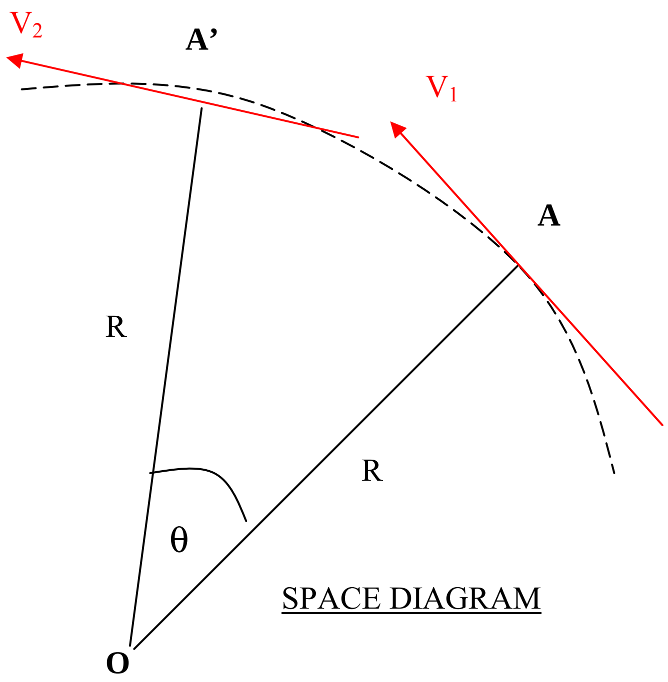
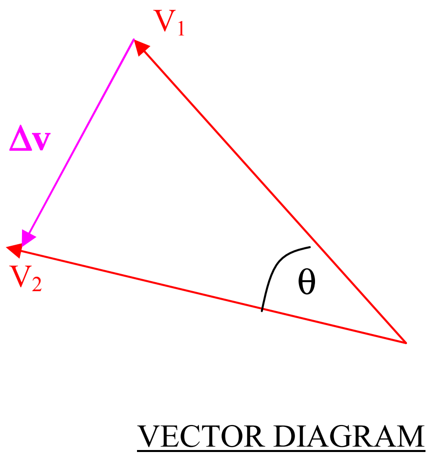
CENTRIPETAL ACCELERATION & CENTRIFUGAL FORCE

Consider a point A moving at uniform angular velocity, in a circular path of radius "R".
Let the point move from A to A' in a time of "t" seconds





From the space diagram, the linear velocity at each point is given by:

$$v_1 = \omega OA = \omega R \quad \text{also} \quad v_2 = \omega OA' = \omega R \quad [OA = OA']$$

Hence $v_1 = v_2$ (in magnitude but **NOT** direction). It is important to note that even if only the **direction** of the linear velocity has changed, this implies a change of velocity, and therefore **acceleration**.

From the Vector Diagram, change in velocity = Δv

$\Delta v = v \times \theta = \omega R\theta$ [for small angles, arc length $R\theta \approx$ straight line]

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}} = \frac{\omega R\theta}{t}$$

Hence:- $a = \omega^2 R$ as $\omega = \theta / t$

also $\omega = \dfrac{v}{R}$ so $a = \dfrac{v^2}{R^2} \times R = \dfrac{v^2}{R}$

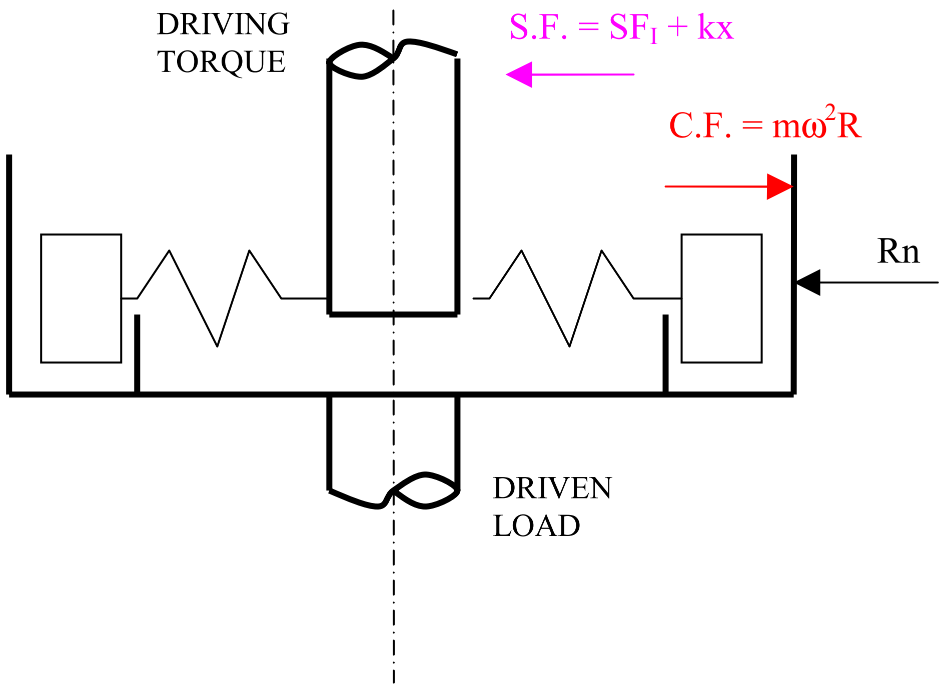
This acceleration is called the "*Centripetal Acceleration*", it is always directed radially inward to the centre of rotation.

From Newton, we know that Force = Mass x Acceleration, so it follows that if we have a rotating mass then there must be a Force = Mass x acceleration = $\mathbf{m\omega^2 R}$ acting radially inwards to constrain the body to move in a circle. This is a definition of **Centripetal Force**, the force necessary to constrain a body to move in a circle. If this force does not act, for instance if you were whirling a mass around in a circle on the end of a wire, and the wire snapped, then, again from Newton, the body would continue in a straight line, that is it would fly off with its instantaneous, (tangential) linear velocity. There will be a reaction to this Centripetal Force, in the opposite direction and therefore radially outwards, and this is known as **Centrifugal Force**. Centrifugal Force is analogous to the radially outward tension in the wire, felt by a person whirling an object round in a circle. It is the radially outwards Centrifugal Force that is used in our calculations, but we should not lose sight of the fact that it exists only as a reaction to Centripetal Force.

Centrifugal Clutches

A common example of the use of centrifugal force is in centrifugal clutches, used, for instance to drive purifier bowls. They act as torque limiting devices to prevent overload of the motor at start up. By "slipping" they limit the maximum Power, and therefore current, taken by the motor and allow the large inertia of the bowl to be more gradually overcome.

Centrifugal clutches are used to transmit torque, via a friction clutch which engages due to centrifugal action on the shoes. It allows the drive unit to start up and gain speed before the shoes start to engage the outer rim of the clutch housing. Once engaged, they will press against the housing, and the friction force produced will cause the output rim to move. The friction force produces a torque on the output. This torque is limited by friction, allowing the clutch to "slip" and limit the Power taken from the drive, for instance during start up or when subjected to shock loads.



First let us consider one shoe. It is held off the outer rim by a spring. These spring usually will have some pre tension. As the shoe begins to move away from the shoe stop, then the spring is extended, causing the spring force to increase.

Evaluating the forces present on the shoe, when the shoe is engaged, let Rn = Normal Force at the Rim

$m\omega^2 r$ – Spring Force (S.F.) – Rn = 0

So if the centrifugal force exceeds the spring force, a reaction force Rn is generated at the contact surfaces.

To evaluate the friction torque generated at the rim.

$T = \mu . R_n . R$, where R is the rim radius, and **not** the radius to the centre of gravity of the shoe, which is used to calculate the centrifugal force. This is an important point, don't forget it!

Finally power out output = **$P = T.\omega$**

Note! As the speed rises from rest there is a particular speed at which engagement just commences. At this speed the spring force ($SF_1 + kx$) is equal to the centrifugal force. As the speed rises above the engagement speed the spring force remains constant but the inertia force increases in value, this increases R_n. The value of R_n governs the friction force between shoe and rim surfaces and hence determines the power transmitted.

Example 1

A centrifugal clutch transmits power when rotating at 600 rev/min. The two 1.2 kg shoes are held at a rotational radius of 300mm by springs of stiffness 15kN/m with an initial spring force of 180N force against the shoe stops. They engage when they have moved 40mm.
The coefficient of friction of the clutch is 0.3, and the rim diameter is 800mm.

Find a) The speed at which the shoes just engage, and
b) The output power of the clutch

Solution

As usual, we should do a sketch. It is usual with these questions to consider just one shoe, then multiply the torque produced by the number of shoes.

Put down the forces acting on the shoe.
When the shoes just engage Rn will equal zero, so $m\omega^2 r$ – Spring Force = 0.
The centrifugal force will have a rotational radius of 300 + 40mm when the shoes engage, so

$$m\omega^2 r = 1.2\,(\omega^2)\,0.34 = 0.408\omega^2$$

Spring force will equal the initial tension, plus the spring extension force,
so S.F. = 180 + (15000 x 0.04) = 780N

So $780 = 0.408\omega^2$, which gives ω = 43.73 rad/sec or 417.5 rev/min ANS a)

b) When the shoes are rotating at full speed Rn will no longer equal zero.

Notice that the spring force is at a maximum once the shoes engage. It cannot increase beyond this value because the shoes cannot move any further once they have contacted the wall. So the spring force at the point of engagement will be the maximum spring force.

At 600 rev/min:

$$m\omega^2 r - S.F - Rn = 0$$

$$1.2\,(2\pi 600/60)^2\,0.34 - 780 - Rn = 0$$

So Rn = 830.7N

The friction torque is $T = \mu .Rn. R$,

so T = 0.3.830.7.0.8/2 = 99.7Nm

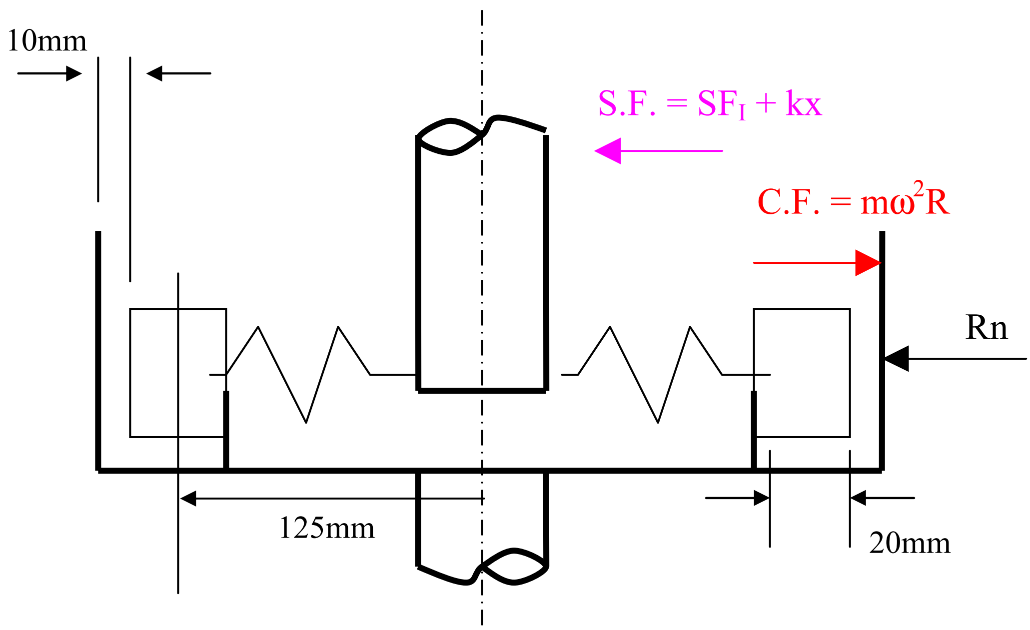
But this is the torque for one shoe, and since we have two shoes this torque is doubled, so output torque = 2 x 99.7 = 199.4Nm

Therefore output power = $(2\pi 600/60)$ x 199.4 = <u>12.53kW ANS b)</u>

<u>Example 2 (Exam standard)</u>

A two shoe centrifugal clutch is shown in the rest position. The stiffness of the springs are 6kN/m, and the initial tension of the springs at rest is 100N. Calculate EACH of the following:

a) the required shoe mass if the clutch just engages at 250 rev/min
b) the maximum power which can be transmitted by the clutch at 500 rev/min, if the coefficient of friction between the shoe and the outer rim is 0.3.



