic energy. Some may be effectively "lost" from the system in the form of heat, light, noise, deformation etc. So, remember that whilst we will get overall conservation of energy, what we do **not** always get is conservation of **useful** forms of energy. Before looking at conservation of energy, let us remind ourselves of the most common forms of energy we come across in Mechanics.

Potential Energy (P.E.)

This is energy possessed by a body by reason of its position relative to some fixed datum.

The work done in raising a body of weight Mg through a height "h" is Mg *x* h (Remembering that work done = Force *x* Distance moved). This energy is stored in the body, being released if the body is allowed to fall to its original datum. It follows that at height "h" the Potential Energy of the body is also Mgh. With Mg in Newton and h in metres, the potential energy of the body will be in [Nm] or [joules], as for work done. We prefer to use the Joule to indicate work or energy as the units of Torque are also Newton-metres.

Kinetic Energy (K.E.)

This is energy possessed by a body by reason of its motion. We can derive an expression for Kinetic Energy in the same way as that used for potential energy, i.e. we can **relate it to work done**.

Remembering that work done = Force *x* Distance moved and that Force = mass *x* acceleration, Consider the work done in accelerating a body of mass M from rest to a speed v [m/s] over a distance s [m].

The average acceleration required is given by:- $v^2 = 2as$

$$a = \frac{v^2}{2s}$$

The average force required to produce this acceleration is:

$P = M \times a$ So substituting for 'a' gives

$$P = M \times \frac{v^2}{2s}$$

The work done by this force P in moving a distance s is

$$P \times s = \frac{M.v^2}{2s} \times s$$

$$= \tfrac{1}{2} M.v^2$$

The expression $\frac{1}{2}M.v^2$ is known as the "kinetic energy" of the body at the speed of "v". It is the energy possessed by the body by virtue of its mass and its speed.

i.e. kinetic energy = $\frac{1}{2} M.v^2$

Kinetic energy is a ***scalar*** quantity since it is not necessary to take the direction of the speed into account.

Example 1

A hammer of mass 14 kg is attached to an arm carried on frictionless pivots and swings in a circle of one metre radius. The hammer is held in a horizontal position and on release strikes a specimen at the lowest point of the swing.

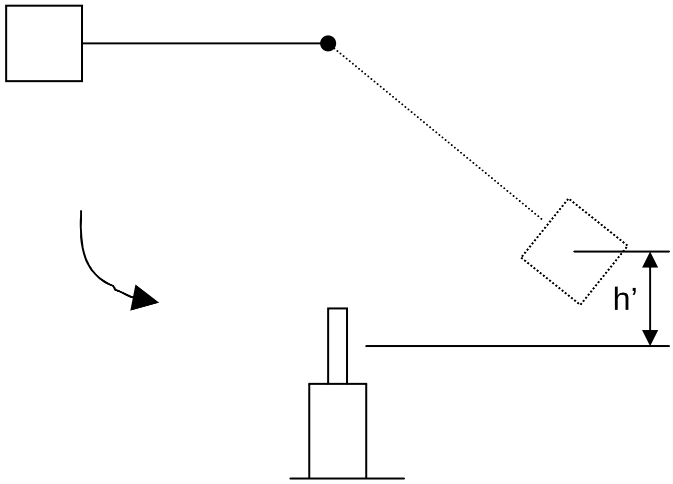
If the specimen requires 60 [J] of energy to fracture, find:

(a) the velocity at impact,

(b) the height to which the hammer will rise after fracturing the specimen.

Solution

First, draw a sketch.




Just before impact:-

loss of potential energy = gain of kinetic energy,

$$Mgh = \tfrac{1}{2}Mv^2$$

$$v = \sqrt{2gh} = \sqrt{(2 \times 9.81 \times 1)} = 4.43 \text{ m/s.}$$

Kinetic energy before impact = initial potential energy

$$Mgh = 14 \times 9.81 \times 1 = 137.3 \text{ [J]}$$

At impact, 60J of energy are lost, so that after impact:-

kinetic energy = 137.3 - 60 = 77.3 [J]

This must also be the amount of energy that is converted to potential energy, Mgh'

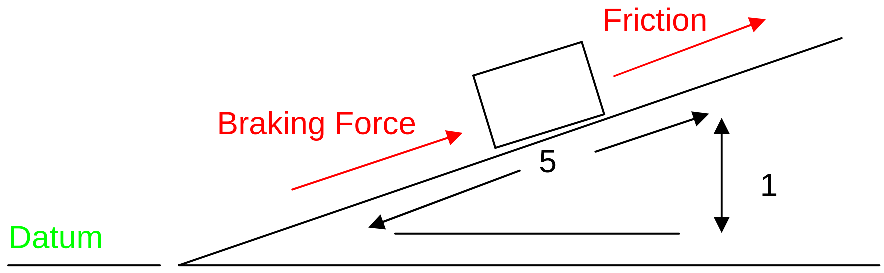
$$\therefore h' = \frac{77.3}{9.81 \times 14} = \underline{0.563 \text{ m}}$$

Example 2

A car of mass 1,125 kg descends a hill of 1 in 5 (sine). Calculate, using an energy method, the average braking force required to bring the car to rest from 72 km/hr in 50 m. The frictional resistance to motion is 250 N.

Solution

First, draw a sketch




The total energy of the car at 72 km/hr is the sum of its kinetic and potential energies. This energy must be lost due to the **work done** (W.D.) by the braking force and frictional resistance force acting through a distance of 50 m. Before getting too involved in the mathematics of a question like this it is always a good idea to make a statement in words about what is happening.

***Work done by braking and friction*** = ***Kinetic and Potential Energy lost***

When calculating Potential Energy, choose the lowest point as your datum and this will avoid any problems of having negative potential energy. From the information given about the slope, (1/5) = (X/50) , so the height corresponding to 50 m on the slope = 10 m.

Initial velocity of the car $v = \dfrac{72 \times 10^3}{3600} = 20$ m/s

P.E. = ½Mg.h = ½ $x$ 1,125 $x$ 9.81 $x$ 10 = 110.25 [kJ]

K.E. = ½M.$v^2$ = ½ $x$ 1,125 $x$ $20^2$ = 225. [kJ]

total energy = 335.25 [kJ]

W.D. by braking force = P $x$ 50 $x$ $10^{-3}$ [kJ]

W.D. by frictional resistance = 250 $x$ 50 $x$ $10^{-3}$ [kJ]

So work done by braking and friction = Kinetic and Potential Energy lost

P $x$ 50 + 250 $x$ 50 = 335.25 $x$ $10^3$

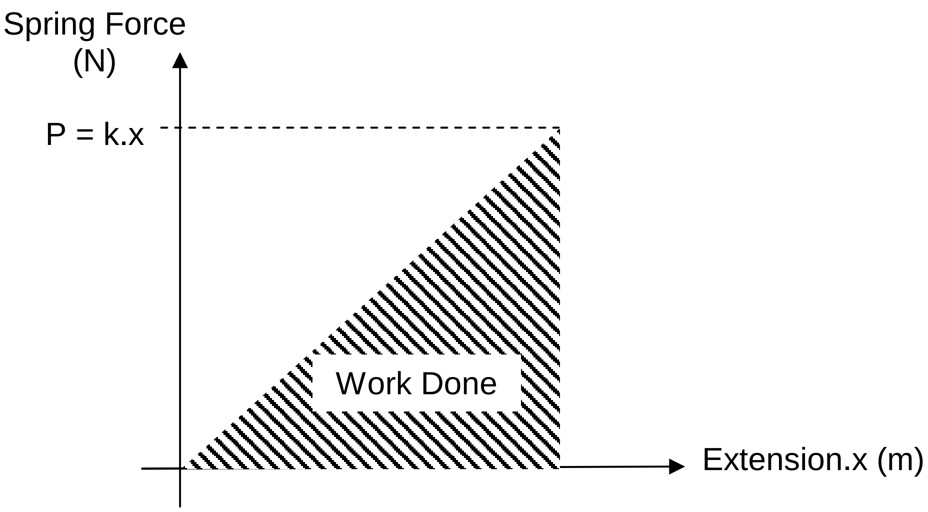
P = 6,455 [N]

## Strain Energy

The work done in compressing or stretching a spring is stored as strain energy in the spring (provided that there is no permanent deformation).
The stiffness of a spring is the load per unit extension and is constant within the working range of the spring.
If "k" is the stiffness, the load P required to produce an extension "x" is given by

$$P = k.\ x$$

This gives a straight-line relationship of P against x




Consider a load gradually applied to a spring so that it varies from zero to a maximum value P and produces a maximum extension x.

$$\text{W.D.} = \text{average load} \times \text{extension}$$

$$= \tfrac{1}{2}\,P\,x \quad = \tfrac{1}{2}\,k.\ x.\ x \quad = \tfrac{1}{2}\,k.x^2$$

Strain Energy [U] = W.D.

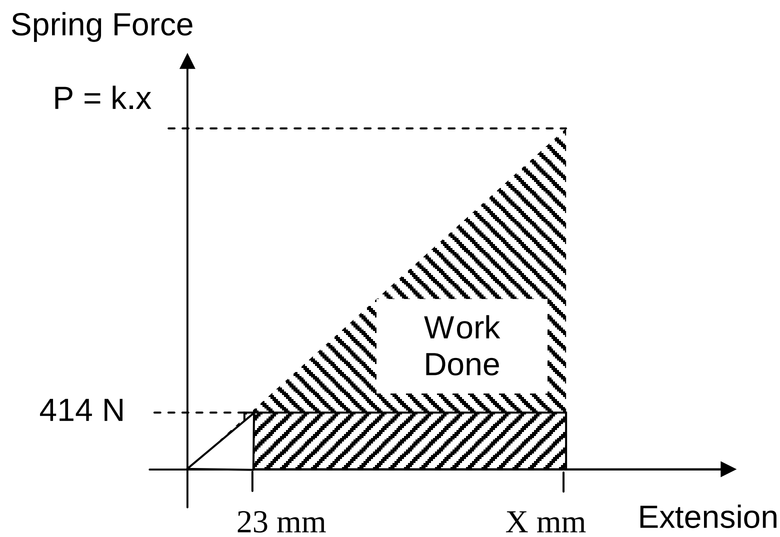
$$U = \tfrac{1}{2}\,.P\,x \;= \tfrac{1}{2}\,k.\ x^2$$