STEP ONE
Draw one half of the clutch, showing ALL the forces and dimensions.

STEP TWO
Put down the forces on one shoe. The clutch is shown at rest, so as the clutch speed increases, then the shoe will move out and engage the outer rim. The shoe will move 10mm before engaging. The first part of the question asks for the speed when the shoe is just touching, so at this speed, the force Rn will be zero.
The spring force will be the sum of the initial tension (given) and the extra spring force due to the spring elongation. (S.F. $= S.F._{INITIAL} + kx$)

$$m\omega^2 r - S.F = 0$$

$$m(2\pi 250/60)^2(0.125 + 0.1) - [(6000 \times 0.01) + 100] = 0$$

Note that "r" changes with speed. When the shoe contacts the surface, the shoe centre of gravity will be 135mm away from the centre of rotation.

So $m\ 26.18^2\ 0.135 - 160 = 0$

Hence <u>m = 1.73kg ANS a)</u>

STEP THREE
When the clutch speed increases, there will now be the surplus centrifugal force pressing against the rim, allowing power to be transmitted.

So $m\omega^2 r - S.F. - Rn = 0$

$$1.73\ (2\pi 500/60)^2(0.135) - [(6000 \times 0.01) + 100] - F = 0$$

$$640.3 - 160 - Rn = 0$$

Thus Rn = 480.3 N

STEP FOUR
Now calculate the friction torque that this force will produce, from $T = \mu.Rn.\ R$. But the value of R is not given, and needs to be determined from the diagram. The centre of gravity of the shoe has already been calculated to be 135mm, and we also know that the shoe is 20mm wide. Thus the radius R to the rim will be 135 + 20/2 = 145mm. Ensure that you can see where this figure has been derived from.

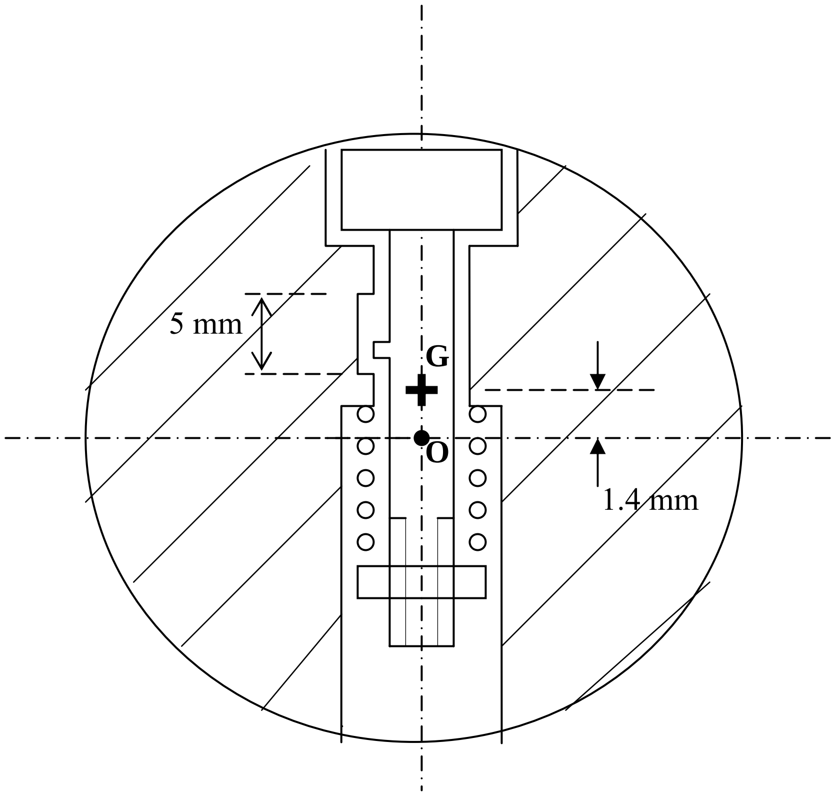
So T = 0.3 x 480.3 x 0.145 = 20.9Nm

Hence the power transmitted will be $P = T\omega = (2\pi 500/60)\ 20.9$ = <u>1094 W ANS b)</u>

<u>Example 3 – Overspeed Trip (Exam Standard)</u>

This question uses some of the principles we have met so far when dealing with Centrifugal Force on Clutches, so is suitable for inclusion here.

The Control Bolt of the steam turbine overspeed trip shown has a mass of 0.2 kg and is held in place by a spring of stiffness 20kN/m, whose initial compression is 100N. The centre of gravity, G, of the bolt is displaced from the centre of rotation, O, by 1.4mm in the rest position.



Calculate EACH of the following:

a) the speed at which the control bolt will start to move out;
b) the additional spring compression required to change the overspeed trip setting to 6600 rev/min (the control bolt offset remains at 1.4mm)
c) the speed at which the control bolt will start to retract from its extended trip position of 5mm travel, when the trip speed is set to 6600 rev/min

STEP ONE
Draw the unit under investigation showing the forces of

- Centrifugal force
- Spring force (both initial and additional)

STEP TWO

Complete part a) of the question. The clue is that the force on the spring is just equal to the initial compression, so $m\omega^2 r$ – initial spring force = 0

Calculate your answer. You should get ω = 597.6 rad/sec or 5706.8 rev/min

STEP THREE

In step two you showed that the trip will start to operate at 5706.8 rev/min. If we wish to increase this speed to 6600rev/min, then an additional spring tension is required by compressing the spring by “x”. **This assumes that the trip will operate as soon as the offset mass starts to move.**

So $m\omega^2 r$ – (kx + initial spring force) = 0

r will remain at 1.4mm (given), as the trip has not yet been activated. You should notice that when the mass does start to move, then it will move quickly to its extreme position as the centrifugal force increases with radius.

Calculate your answer. You should get x = 1.7mm

STEP FOUR

Once the trip has been activated, it will move out by 5mm. This increase in centrifugal force means that the trip must slow down before the offset mass will reset. This is a safety measure, and allows the turbine to slow down. This time the “r” value will be 5 + 1.4mm = 6.4mm.

The spring tension will be the sum of (initial spring tension + tension to increase setting to 6600 rev/min + tension resulting from 5mm travel). The total spring tension should be 233.8N. Ensure you have this figure before proceeding.

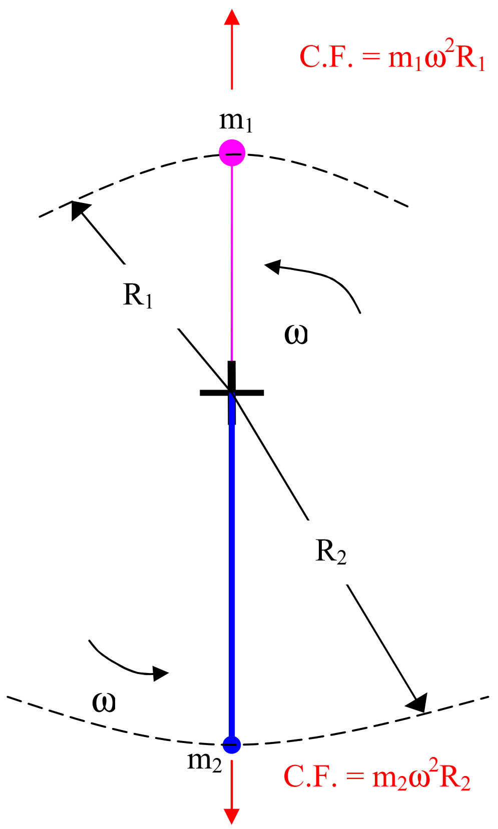
From $m\omega^2 r$ – (total spring force) = 0, the new value of ω can be calculated.

Calculate your answer. You should get 4081 rev/min.

BALANCING

Balancing of Co-Planar Masses

Consider a mass "m" rigidly attached to a shaft by means of an arm of negligible mass and of radius R. If the shaft rotates at a constant angular speed (ω), a centripetal force must act on the mass and an equal and opposite force acts on the shaft bearings. For such a system the result of this unbalanced force would be considerable wear at the bearings.



$m_1\omega^2R_1 = m_2\omega^2R_2$

Since ω^2 is common to both sides then this becomes:

$m_1R_1 = m_2R_2$

A balance force can be achieved by adding an equal mass m at an identical radius R, diametrically opposite the original mass, or by adding a smaller mass at a greater radius.

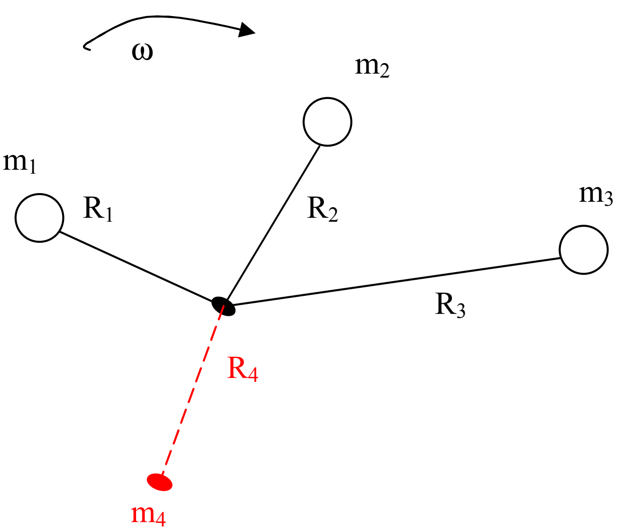
Several Masses;-

If m_1, m_2, m_3 etc are the masses at radii R_1, R_2. and R_3 on a common shaft, for dynamic force balance the vector sum of the individual centrifugal forces must be zero.

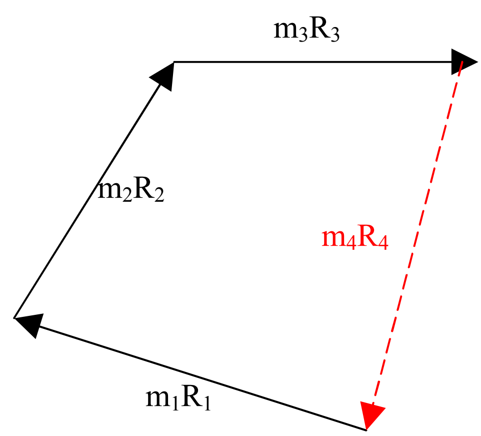
For balance $\Sigma\ (m.\ \omega^2\ R)$ = zero

or $\Sigma\ (m.R)$ = zero .... as ω is common

For convenience, we usually draw an 'mr' diagram rather than the true vector diagram which would use the $m\omega^2R$ values. Remember that the direction of the forces will be RADIALLY OUTWARDS for the instantaneous position of the rotating mass.



SPACE DIAGRAM



"mR" DIAGRAM

Hence a polygon of m.R. values would "close" if there were dynamic balance. So having drawn the "mR" diagram for the three masses, in order to get the vector diagram to close we need an Equilibrant "mR" value "m_4R_4" . Having obtained this value, we have the choice of the relative values of mass and radius to give us the "mR" product.

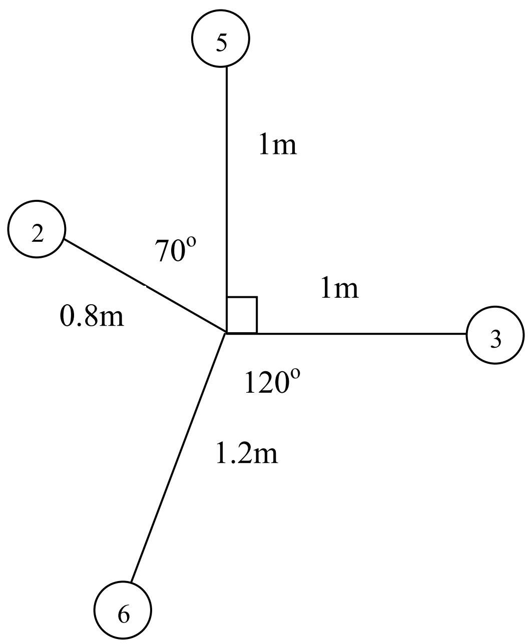
If we know the radius, then $m = \frac{mR}{R}$ Now lets try a simple example

Example 4 :-

Four masses are rigidly attached to a shaft that rotates at 125 rev/min.

The masses are all in the same plane of rotation and their magnitudes and radii are shown.

Find the out of balance force on the shaft when it rotates at 125 rev/min, and the position and radius required for a 3kg balance mass.



Solution:

By way of example, we will do this by both possible methods, i.e.

- Drawing
- Calculation

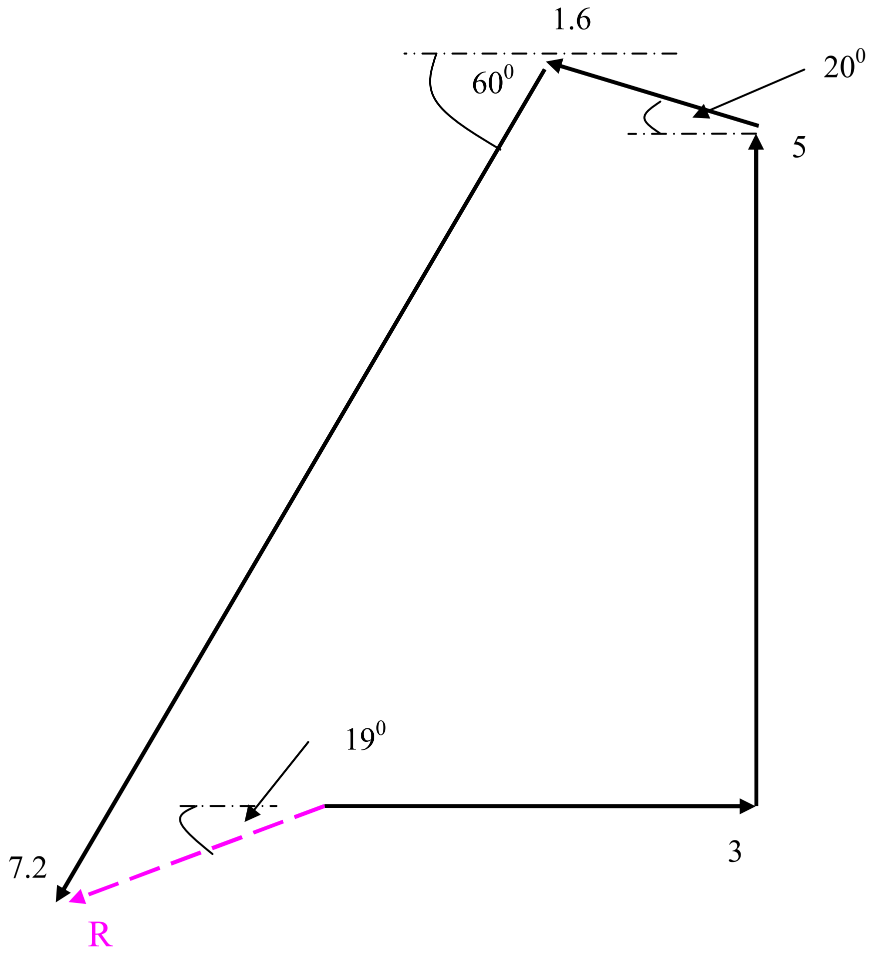
Either way, when tackling problems on balancing, a useful first step is to draw up a table of 'mr' values. Since we are dealing with a rotating mass system, the position of any force can only be an instantaneous one, so we will take one of the masses as a reference. It will be easiest if our reference 'mr' is shown in the horizontal or vertical position.

Here, lets take the 3kg mass as the reference, so our table of 'mr' values would look like this:

m (kg)	R (m)	mr (kgm)	Position
3	1	3	Reference
5	1	5	90^0 Anti Clockwise
2	0.8	1.6	160^0 Anti Clockwise
6	1.2	7.2	120^0 Clockwise

Whether we are going to solve the problem by scale drawing or calculation, we should now draw the 'mr' diagram to scale. A drawing done for a scale drawing approach obviously needs to be larger (half a side of A4 at least) and more accurate than a drawing done for a calculation.

The rules for drawing an 'mr' diagram are the same as for a force vector diagram. We are adding the 'vectors' , so they should be drawn nose-to-tail and the resultant will go from start to finish. We can draw the vectors in any order, but for clarity we will start with our reference and then work our way around in an anti-clockwise direction.



From the above drawing, by scale, the **resultant** 'mr' is 2.15kgm, 161^0 clockwise of the 3 kg reference mass.
The out of balance force = $mr\omega^2 = 2.15 \times (125 \times 2\pi/60)^2 = \underline{368\ N}$

This means that the **equilibrant** 'mr' would be in the opposite direction, that is, <u>19^0 anti-clockwise</u> from the 3 kg reference mass.

If the balance mass is to be 3kg, then the radius, R = mR/m = 2.15/3 = <u>0.72</u>

To ***calculate*** a value of resultant 'mr', we draw the diagram and the summate the components of them in two mutually perpendicular directions.
Don't expect them to summate to zero, as we are looking for a resultant!!

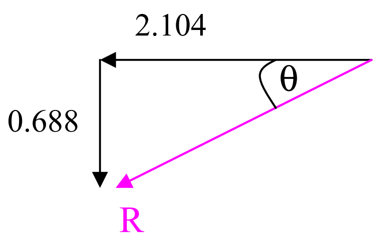
Σ 'mr' in the 'X' direction, → positive,

$\Sigma = 3 + 0 - 1.6 \cos 20^0 - 7.2 \cos 60^0 = -2.104$ kgm. (the minus sign indicates it will be from right to left)

Σ 'mr' in the 'Y' direction, ↑ positive,

$\Sigma = 0 + 5 + 1.6 \sin 20^0 - 7.2 \sin 60^0 = -0.688$ kgm. (the minus sign indicates it will be downwards)

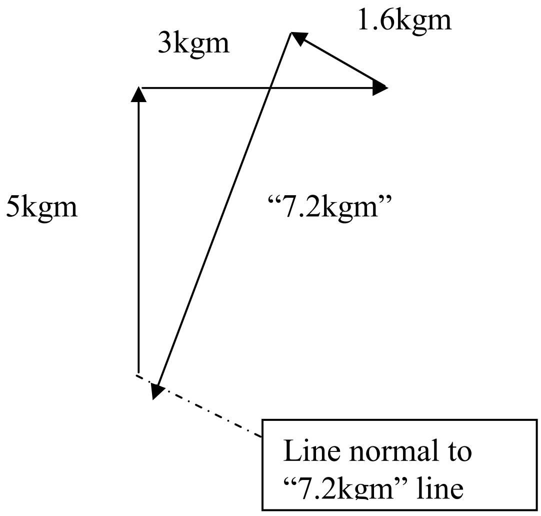
Using Pythagoras, the two components can be combined to give Resultant = 2.214 kgm



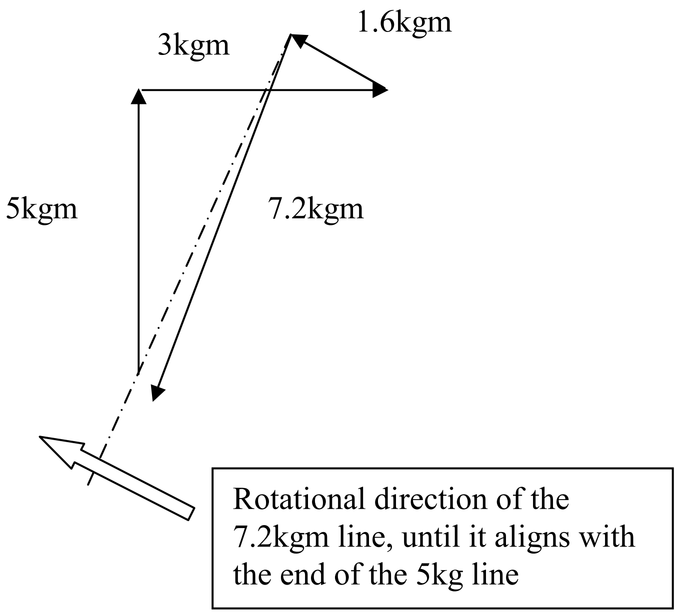
And Tan θ = 0.688/2.104, which gives θ = 18.1 degrees. These answers are of course very similar to those obtained by drawing. If you had any difficulty resolving or re-combining these components then you should look again at the section on Forces and Moments.

Rather than insert a new mass, many engineering systems are balanced by removing mass, i.e. by grinding. This method is typically preferred for balancing fans and turbines. So for the **original** system shown, we could reduce the out of balance of the system by removing some of the 6kg mass, effectively shortening the "7.2kgm" vector so that it ends as near as possible to the start point.

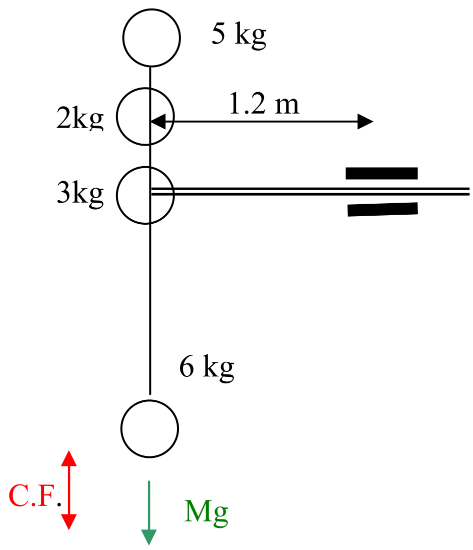
The easiest and quickest method of achieving this would be by drawing the various "mr" values, as shown. Try it yourself!



Another method that could be used to improve the balance of the system would be to **rotate** one of the masses. Look at our original "mr" values and decide how much the 6kg mass should be moved to achieve an improvement in the system balance.



Out of Balance forces acting upon a bearing



In the previous example, we showed that the resultant out of balance centrifugal force was 368N. Consider now that the arrangement is supported in a bearing as shown.

The 368N is a **rotating force**. However there is also the gravitational force of all the other masses to consider. This will equal (5+2+3+6)g = 157 N

Hence the **maximum** force acting on the bearing will be at the instant when the out of balance force and gravitational force both act in the same direction.

Max force = 157 + 368 = 525N
Min force = 157 – 368 = -211N (UPWARDS)

The force on the bearing will also result in a moment on the bearing, as the bearing support and the centrifugal and gravitational forces are not in the same plane.

Hence maximum moment on the bearing will be 525 x 1.2 = 630Nm

The shaft will lift when the moment about the bearing is equal to zero. In this case, neglecting the weight of the shaft itself, this occurs when the centrifugal force is equal and opposite to the gravitational force.