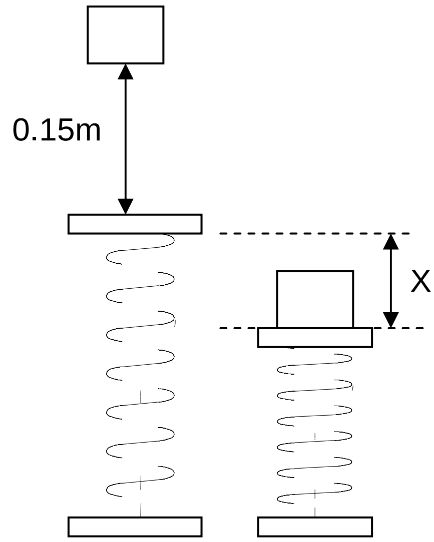
Example 3

A spring of stiffness 18 kN/m is installed between plates so that it has an initial compression of 23 mm. A body of mass 4.5 kg is dropped 150 mm from rest on to the compressed spring. Find the further compression of the spring neglecting loss of energy at impact.

Solution

In a question like this it is best to draw both ***a sketch and a graph*** of spring force against extension, as this will help us to obtain an expression for the work done.




Let X = further compression of the spring [m]

initial spring force = k *x* initial compression

$= 18 \times 10^3 \times 0.023 = 414$ N

maximum force = $414 + (18 \times 10^3 \times X)$ N

Putting down in words what the exchange of energy is,

loss of Potential Energy of the 4.5 kg mass = Work done on the spring

Loss of P.E. = Mg.h = $4.5 \times 9.81(0.15 + X)$ [joule] ……………………..1

From the graph,

work done on the spring = area OABC = 414 $x$ X + ½ (18. $x$ $10^3$ $x$ X ) $x$ X……2

This is the area of the shaded rectangle plus the shaded traiangle.

Equating the loss of potential energy to the work done, equations 1 and 2 then:

$$4.5 \times 9.81(0.15 + X) = 414X + 9000X^2$$

Rearranging this equation gives a quadratic, $X^2 + 0.0411X - 0.000736 = O$

Solving this quadratic in the usual way gives X = 0.0136 m = <u>13.6 mm</u>

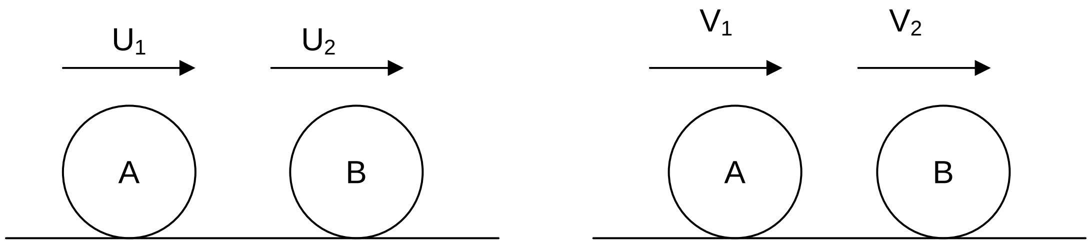
CONSERVATION OF MOMENTUM

Linear momentum is the product of mass and velocity. Momentum is **not a force**, and it is **not a form of energy**.
**Momentum is the product of mass and velocity**.

Principle of Conservation of Momentum.

This states that ***the total momentum of a system of masses in any given direction will remain constant unless the system is acted upon by an external force in that direction***. This comes directly from Newton's Second Law of motion, which said that the rate of change of momentum is proportional to the applied force.

Consider two bodies A and B of masses $m_1$ and $m_2$, moving with velocities $u_1$ and $u_2$ . If $u_1$ is greater than $u_2$, A will collide with B. If after collision the final velocities are $v_1$ and $v_2$, then $v_1$ will be less than $u_1$ and $v_2$ will be greater than $u_2$.




For A, loss of momentum = $m_1(v_1 - u_1)$.
For B,  gain of momentum = $m_2(v_2 - u_2)$.

At impact, we will have Conservation of Momentum unless an **external** force is applied.  When two bodies collide as above there are three possible outcomes, provided they remain in contact with the ground:

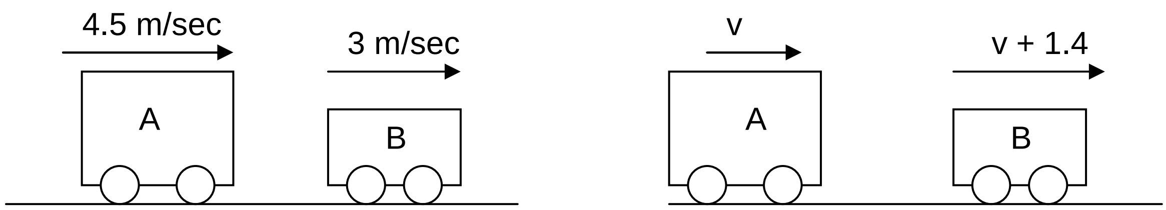
1) One body rebounds back from the other one
2) They move away together at constant velocity
3) They move away in the same direction but at different speeds.

Note that the principle of conservation of momentum applies equally to all three cases. Conservation of momentum depends only on the absence of an EXTERNAL force.