So $157 = mr\omega^2 = 2.15 \times \omega^2$

Which gives a speed for lifting of 8.55 rads/sec. Of course, the shaft will only lift for an instant. As the out of balance centrifugal force rotates, at the other extreme the shaft will be pushed onto its bearings by both its own weight and Centrifugal Force, thus increasing the maximum load on the bearings. In between these two extremes, the shaft will tend to move from side to side. The effects of being out of balance are thus:

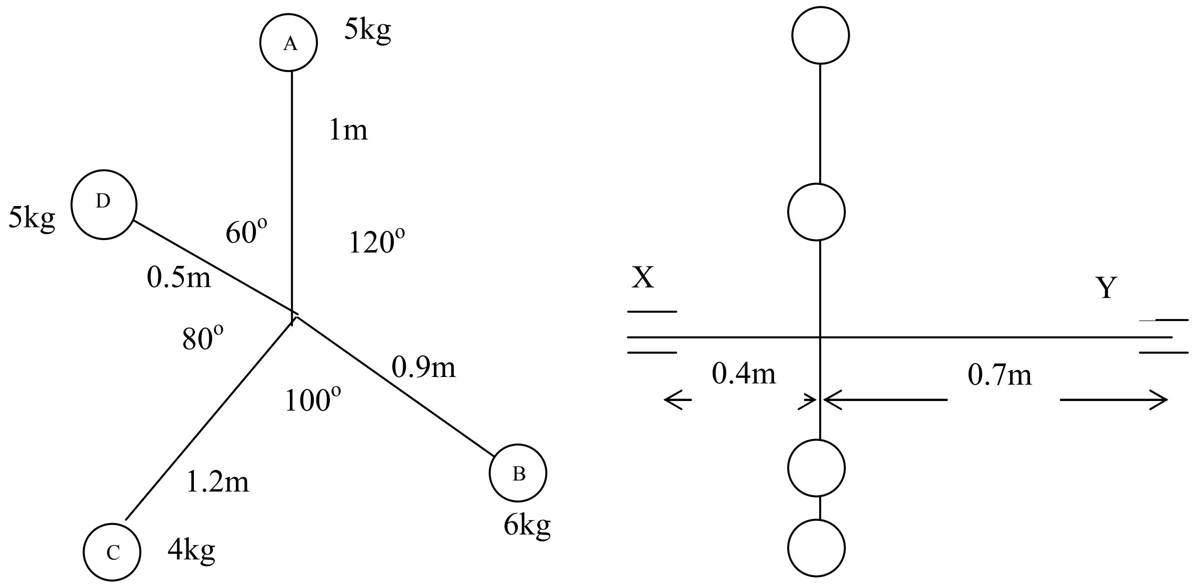
a) Increased maximum load on the bearings, and hence wear
b) Cyclic loading of bearings leading to possible fatigue failure
c) Vibration and noise

Example 5 (Exam standard)

Four masses are arranged on a common shaft as shown below. The system is supported between two bearings X and Y, as shown. The arrangement rotates at 160 rev/min.

(a) Determine the angular adjustment of mass C required to achieve minimum out of balance.
(b) Determine each of the following:

i) the minimum out of balance force achievable after the adjustment of part (a)
ii) the maximum reaction force of bearings X and Y.



STEP ONE

Draw up a table of "mr" values, and since we are going to move "C", we will put this one last.

	m	R	mR kgm
A	5	1	5 Reference
B	6	0.9	5.4 120^0 CW
D	5	0.5	2.5 300^0 CW
C	4	1.2	4.8
Total	20		

STEP TWO

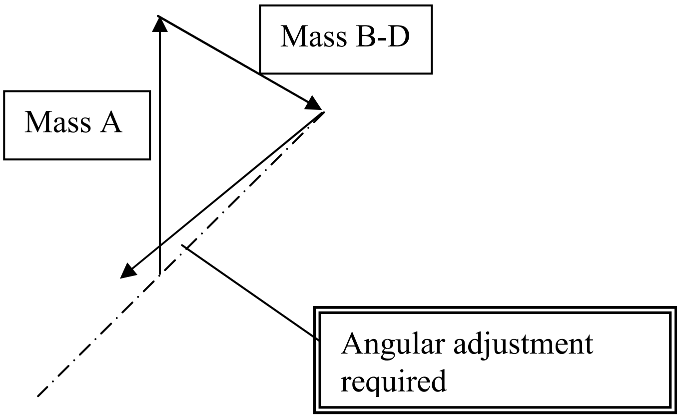
Draw the position of the 5kg and 6kg mass's "mr" value, remembering that the direction of each "mr" value will be radially outwards for the instant considered. Did you notice that two of the masses are opposite one another?? Why not use the resultant of these two masses alone, so that only two lines for mass A and B/D combined need to be drawn!

STEP THREE

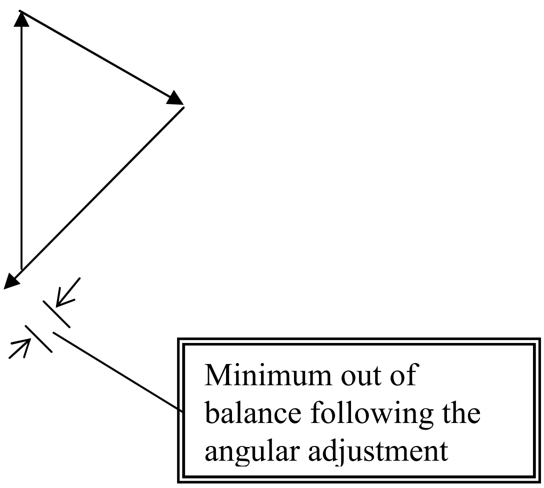
Once the three lines are drawn, then draw a line to connect the two ends. This is the required position of the 4.8 kgm vector.

STEP FOUR

Measure the angle between the original and new position of the 4.8kgm line. Your answer should be 4.5^{o}



STEP FIVE



STEP SIX

Measure the minimum out of balance (my figures gave 0.45 kgm), thus the out of balance force is $0.45\ \omega^2$

$= \quad 0.45 \times 16.8^2$

$= \quad 127N$

Once the minimum out of balance has been calculated, then this force needs to be combined with the gravitational force of the four masses. The total mass is (5+6+4+5) = 20kg = 196.2N

The maximum force will occur when both gravitational force and out of balance forces are combined, and the minimum force will occur when the out of balance force acts vertically up (i.e. opposing the gravitational force).

Hence for this answer maximum force = 127 + 196 = 323N

By taking moments about each bearing, as was demonstrated earlier, then

R_x = 205N and R_y = 117N.

GOVERNORS

A governor is used to try to keep the speed variation of a machine to a minimum. The governors considered here use either:-

(i) Control by gravity (Used by the "Porter" governor)
(ii) Control by spring force (Used by the "Hartnell" governor)

In order to achieve its purpose of maintaining a constant speed regardless of load changes, it is necessary to have a significant mechanical movement in the device, in response to fairy small speed changes. The smaller the speed change to produce a given mechanical movement, the more "Sensitive" the governor.

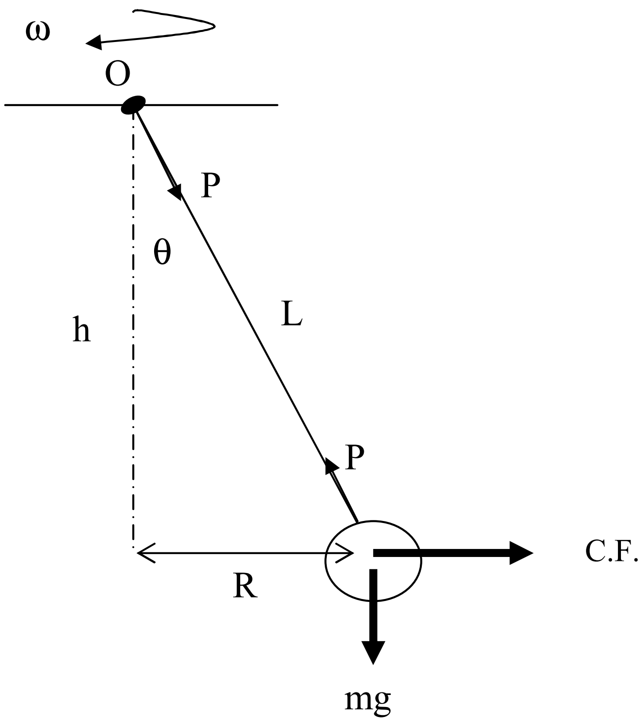
It is usual to specify governor sensitivity in terms of engine speed variations, and governor sensitivity is defined by the ratio:-

$$\frac{\omega_{mean}}{\omega_{max} - \omega_{min}}$$

In other words, the average speed divided by the change of speed, the units cancelling out to give a number only. You will need to remember this formula. If a governor is over sensitive, the engine will be subjected to constantly changing throttle conditions from maximum to minimum, i.e. it will hunt excessively

THE CONICAL PENDULUM

The simple conical pendulum is the basis for the elementary speed governor. It can be represented by a small mass "m" on the end of a light arm (mass negligible), the other end of the arm being pivoted.
The mass is made to revolve at constant angular velocity, in a circular path of radius "R".



For dynamic equilibrium:- Sum moments about O = zero

$$mg \times R - m\,\omega^2 R \times h = \text{zero}$$

$$g = \omega^2 h$$

$$h = \frac{g}{\omega^2} \qquad \therefore \quad \omega = \sqrt{\frac{g}{h}}$$

The angular velocity is therefore independent of the mass "m"

We could also derive this expression by considering the other condition for dynamic equilibrium:- Sum forces along any axis = zero

Vertical:- $P \cos\theta - mg = \text{zero}$

$$P \cos\theta = mg \qquad \text{(i)}$$

Horizontal:- $P \sin\theta - m\,\omega^2 R = \text{zero}$

$$P \sin\theta = m\,\omega^2 R \qquad \text{(ii)}$$

Dividing (ii) by (i) gives

$$\tan\theta = \frac{\omega^2 R}{g}$$

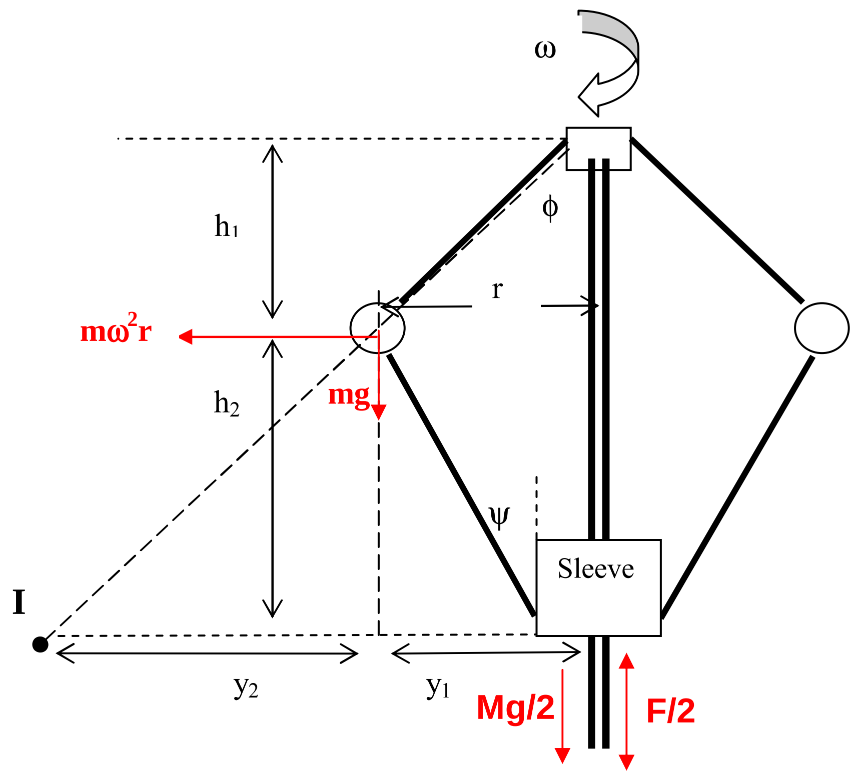
but $\tan\theta = \dfrac{R}{h} = \dfrac{\omega^2 R}{g}$

So $\omega = \sqrt{\dfrac{g}{h}}$ as before.

The conical pendulum was the principal behind the Watt governor. The problem in making practical use of the Watt governor lies in the expression derived above. At low speeds, say between 60 and 100 rpm, we would have a change in governor height of 159mm (try the figures yourself!). This movement could easily be used to alter a fuel linkage to regulate the speed of the engine. At high speeds however, we do not get enough movement for a practical device. For instance, between 500 and 1000 rpm there is less than 2mm change of height. We therefore need to change the characteristics of the basic conical pendulum type governor, and we can do this by either adding a central mass (the Porter governor) or by the use of spring force (the Hartnell governor).

The Porter Governor

The first type the "Porter" governor is illustrated. The arms are sometimes, but not always, of equal length and carry small masses called "bobs".
The arms are pivoted to the upper end of a vertical spindle, and a sleeve which carries the load mass "M" is free to slide on the spindle. The sleeve is connected to the upper arm assemble by the lower arms.
The spindle is driven by the engine and as the speed rises the bobs fly outward, giving an upward motion to the sleeve. This movement is transmitted to the engine throttle by a linkage.



When the speed changes, the following actions will occur. If the rotational speed increases, then the centrifugal force of the sensing balls will increase causing the ball radius "r" to increase, and this results in the sleeve moving **up**. This movement of the sleeve is transmitted to the speed control valve, and regulates the speed of the prime mover.