Note also that although there is overall conservation of energy, there is often a loss of **useful** forms of energy, such as Kinetic Energy, on collision. We will see this loss of kinetic energy in most of the examples we do.

Example 4

A truck "A" of mass 6 tonne travelling with a velocity of 4.5 m/sec collides with a truck "B" of mass 4 tonne travelling with a velocity of 3 m/sec in the same direction. If the relative velocity of the trucks after impact is 1.4 m/sec, find the final velocity of each truck.




Momentum truck A = M *x* u = 6000 x 4.5 = 27000 kgm/sec

Momentum truck B = 4000 x 3 = 12000 kgm/sec

Total Momentum (initial) = 39,000 kgm/sec.

let v = final velocity of A [m/sec]:-

Then the final velocity of B = v + 1.4 [m/sec].

Final Momentum A = M *x* v = 6000v kgm/sec

Final Momentum B = 4000(v + 1.4) = 4000v + 5600 kgm/sec

So total final Momentum = 10000v + 5600 kgm/sec

From the principle of conservation of momentum,

Total final Momentum = Total initial Momentum

10000v + 5600 = 39,000

$\therefore$ v = 3.34m/sec (velocity of truck A)

For B final velocity = 3.34 + 1.4 = 4.74 m/sec.

Note that we do not have conservation of ***kinetic*** energy.

K.E. before impact $= \frac{1}{2}\, 6000 \times 4.5^2 + \frac{1}{2}\, 4000 \times 3^2 = 78{,}750$ J.

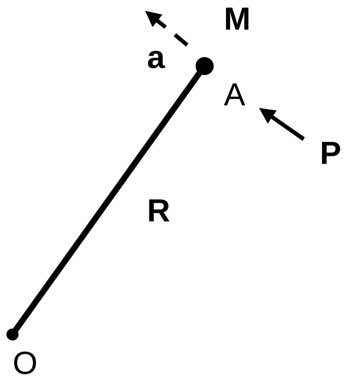
K.E. after impact $= \frac{1}{2}\, 6000 \times 3.34^2 + \frac{1}{2}\, 4000 \times 4.74^2 = 78{,}402$ J.

$\therefore$ Loss of kinetic energy = 348 J

MOMENT OF INERTIA

We have already seen that in accordance with Newton's second law of motion, any change in linear motion requires an applied force. The same principle applies to angular motion, except that here ***any change in angular motion will require a driving torque***.

Consider a concentrated body of mass M, attached to end A of a light arm OA of length R. For a force P applied at A perpendicular to OA. the mass M would move, the movement being constrained in a circular path.




From:- Force = Mass *x* Acceleration, $P = M \times a$

and since $a = \alpha R$ then $P = M.\alpha R$

Multiplying both sides by R gives:

$$P \times R = M.\alpha R \times R \quad \text{( multiplying both sides by R)}$$

P *x* r is the Torque (T) on the mass;- $\therefore \quad T = M.R^2 \alpha$

The term "$MR^2$" is known as the "Moment of Inertia" (I) of the mass about point "O". Moment of Inertia is also known as the second moment of mass. The units of "I" are kg-$m^2$.

$$T = I\alpha \text{ (analogous to } F = M \times a)$$

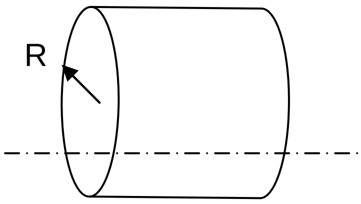
Where the mass is **not concentrated** at a given radius, the radius at which the mass is considered to act is known as the "Radius of Gyration" of the mass about O, denoted by k.

$$\text{hence :- } I = M k^2 \text{ and } T = I \alpha = M k^2 \alpha$$

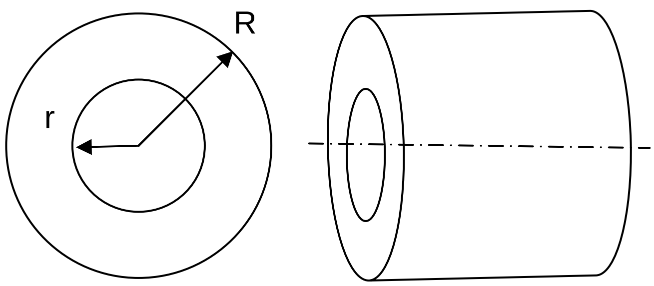
Remember that the ***Inertia of a body is its reluctance to change motion***. For a body moving with linear motion, the inertia depends upon the mass. From the above we can see that for a rotating body the inertia depends not only on the mass but also on the radius of gyration. Thus if we want a rotating body to have a high value of Inertia we should put most of the mass at the greatest radius so as to increase the Radius of Gyration. This would apply, for instance, to a flywheel, where most of the mass is concentrated near the rim to increase the value of ‘k’ and hence increase the moment of Inertia.

The radius of gyration is often stated in a question, but we can quickly calculate it for some common shapes using the following formulae:

For a solid cylinder, $k^2 = \frac{R^2}{2}$




For a hollow cylinder, $k^2 = \frac{R^2 + r^2}{2}$




CONSERVATION OF ANGULAR MOMENTUM

Angular momentum is given by the product of I *x* ω (similar to linear momentum m.v). If we wish to change the angular momentum of a rotating body then the rate of change of momentum would be given by:

$$\frac{I(\omega_2 - \omega_1)}{t} = I\alpha$$

But we have just derived an expression for torque that says $T = I\alpha$

Hence **Torque = rate of change of angular momentum**.

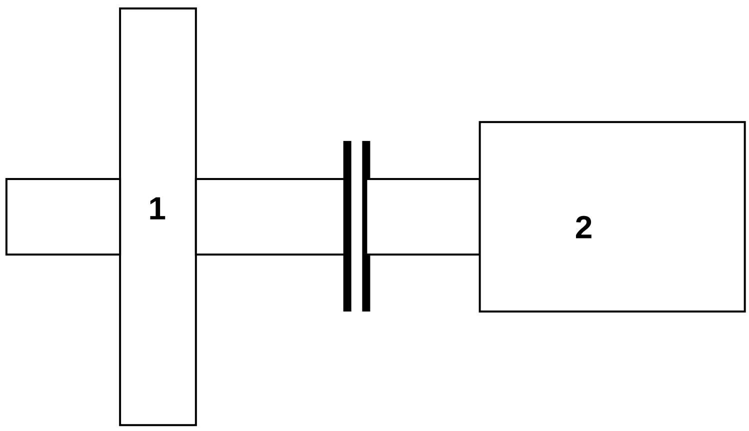
This means that **if there is no externally applied torque, the angular momentum of the system remains constant.** Notice the similarity of this statement to that for linear momentum.

Example 5

A flywheel and its shaft have a mass of 300kg and a radius of gyration of 600mm. It is running freely at 720 rpm when it is connected to a second shaft by a clutch. The second shaft is at rest when the clutch is engaged and it has a moment of inertia of 40 $kgm^2$.

Determine the final speed of rotation of the two shafts and the energy lost during the engagement period.

Solution:




For the flywheel, $I_1 = 300 \times 0.6^2 = 108\ kgm^2$. $\omega_1 = \dfrac{720 \times 2\pi}{60} = 75.4$ rads/sec

For the second shaft, , $I_2 = 40\ kgm^2$. $\omega_2 =$ Zero

Since we have Conservation of Momentum (No external torque), then

Momentum before engagement = momentum after engagement

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_3$

$108 \times 75.4 + 0 = 148.\omega_3$

$\therefore \omega_3 = 55.02$ rads/sec

Note that, as with linear systems, ***conservation of momentum does not mean conservation of useful forms of energy*** such as Kinetic Energy. We will prove this very shortly, but for now we can just accept that angular Kinetic Energy = ½ $I\omega^2$

Energy lost at engagement = K.E before engagement - K.E after engagement

$$= \tfrac{1}{2} I_1\omega_1^2 - \tfrac{1}{2}(I_1 + I_2)\omega_3^2$$

$$= (\tfrac{1}{2} .108 \times 75.4^2 + 0) - (\tfrac{1}{2} .148 \times 55.02^2)$$

$$= 82.97 \text{ kJ}$$

So, as with linear systems, we have **conservation of momentum in the absence of an external torque**, but **not conservation of useful forms of energy**.
We can also apply D'Alembert's principle to angular motion in a similar way to that used for linear motion. With linear systems, wherever we see acceleration, we put a force "m.a" in the opposite direction. **With angular systems, wherever we see acceleration, we put a TORQUE "$I\alpha$" in the opposite direction**.

Example 6
A cylinder has a mass of 600 kg, an inside diameter of 600mm and an outside diameter of 800mm. It rotates about its longitudinal axis.

a) Determine the Moment of Inertia of the cylinder

b) What torque would be required to accelerate the cylinder from rest to 300 rpm in 8 seconds if there is a constant resisting torque of 50Nm.

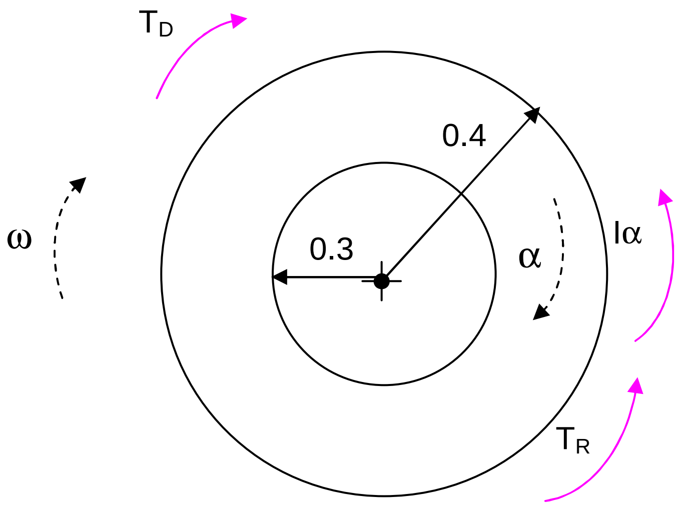
Solution

For a hollow cylinder, $k^2 = \frac{R^2 + r^2}{2}$

$\therefore k^2 = \frac{0.4^2 + 0.3^2}{2}$

$\therefore k^2 = 0.125$

$\therefore I = M k^2 = 500 \times 0.125 = 62.5 kgm^2$




$T_D$ = Driving Torque
$T_R$ = Resisting Torque

We should use the sketch above to show the direction of all the torque's, remembering that:

1.Friction always opposes motion
2.The Inertia Torque always opposes the acceleration

We need to work out the acceleration,

$\alpha = \frac{\omega 2 - \omega 1}{t} = \frac{300 \times 2\pi - 0}{60 \times 8} = 3.927$ rads/sec

We can now summate the torques,

$\Sigma$ Torque ↻ +ve, $\Sigma = 0$

$$T_D - I\alpha - T_R = 0$$

$$\therefore T_D = 62.5\alpha + 50$$

$$\therefore T_D = 295.4 Nm$$

## Kinetic Energy of Rotation

We will now derive an equation for the Kinetic Energy of rotation. We will be using the same principal as that used for linear Kinetic Energy. We will equate the work done in accelerating a body to the Kinetic Energy gained. Consider the work done in accelerating a shaft of moment of inertia "I", from rest to a speed $\omega$ while turning through an angle $\theta$ rad.

The average angular acceleration $\alpha$ is given by:-

$$\omega_2^2 = \omega_2^2 + 2\alpha\theta$$

$$\alpha = \frac{\omega^2}{2\,\theta}$$

From the expression for torque:-

$$T = I\,\alpha \qquad = I \times \frac{\omega^2}{2\,\theta}$$

The work done by the torque T in rotating the shaft through $\theta$ radians is given by:-

$$T\,\theta \qquad = I \frac{\omega^2}{2\,\theta} . \theta$$

$$\text{k.e.} = \tfrac{1}{2} I\omega^2$$

Note the similarity between the expressions for kinetic energy of translation (linear motion) and kinetic energy of rotation. For rotational kinetic energy we have "I" instead of "m" and "$\omega$" instead of "v".

$$\text{k.e. of translation} = \tfrac{1}{2} M v^2$$

$$\text{k.e. of rotation} = \tfrac{1}{2} I \omega^2$$

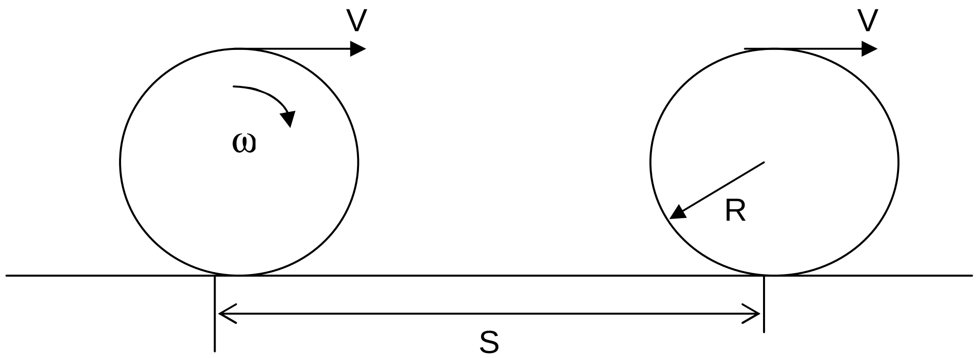
Total Kinetic Energy of a Rolling Wheel

If a wheel rolls then the total kinetic energy is made up of **two** parts--

1. The (linear) kinetic energy of translation of the centre of mass.

2. The (angular) kinetic energy of rotation about the centre of mass.

If v is the linear velocity of the wheel and I is the moment of inertia about its axis of rotation:-




Total kinetic energy = $\frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2} M v^2 + \frac{1}{2} M k^2 \omega^2$$

Where M is the mass and k the radius of gyration of the wheel. At first glance there appear to be four variables in this equation. This may not always be the case however, since for rolling (without slip) $\omega = v/R$, where R is the wheel radius. Our equation then reduces to:

$$\text{k.e. of wheel} = \frac{1}{2} M v^2 + \frac{1}{2} M k^2 \frac{v^2}{R^2}$$

$$\text{k.e. of wheel} = \frac{1}{2} M v^2 \left( 1 + k^2/R^2 \right)$$

We do not need to remember this equation in particular. We can always derive it from first principles, and it only applies to a wheel rolling **without** slip. If the relationship between “k” and “R” is known it would be possible to reduce the equation still further.

Now for a few examples on angular kinetic energy.

Example 7.

A shaft of moment of inertia 34 $kg.m^2$ is initially running at 600 r.p.m. It is brought to rest in eighteen complete revolutions by a braking torque. The friction torque is 160 Nm throughout. Find, using an energy method, the braking torque required.