Similarly, when the rotational speed decrease, then the centrifugal force of the sensing balls will decrease causing the ball radius "r" to decrease, and this results in the sleeve moving **down**. This movement of the sleeve is transmitted to the speed control valve, and regulates the speed of the prime mover.

There are two methods that we can use to evaluate the forces on the Porter Governor, a graphical solution or by calculation

Calculation method

For this method we shall examine the forces on the governor using moments. We shall examine only one side of the governor, hence **the sleeve forces that are common to both balls will be halved**. To simplify the calculation, we shall use the point "I" as the datum for the moments.

There are four forces present

1. Centrifugal force ($m\omega^2 r$) acting radially outwards from the ball centre
2. Gravitational force (mg) acting downward from the ball centre
3. Sleeve gravitational force (Mg/2) acting downward from the sleeve centre
4. Sleeve friction force (F/2) acting opposite to the direction of motion of the sleeve

Taking moments about the point I, and clockwise moments are +ve

$$(mg \times y_2) + (Mg/2 \times (y_2 + y_1)) \pm (F/2 \times (y_2 + y_1)) - (m\omega^2 r \times h_2) \quad = \quad 0$$

Note that if the arms are of equal length, then $h_1 = h_2 = h$ and $y_1 = y_2 = r$. This causes the equation above to become:-

$$(mg \times r) + (Mg/2 \times 2r \pm (F/2 \times 2r - (m\omega^2 r \times h) \quad = \quad 0$$

"r" is thus common to every term, and the "2's" cancel, to give:-

$$mg + Mg \pm F - m\omega^2 h = \quad 0$$

Which can be rearranged to give $\omega = \sqrt{\dfrac{mg + Mg \pm F}{mh}}$ or $h = \dfrac{mg + Mg \pm F}{m\omega^2}$

Note that these last two expressions can only be used if the arms are of equal length, and that both "M" and "F" are the **total** sleeve mass and frictional force.

The Effect Of Friction.

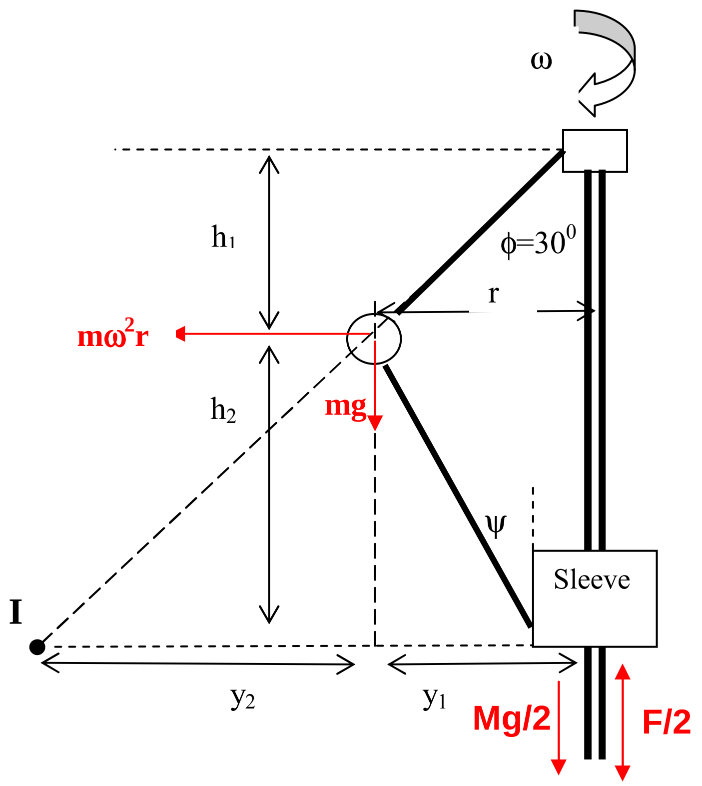
Friction will exist when the governor is altering position, it will exist at the pins and at the sleeve.

It is usual to allow for the combined effect of friction by a single force acting at the sleeve. This friction will always oppose motion, it will be directed upward when the sleeve moves down and directed down when the sleeve moves up. A careful look at the expressions derived above shows that the gravitational forces (mg and Mg) have both turned out to be positive in these equations. It follows that friction will therefore also be positive if in the same, downwards, direction. Friction will be downwards if movement of the sleeve is upwards, which happens when the speed is increasing.

Example 6

Consider a Porter Governor in which the upper arm is 200mm long and the lower arm is 250mm long. The radius of the ball rotation is 100mm. The ball mass is 2kg, and the sleeve mass is 15kg. Sleeve friction is constant at 49N. If the angle between upper arm and rod is 30^o, calculate the rotational speed of the governor if the governor speed is rising.

STEP ONE
Draw the governor, with all forces and dimensions clearly shown



STEP TWO
As we are given the angle ϕ, we are able to calculate the dimension h_1.

$h_1 = \cos 30^o \times 200 = 173.2\text{mm}$

From Pythagoras, $h_2 = \sqrt{250^2 - 100^2}$ = 229.1mm

From similar triangle rule h_2/h_1 x 100 = y_2 = 229.1/173.2 x 100 = 132.3mm

Therefore $y_2 + y_1$ = 0.1323 + 0.1 = 0.2323

STEP THREE
Evaluate the moments about the point I. (Note that friction will act down, as the sleeve is about to rise)

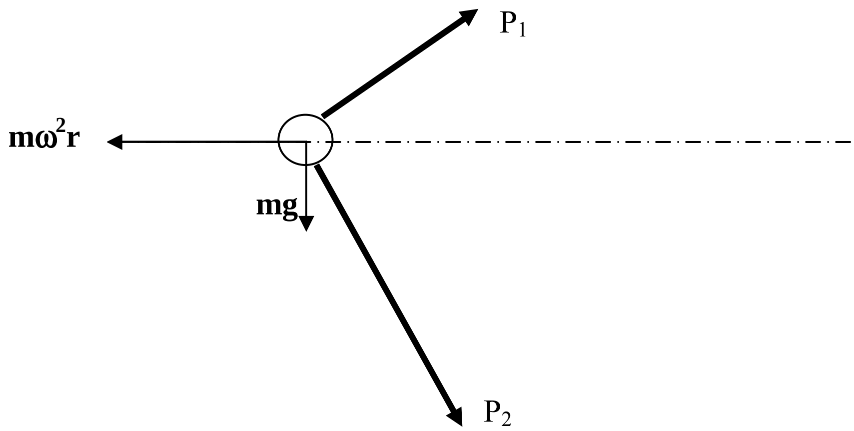
$$(mg \text{ x } y_2) + (Mg/2 \text{ x } (y_2 + y_1)) + (F/2 \text{ x } (y_2 + y_1)) - (m\omega^2 r \text{ x } h_2) = 0$$

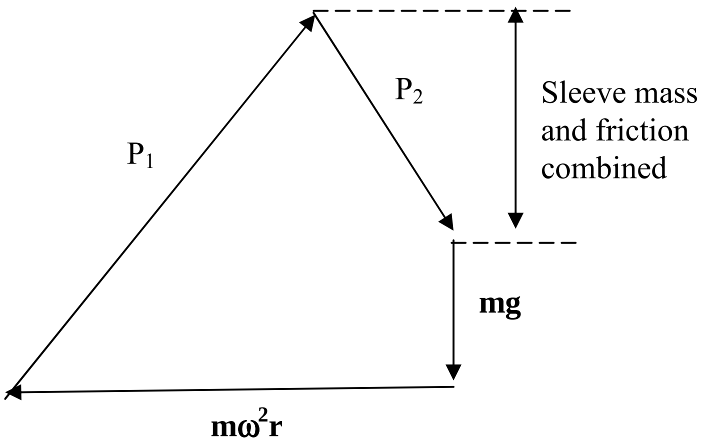
$$(2g \text{ x } 0.1323) + (15g/2 \text{ x } (0.2323)) + (49/2 \text{ x } (0.2323)) - (2\omega^2 \text{ x } 0.1 \text{ x } 0.229) = 0$$

$$2.596 + 17.09 + 5.691 - 0.0458\omega^2 = 0$$

Therefore ω = **23.54 rad/sec** with the speed rising

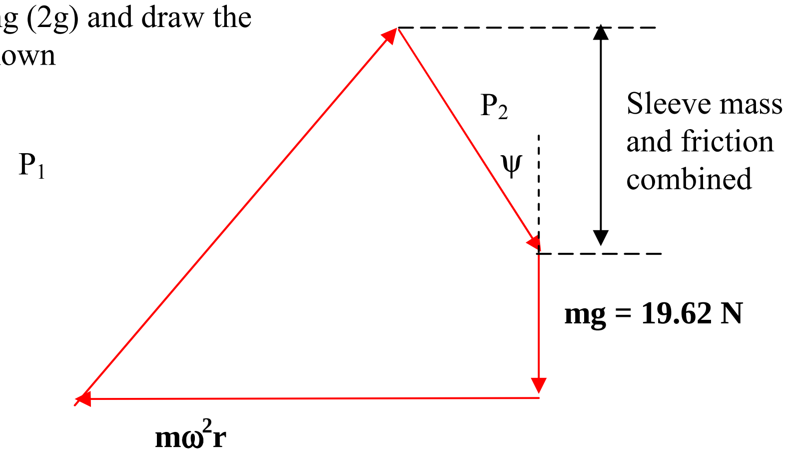
This question can also be solved using a graphical method. Here, we will look at all the forces acting **at** the sensing ball. When taking moments, the arms are ties and the forces at each end cancel each other out. When considering the forces at the ball, however, we must include the tension at the end of each arm.





We do not know the magnitude of the tension in the sensing arms P_1 and P_2 but we do know the angles of the arms. We also know that **the vertical component of P_2 must be equal in magnitude to the weight of the sleeve mass plus friction**, as **these are the vertical forces at the other end of this lower arm**. Remember that the friction force will allows oppose the motion of the sleeve, so when the speed is falling and the sleeve is moving down, then the friction force will act upwards. We can now draw the force diagram.

STEP ONE
Calculate the gravity force mg (2g) and draw the force vector as 19.62N as shown



STEP TWO
Calculate the angle ψ from the dimensions given . Here, $\psi = \sin^{-1} 100/250 = 23.6^o$

STEP THREE
Calculate the vertical component of P_2. As the speed is rising and the sleeve is about to move up, then this will equal the sleeve gravitational force + the sleeve frictional force combined.
So vertical component = $(15g + 49)/2 = 98.08$ N

STEP THREE
Draw the line P_2 at an angle of 23.6^o from the vertical, until the vertical dimension equals 98.08 N.

STEP FOUR
Once line P_2 is drawn, then draw a line down from this point at 30^o to the vertical. Where this line crosses the horizontal line of $m\omega^2r$, then the force due to centrifugal force and P_1 can now be found.

STEP FIVE
My drawing gave a figure for centrifugal force as 109 N. Thus $\omega = 23.3$ rad/sec.

Whether you choose the graphical or calculation method will depend upon the question and your personal preference, but you would be advised to **practice both** methods, as the skills you will pick up you be needed in many areas of the subject material.

Example 7 (Exam standard)

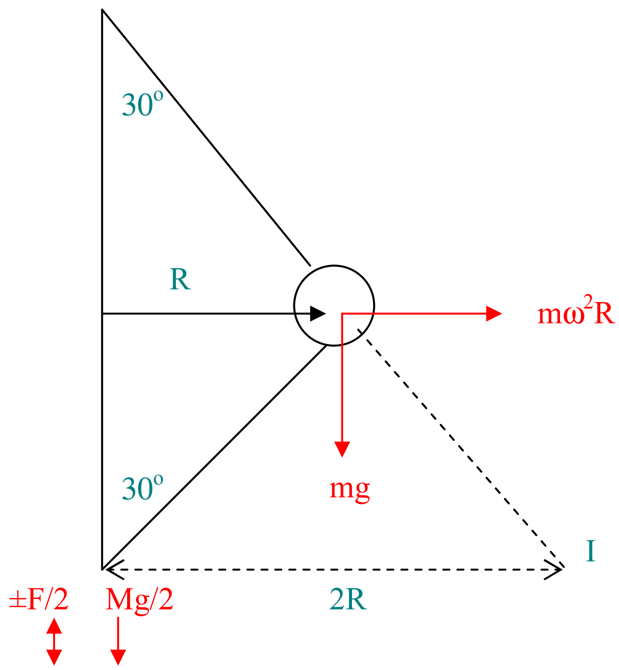
A Porter governor has arms of equal length 300mm and rotating masses of 3kg. At the mean spee of 180rev/min, with the speed rising, both sets of arms are at 30 degrees to the vertical. Friction of the central sleeve is constant at 30N.