Solution

$$600 \text{ r.p.m.} = \frac{2.\pi}{60} \times 600 = 62.83 \text{ rad/s}$$

$$\begin{aligned} \text{initial kinetic energy} &= \tfrac{1}{2} I \omega^2 \\ &= \tfrac{1}{2} \times 34 \times 62.83^2 \\ &= 67{,}000 \text{ [J]} \end{aligned}$$

Work is done by the friction torque $T_f$:-

$$\begin{aligned} T_f \times \theta &= 160 \times (2\pi \times 18) \\ &= 18{,}100 \text{ [J]} \end{aligned}$$

Let T = braking torque:-

$$\begin{aligned} \text{W.D. by T} &= T \times \theta \\ &= T \text{ x } (2\pi \times 18) \\ &= 113.T \text{ [J]} \end{aligned}$$

Putting down in words what energy conversion is taking place:

W.D. by braking and friction = kinetic energy of rotation lost

$$113\,T + 18{,}100 = 67{,}000$$

$$T = 433 \text{ Nm}$$

Example 8. (Class One Standard)

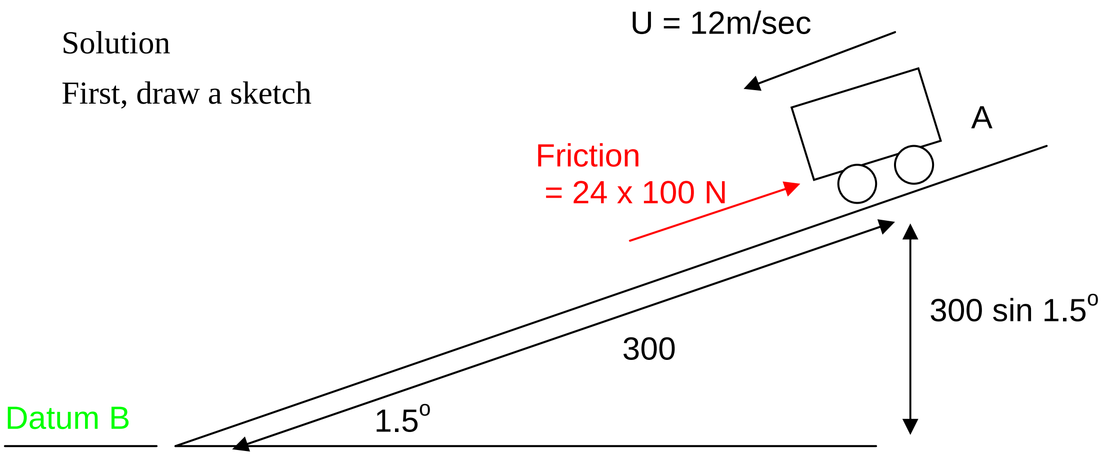
A railway wagon has a total mass, (including its wheels), of 24 tonne. Each wheel has a mass of 900kg, a diameter of 1.2m and a radius of gyration of 0.52m. At position A, 300m from the bottom of a $1.5^0$ slope, the wagon is moving downward with an instantaneous velocity of 1.2m/s. Resistance to motion is constant at 100N/tonne.

(a) Calculate the energy lost by the wagon in overcoming resistance as it travels from position A to the bottom of the slope (position B).

(b) Calculate the velocity of the wagon at the bottom of the slope.

Solution

First, draw a sketch




The total energy of the wagon at position A, compared to the datum at the bottom of the slope, will be the sum of its kinetic and potential energies. Since there will be both translational and rotational kinetic energy, we will need to work out the total Moment of Inertia 'I' for the wheels.

$$I = M k^2 = 900 \times 0.52^2 = 243.4 \text{ kgm}^2$$

So total for 4 wheels is 4 x 243.4 = $973.6 \text{ kgm}^2$

Part a) of the question is relatively straightforward.

Energy lost by wagon due to friction = work done by friction.

Work done = force x distance moved = 2400 x 300 = 720 kJ.

For part b) we should first put down, in words, what is happening.

Energy at B = Energy at A – WD by friction

Rotational $K.E_B$ + Linear $K.E_B$. = Rotational $K.E_A$. + Linear $K.E_A$ + $P.E_A$. – 720 kJ

We can work out the Energy at A from the information given, remembering that $\omega = v/R$, where R is the wheel radius. Hence $\omega = 12/0.6 = 20$ rads/sec.

So energy at A = Rotational K.E. + Linear K.E + P.E

$$= \tfrac{1}{2} M v^2 + \tfrac{1}{2} I\omega^2 + Mgh$$
$$= \tfrac{1}{2}\, 24{,}000 \times 12^2 + \tfrac{1}{2}\, 973.6 \times 20^2 + 24{,}000 \times 9.81 \times 7.853$$
$$= 3772 \text{ kJ}$$

So Energy at B = 3772 – 720 = 3052 kJ

So at B, Rotational K.E. + Linear K.E. = 3052 kJ

$$\therefore \tfrac{1}{2} M v^2 + \tfrac{1}{2} I\omega^2 = 3052 \times 10^3$$

$$\therefore 12{,}000\, v^2 + \tfrac{1}{2} \times 973.6\, \omega^2 = 3052 \times 10^3$$

And using $\omega = v/0.6$ we can now write

$$\therefore 12{,}000\, v^2 + 486.8\, (v^2/0.36) = 3052 \times 10^3$$

$$\therefore 12{,}000\, v^2 + 1352\, v^2 = 3052 \times 10^3$$

$$\therefore v^2 = \frac{3052 \times 10^3}{13{,}352}$$

$$\therefore v = 15.12 \text{ m/sec}$$

Self Assessed Questions for you to try. (Answers given)

1. A body with a mass of 100 kg moving at 4 m/s due East collides with another body of 150 kg moving at 2.5 m/s due West. Calculate the magnitude and direction of the two bodies immediately after impact if they remain locked together. Determine also the loss of kinetic energy.