Determine each of the following:

(a) the central sleeve mass;
(b) the speed that would cause the sleeve to rise 20mm;
(c) the speed that would cause the sleeve to fall 20mm

Solution:

In this question, the length of each arm is equal, which makes the calculation of the various dimensions slightly easier. At the datum speed of 180 rev/min the arms are at 30^o to the vertical, so the ball radius can be calculated. But first, a sketch.



STEP ONE
Draw the governor will ALL the forces and dimensions shown

STEP TWO
Ball radius $= 300\sin 30^0 = 150$mm
$h_1 = h_2 = 300\cos 30^o = 260$mm
$y_1 = y_2 =$ ball radius $= 150$mm

STEP THREE

Specify the direction of the friction force F. As the initial speed is rising, then the sleeve friction will act down.

STEP FOUR
Take moments about point I
$(mg \times R) + (Mg/2 \times 2R) + (F/2 \times 2R) – (m\omega^2 r \times h_2) = 0$
$(3g \times 0.15) + (Mg/2 \times (0.3)) + (30/2 \times (0.3)) – (3 \times (2\pi180/60)^2 \times 0.15 \times 0.26) = 0$
$4.4 + 1.47M + 4.5 – 41.57 = 0$

Thus M = 22.2kg

STEP FIVE
When the sleeve moves up 20mm, then this will alter the dimensions of h_1 and h_2. Both will change equally, and reduce by the **total** amount 20mm, **hence both will reduce by 10mm**. So $h_1 = h_2 = 250$mm. With the speed and sleeve both rising then friction will act down.

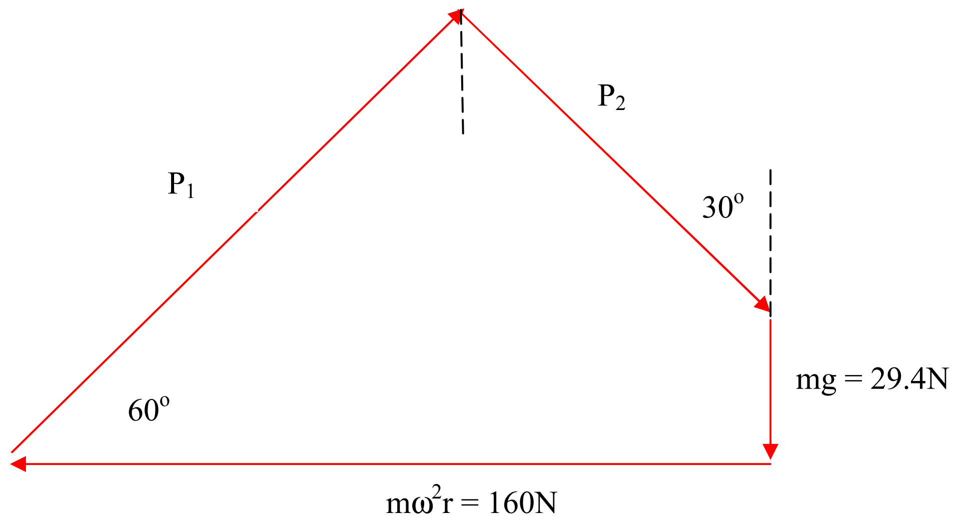
STEP SIX
Take moments about point I
$(mg \times y_2) + (Mg/2 \times (y_2 + y_1)) + (F \times (y_2 + y_1)) – (m\omega^2 r \times h_2) = 0$
$(3g \times 0.15) + (22.2g/2 \times (0.3)) + (30/2 \times (0.3)) – (3\omega^2 \times 0.15 \times 0.25) = 0$
$4.4 + 32.7 + 4.5 – 0.1125\omega^2 = 0$
Therefore ω = 19.2 rad/sec with the speed rising

STEP SEVEN
When the sleeve moves down 20mm, then this will again alter the dimensions of h_1 and h_2. Both will change equally, and increase by the **total** amount 20mm, **hence both will increase by 10mm**. So $h_1 = h_2 = 270$mm. With the speed and sleeve both rising then friction will act UP.

STEP EIGHT
Take moments about point I
$(mg \times y_2) + (Mg/2 \times (y_2 + y_1)) + (F \times (y_2 + y_1)) – (m\omega^2 r \times h_2) = 0$
$(3g \times 0.15) + (22.2g/2 \times (0.3)) + (30/2 \times (0.3)) – (3\omega^2 \times 0.15 \times 0.27) = 0$
$4.4 + 32.7 - 4.5 – 0.1215\omega^2 = 0$

Therefore ω = 16.4 rad/sec with the speed falling

This question could also be answered using the graphical method. The ball weight is known at 3g or 29.4N, and the datum centrifugal force is known as $m\omega^2 r = 160N$.



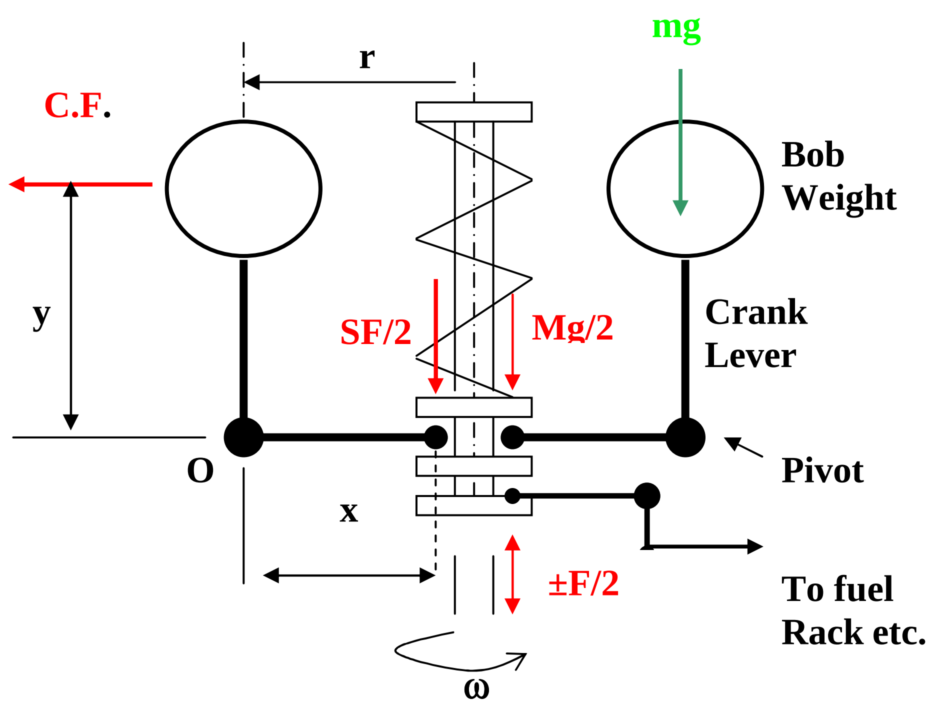
The lines P_1 and P_2 can now be drawn, using the known angles of 30^o for the arm P_2 and 60^o for the arm P_1.

The vertical dimension of P_2 can now be found by measurement. This dimension is the resultant of the sleeve friction and the sleeve mass.

To determine the other two parts of this question, requires the student to re-calculate the change in the angles of the arms. This angle will change for each speed change.

THE HARTNELL GOVERNOR

The Hartnell governor uses spring force to resist the tendency to movement caused by centripetal acceleration.



Let

m = mass of one bob weight (kg)

SF = Total Spring Force exerted on sleeve (N)

x = length of sleeve arm (m)

y = length of ball arm (m)

ω = angular velocity (rads/sec)

M = Central mass of sleeve (sometimes negligible)

F = total friction force (referred to sleeve)

The bob weights are attached to a carrier which causes them to rotate at the same speed as the driving shaft. In use, the governor operates with the arms only slightly away from the horizontal or vertical positions, so that the moment of the bob weights about the pivot may be neglected, as it is small compared to the moments of the centrifugal force and spring force.

Taking moments about the pivot "O", and considering only one bob weight, then

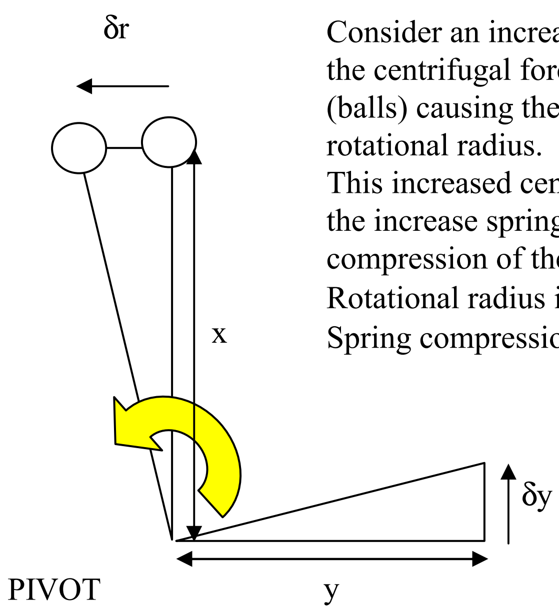
$$(SF/2\ (x)) + (Mg/2\ (x)) +/- (F/2\ (x)) - (m\omega^2 r\ y) = 0$$

Note that as we have considered only one flyweight, each term of the spring force, sleeve mass and friction is halved.

Now consider the changes that occur when the governor speed **falls**.

1. Spring Force (SF) will decrease as the spring tension is reduced. This reduction will be dependant upon the spring stiffness (k) and movement of the sleeve (δy). So change in spring force will be ($k\delta y$).
2. The rotational radius of the ball masses will reduce
3. The pivot levers x and y will also change, but these changes are small, and effect the Centrifugal Force and Spring Force moments equally, so that they can be ignored, and therefore x and y will be assumed constant for all conditions.
4. The friction within the sleeve will now act UP as it is opposing the motion of the falling sleeve.

In Hartnell governors, the change in compression of the central spring is directly related to the change in the ball radius. These two variables are linked by the proportions of the dimensions x and y, as they are connected at the common pivot point.



Consider an increase in speed. This will increase the centrifugal force on the sensing flyweights (balls) causing the balls to move out to a larger rotational radius.
This increased centrifugal force is BALANCED by the increase spring force that occurs from the compression of the springs.
Rotational radius increases by δr
Spring compression increases by δy

Example 8

A Hartnell governor is used to control the speed of an engine operating at 400 rev/min. The governor details are:

Ball arm length	40mm
Sleeve arm length	60mm
Flyweight mass	2 kg
Inner sleeve mass	2 kg
Spring stiffness	10 kN/m
Spring tension at 400 rev/min	330N

Assume speed is falling at 400 rev/min and that friction is constant at 12N

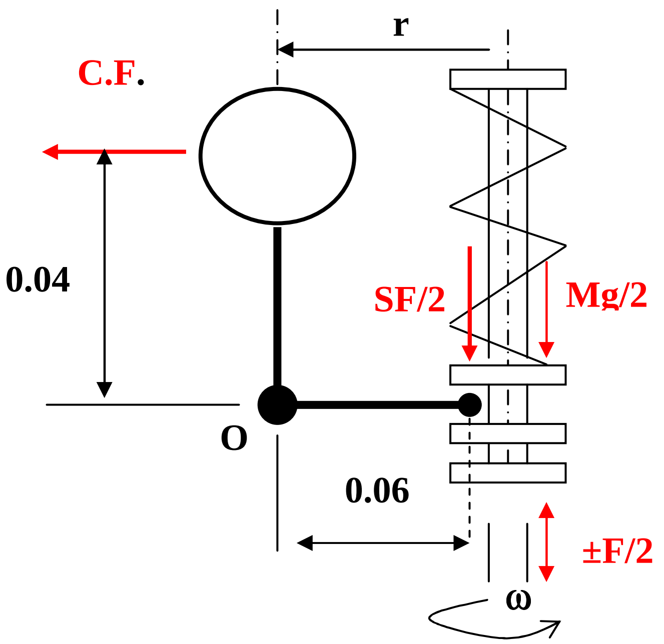
Calculate the movement of the inner sleeve from the datum speed of 400 rev/min, when the engine speed changes to:

a) 425 rev/min

b) 380 rev/min

STEP ONE

Draw the simple view of one crank, showing the centrifugal forces, and spring and friction forces. Show the dimensions of the flyweight arms



STEP TWO

Evaluate the moments about the pivot for the various speeds. Remembering to ensure that you have friction in the correct direction, opposing motion.

From (SF/2 x 0.06) + (Mg/2 x 0.06) +/- (F/2 x 0.06) – ($m\omega^2 r$ x 0.04) = 0

At 400 rev/min, $\omega = 41.89$ rad/sec

So $(330/2\ (0.06)) + (2g/2\ (0.06)) - (12/2\ (0.06)) - (2.\ 41.89^2\ r\ 0.04) = 0$
By putting in all the known data we can see that we do not know the rotational radius of the balls, so calculating the unknown r:

$9.9 + 0.589 - 0.36 - 140.4\ r = 0$

Therefore r = 72.1mm

STEP THREE
When the speed changes so will the radius r, and the spring force. If the speed increases then the spring is compressed as the sleeve rises, so the friction will act down.

When the speed increases to 425 rev/min (44.51 rad/sec), the spring force will change to:

Initial spring force + spring compression change
$= 330 +$ spring stiffness (k) x δy but $\delta y = \delta r\ x/y$
$= 330 + 10000 \text{ x } \delta r\ 60/40$
$= 330 + 15000 \text{ x } \delta r$

The new centrifugal force will change to $m\omega^2\ (r + \delta r)\ y$
$= 2 \text{ x } 44.51^2 \text{ x } (0.0721 + \delta r)\ 0.04$
$= 158.5\ (0.0721 + \delta r)$
$= 158.5\ \delta r + 11.43$

Placing both of these new values in our original evaluation of moments about the pivot point gives:
$(SF/2\ (0.06)) + (Mg/2\ (0.06)) + (F/2\ (0.06)) - (m\omega^2\ (r + \delta r)\ y) = 0$

$$\frac{(330+15000\times\delta r)\times 0.06}{2}+\frac{(2g)\times 0.06}{2}+\frac{(12)\times 0.06}{2}-(158.5\times\delta r+11.43)=0$$

$9.9 + 450\delta r + 0.589 + 0.36 - 158.5\delta r - 11.43 = 0$

$-0.581 + 291.5\delta r = 0$

Therefore δr = 2mm

So the radius will increase by 2mm, and the sleeve will move UP by
$\delta y = \delta r\ x/y = 2.60/40 = 3$mm

STEP FOUR
Now consider what occurs when the speed falls. Under these conditions the spring compression will reduce, and the ball rotational radius will decrease. As the sleeve is falling then friction will act UP.

When the speed decreases to 380 rev/min (39.79 rad/sec), the spring force will change to:

Initial spring force - spring compression change

$= 330 - \text{spring stiffness (k)} \times \delta y$ but $\delta y = \delta r\ x/y$

$= 330 - 10000.\ \delta r\ 60/40$

$= 330 - 15000.\ \delta r$

The new centrifugal force moment will change to $m\omega^2\ (r - \delta r)\ y$

NOTE THE –ve VALUE for δr

$= 2 \times 39.79^2 \times (0.0721 - \delta r)\ 0.04$

$= 126.7\ (0.0721 - \delta r)$

$= 126.7\ \delta r - 9.13$

Placing both of these new values in our original evaluation of moments about the pivot point gives:

$(SF/2\ (x)) + (Mg/2\ (x)) - (F/2\ (x)) - (m\omega^2\ (r - \delta r)\ y) = 0$

(NB FRICTION ACTS DOWN)

$$\frac{(330+15000\times\delta r)\times 0.06}{2}+\frac{(2g)\times 0.06}{2}-\frac{(12)\times 0.06}{2}-(126.7\times\delta r-9.13)=0$$

$9.9 - 450\delta r + 0.589 - 0.36 - 126.7\delta r + 9.13 = 0$

$19.26 - 576.7.\ \delta r = 0$

Therefore $\delta r = 33.4$mm and $\delta y = \delta r\ x/y$, so $\delta y = 50$mm

Example 9 (Exam standard)

A Hartnell governor is fitted with two flyweight rotating balls of mass 0.5 kg. The balls orbital radius is 80mm, and the balls are vertical when the governor input speed is 4320 rev/min and rising. The sleeve friction can be assumed constant at 15N. The length of the ball arms are 60mm, and the sleeve arms are 50mm.
Ignore the gravitational effects of the flyweight balls.

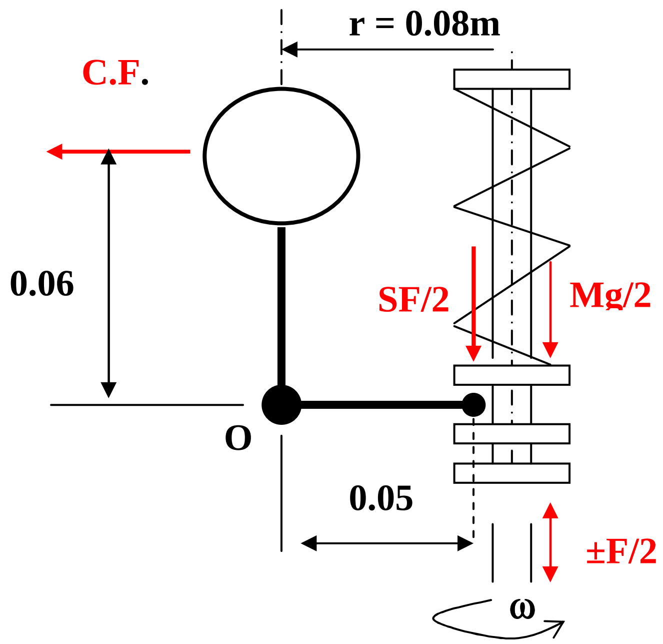
Calculate EACH of the following:

a) The spring stiffness, if the sleeve is measured to rise 3 mm, when the speed increases to 5000 rev/min (10)

b) The fall in the sleeve when the speed falls from 4320 to 4000 rev/min (6)

STEP ONE

Draw one half of the flyweights showing all forces that are present. Include your dimensions at this stage. **Remember the larger your sketch, the less likely you are to make a simple mistake that will cost you marks and time in the examination**.



STEP TWO
Evaluate the moments about the pivot.
$(SF/2\ \ (x)) + (Mg/2\ (x)) + (F/2\ \ (x)) - (m\omega^2 r\ \ y) \quad = \quad 0$

Write down the various forces that are present
Spring force = unknown
Sleeve mass = **not given**, so we must assume we can **ignore** this. It is good practice **to state any assumptions** that you make during your analysis.
Sleeve friction = 15N and acting down as the sleeve is just rising
Centrifugal force = $m\omega^2 r$ $\quad \omega = 452.39$ rad/sec

Thus $(SF/2\ \ (0.05)) + (15/2\ \ (0.05)) - (0.5.\ 452.39^2.0.08\ .0.06) \quad = \quad 0$

$0.025SF + 0.375 - 491.18 = 0$

So $SF_1 = 19.63$ kN

This only gives the spring force at the datum speed of 4320 rev/min. We now need to calculate the spring force at the new speed of 5000 rev/min.

STEP THREE

When the governor changes speed, then both the ball rotational radius and the spring compression will change. As we have been given the amount the sleeve rises, we can calculate the new rotational radius.

Write down the various forces that are present
Spring force = unknown but larger than before
Sleeve friction = 15N and still acting down as the sleeve is still rising

Centrifugal force = $m\omega^2(r + \delta r)\ y$ $\omega = 523.6$ rad/sec. From $\delta y = \delta r\ x/y$, then $\delta r = \delta y\ y/x = 0.003.60/50 = 0.0036$m.
Evaluate the moments about the pivot.
$(SF/2\ (x)) + (Mg/2\ (x)) + (F/2\ (x)) – (m\omega^2 r\ y) = 0$

$$\frac{(SF)\times 0.05}{2} + \frac{(15)\times 0.05}{2} - (0.5\times 523.62\times(0.08+0.0036)\times 0.06) = 0$$

$0.025SF + 0.375 – 687.6 = 0$

So $SF_2 = 27.49$ kN

STEP FOUR
Now we have the spring force at two different speeds. We also know that the spring has been compressed by 3mm to achieve this change in spring force, so

Spring compression = $k = (SF_2 – SF_1)/x$
$= (27.49 – 19.63)/0.003$
$= 2620$ kN/m

STEP FIVE
Once we have calculated the spring stiffness, we can now complete the final part of the question where the speed is falling to the new speed of 4000rev/min.

Evaluate the moments about the pivot.
$(SF/2\ (x)) - (F/2\ (x)) – (m\omega^2\ (r - \delta r)\ y) = 0$

Write down the various forces that are present
Spring force $= SF_1$ - spring compression change
$= 19630 – 2620{,}000 \times \delta y$ but $\delta y = \delta r\ x/y$
$= 19630 – 2620{,}000 \times \delta r\ 50/60$
$= 19630 – 2183{,}333 \times \delta r$
Sleeve friction = 15N and acting UP as the sleeve is falling
Centrifugal force = $m\omega^2 r$ $\omega = 418.88$ rad/sec

$$\text{So } \frac{(19630 - 218333\delta r)}{0.1} - \frac{(15)\times 0.05}{2} - 0.5\times 418.88^2\times(0.80-\delta r)\times 0.06 = 0$$

$490.75 – 54583.3.\ \delta r – 0.375 – 5263.8\ (0.08 - \delta r) = 0$

$490.75 – 54583.3.\ \delta r – 0.375 – 421.1 + 5263.8\delta r = 0$

$69.27 – 49319.5.\ \delta r = 0$

Therefore $\delta r = 1.4$mm, so $\delta y = \delta r.\ 50/60 =$ **1.17mm**.

So the sleeve moves DOWN by 1.17mm.

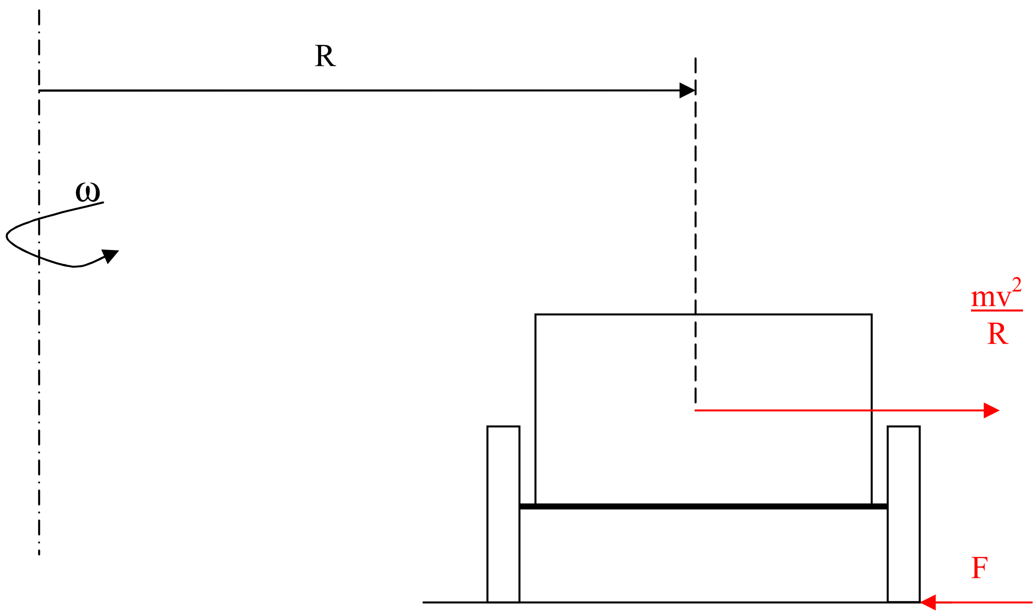
SIDE SKIDDING ON CURVED TRACKS.