Ans: 0.1 m/s due East and 1.268 kJ

2. A bullet weighing 30 gramme has a velocity of 700 m/s strikes a baulk of timber and penetrates it to a depth of 350 mm. Find the average resisting force. If the timber had been 200 mm thick, what would have been the velocity of the bullet just after passing through?

Ans: 21 kN 458 m/s

3. A pile driver with a mass of 800 kg falls through a height of 4 m on to the head of a pile with a mass of 500 kg. If the pile is driven into the ground a distance of 10 mm and there is no rebound at impact, find the penetration resistance of the ground, assuming it remains constant, and the energy expended in deformation of the head of the pile.

Ans: 1.943 MN and 12.12 kJ

4 . A truck with a mass of 6 tonne travelling at 7.2 km/h collides with a stationery truck of mass 10 tonne. Find the common velocity of the trucks at the instant they move together and the loss of kinetic energy at impact. If this loss of kinetic energy is utilized in compression of the four buffer springs, determine the maximum compression of the springs if they have a stiffness of 250 N/mm.

Ans: 0.75 m/s 7.5 kJ 122.5 mm

5 . In an Izod impact testing machine the hammer head has a mass of 20 kg and it moves in a vertical circular arc of 1.2 m radius. If it is released at a point $60^o$ from the lowest position, find the velocity and kinetic energy of the head at the bottom of its swing. In breaking a standard specimen it moves through an angle of $45^o$ on the other side. Find the impact value of the metal under test neglecting the effects of friction.

Ans: 3.435 m/s, 118 J, 49 J

6 . A machine cuts a hole of 80 mm diameter in plate of thickness 14 mm and the energy expended for each 10 $mm^2$ of the cut is 50 J. The flywheel of the machine has a radius of gyration of 900 mm and a normal running speed of 120 rev/min. During the cutting operation the speed falls to 85% of its normal value and the loss of kinetic energy may be assumed to be fully absorbed by the cutting operation. Determine the mass of the flywheel.

Ans: 994 kg

7 . A four wheeled truck has a total mass of 2.5 tonne, each wheel is 800 mm diameter of mass 125 kg and radius of gyration 300 mm. It is ascending an incline of 1 in 10 at 27 km/h when

the brakes are applied and the truck is brought to rest over a distance of 16 m. Calculate the dissipation of energy in braking. Ans: 39.06 kJ

8. A wheel and shaft of total mass 15 kg run in horizontal bearings. The shaft is 50 mm diameter and has a cord wound round it with a hook on the free end. A force of 3.5 N at the end of the cord is just sufficient to overcome friction whilst a force of 22 N gives a fall of 1 m in 8 s from rest. Find the radius of gyration of flywheel and shaft.

Ans: 155 mm

9 . The rotor of a steam turbine has a mass of 20 kg and radius of gyration 40 mm. The critical speed range of the rotor is from 20,000 rev/min to 18,000 rev/min. Calculate the minimum breaking torque required when the power is shut off to ensure that it takes no longer than 2 s to pass through this range.

Ans: 3.35 Nm

10. A flywheel running at 1200 rev/min has a mass of 25 kg, its outside diameter is 500 mm and radius of gyration 200 mm. If braking force of 100 N is applied for 10 s and then 30 N until the flywheel comes to rest, find the angular retardation in each case and the total time taken.
$\mu$ between flywheel and brake is 0.4 and the brake is applied at the flywheel rim.

Ans: 10 rad/s$^2$ 3 rad/s$^2$ and 18.57 s

11. In a conical friction clutch the effective diameter of the contact surfaces is 80 mm, the semi-apex angle of the cone is 15$^o$, $\mu$ is 0.3 and the axial force transmitted 180 N. The clutch is employed to connect an electric motor running at 1000 rev/min to a flywheel which is initially at rest. The mass of the flywheel is 15 kg and its radius of gyration 150 mm. Calculate the time taken for the flywheel to reach full speed.

Ans: 4.24 s

12. A motor drives a machine through a friction clutch which transmits a torque of 14 Nm whilst slip occurs during engagement. The motor has a mass of 60 kg and radius of gyration 150 mm and the machine a mass of 30 kg and radius of gyration 80 mm. If the motor is running at 750 rev/min and the machine is initially at rest, find their common speed after engagement. Find also the time taken to reach this speed and the energy absorbed during engagement.

Ans: 657 rev/min, 0.94 s and 519 J

13. A truck of 8 tonne mass is hauled up an incline of 1 in 20 by means of a rope parallel to the incline, and passing around a drum at the top of the incline. The drum is 2.4 m effective diameter and a mass of 5 tonne with a radius of gyration of 900 mm. Tractive resistance is 120 N per tonne mass and there is a friction torque of 170 Nm at the drum shaft. If the torque exerted at the drum is 34.25 kN m, find the acceleration of the truck up the incline.

Ans: 2.175 m/s$^2$

14. A pulley of mass 100 kg and diameter 1.2 m has a resistance torque of 1.5 Nm due to bearing friction. A cord is wrapped around the pulley with a 1 kg mass attached at its free end and when released from rest, it is observed to descend 1 m in 2 seconds. Calculate the radius of gyration for the pulley.

Ans: $k = 0.2215$ m