When a vehicle travels round a bend in the road there is a tendency to side slip. The tendency to slip is dependent on the centrifugal force exerted and the friction resistance at the wheels.

If the friction grip is sufficient to prevent side slip there is a danger of overturning by pivoting about the outer wheels. Overturning depends on the centrifugal force and the width of the wheelbase

Example 10

A car of mass 2 tonne rounds an un-banked curve of 60 m radius at 72 km/h. Calculate the side thrust on the tyres.



Solution

The radial forces acting on the car are:

(a) the **radially** outward inertia (centrifugal) force, $\dfrac{mv^2}{R}$

(note we use this form for the expression, rather than its other form, $m\omega^2 R$, since we are more likely to know a vehicles linear speed, v)

(b) the inward force F exerted by friction at the road on the tyres,

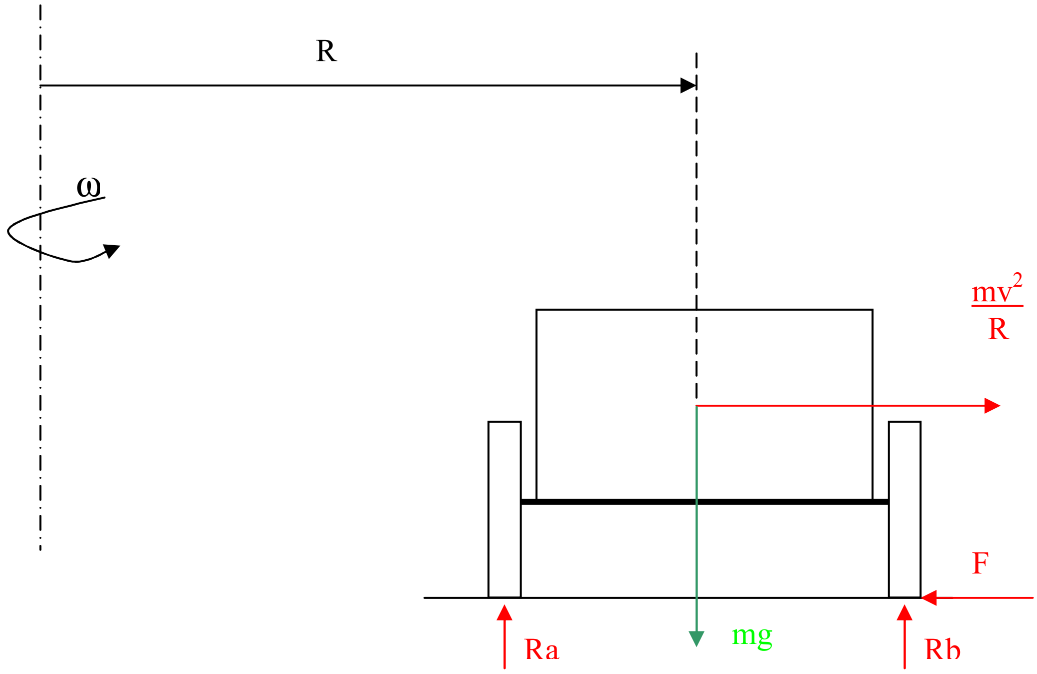
As these are the only radial forces present, they must be equal and opposite, hence

$$F = \frac{mv^2}{R}$$

$$= 2 \times 10^3 \times \frac{(72 \times 10^3)^2}{3600} \times \frac{1}{60} \qquad = 13{,}330 \text{ N}$$

Example 11:-

A four wheeled vehicle of mass 4 tonne traverses an un-banked curve of 100 m radius at 12 m/s. The wheel track width is 2 m and the centre of gravity of the vehicle is 750 mm above the road. calculate the normal reactions at each wheel.



Sum moments about inner wheel track A = zero clockwise +ve

$$\frac{M.v^2}{R} \times 0.75 + Mg \times 1 - R_b \times 2 = \text{zero}$$

$$R_b \times 2 = \frac{4 \times 10^3 \times 12^2 \times 0.75}{100} + 4 \times 10^3 \times 9.81 \times 1$$

$$= 4{,}320 + 39{,}200 \qquad R_b = 21{,}760 \text{ N}$$

$\therefore$ force on **each** outer wheel (remember to read the question, there are two of them!)

$= 10{,}880$ N

Summing vertical forces

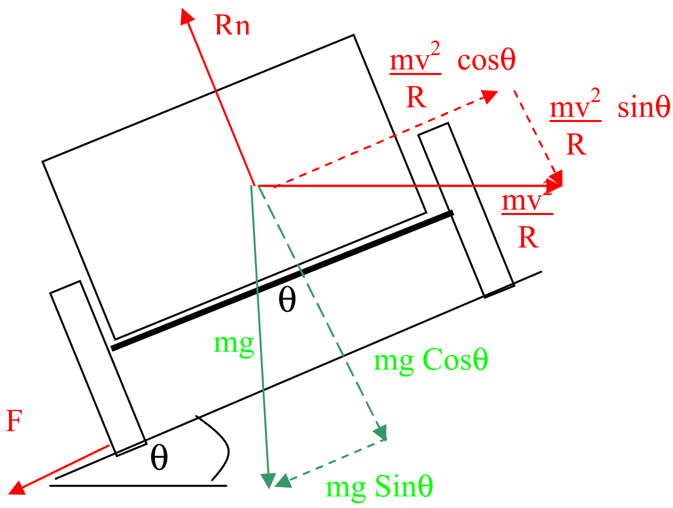
$$R_b + R_a = Mg$$

$$R_a + 21{,}760 = 4 \times 1{,}000 \times 9.81 \quad = 17{,}440 \text{ N}$$

Force on **each** inner wheel = 8,720 N

Example 12.

Calculate the angle of banking on a bend of 100 m radius so that vehicles can travel round the bend at 50 km/hr without side thrust on the tyres. For this angle of banking what is the value of the coefficient of friction if skidding outwards just commences for a car travelling at 120 km/hr.



Summating forces parallel to the slope;-

$$\frac{M.v^2}{R} \text{ Cos } \theta \ - \ F - \ Mg \text{ Sin } \theta \ = \ \text{zero}$$

For no side thrust, then the frictional force = 0 (no tendency to slide either way),

$$\text{Hence:- } \tan\theta = \frac{v^2}{g.R} = \frac{13.89^2}{9.81 \times 100} \qquad \theta = 11.2^o$$

When skidding outwards commences we are at the limiting value of friction, so

$$F = \mu \times R_n \qquad = \mu \left(\frac{mv^2}{R} Sin\theta + MgCos\theta \right)$$

Summating forces parallel to the slope;-

$$\frac{mv^2}{R} \text{ Cos } \theta - \mu \left(\frac{mv^2}{R} Sin\theta + MgCos\theta \right) - \text{ Mg Sin } \theta = \text{ zero}$$

$$\mu \frac{33.33^2}{100} x\, 0.194 + 9.81\, x\, 0.981 + 9.81\, x\, 0.194 = \frac{33.33^2}{100} x\, 0.981$$

$\underline{\mu = 0.765}$

An alternative, and quicker way of doing this example would be by the friction angle method.

Try it!

Example 13 (exam standard)

A vehicle travels around a banked track at a constant speed of 30m/sec. The vehicle has a wheelbase width of 1.3 metres and centre of gravity 1.8 metres above the track surface, and travels on an effective radius of 150metres.

Calculate each of the following:

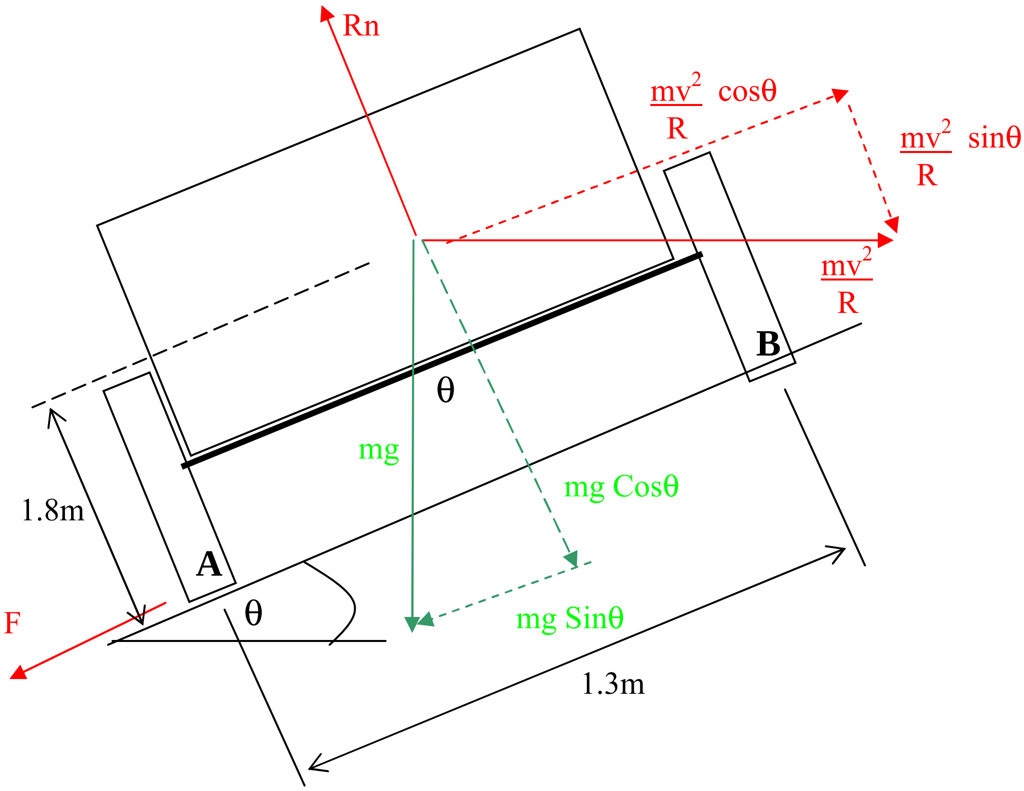
(a) the angle of banking required to prevent the vehicle from overturning;

(b) the minimum coefficient of friction between track and the to prevent the vehicle from sliding at this constant speed

Solution:

STEP ONE

Draw the vehicle in place, with ALL forces and dimensions shown. Ensure your drawing size is **at least** half a page, as this will reduce the chance of an error.



STEP TWO

With this type of question, we should evaluate the forces and moments present. So, do your sketch, put down the forces present, then produce your equations for the forces parallel to the plane, and the moments about the outer wheel B (the wheel that remains on the ground).

Summating forces parallel to the plane, (up the plane is +ve)

$m\omega^2 r \cos\theta - \mu\ (mg\cos\theta\ + m\omega^2 r\sin\theta - mg\sin\theta) = 0$

Summate the moments about the outer wheel B, clockwise positive. Remember, at the moment of overturning the inner wheel is just about to lift from the ground, so the reaction there is zero.

$(mg\cos\theta + m\omega^2 r\sin\theta)(1.3/2) - (m\omega^2 r\cos\theta - mg\sin\theta)1.8 = 0$

If you have drawn your diagram **clearly and correctly**, so should have these two equations. If your equations differ, look back to find where you have made the error. Do not proceed unless you have identified any errors, as you will often make the same mistake again.

Note that when we split a force into its' components, as we have done here, **the components still act at the point of application of the force**.

Note also that we have chosen to take moments of the **four component forces**, rather that just the two forces (centrifugal and gravity). Why? Well, remember moment is force times **perpendicular** distance, and here ***we know the perpendicular distances to the four components***, but we **do not** know the perpendicular distances to the line of action of the two main forces. This is an important principle, make sure you understand and remember it.

STEP THREE

Looking at our two equations, and then studying the information that we have been given, shows that we do not know θ or μ. In fact both of these variables have been asked for in the question. So to find θ, we shall use the moment equation, as θ is the only variable that is unknown. Note that the m value is common to all variables in the equation and will therefore cancel out.

ω is equal to v/r = 30/150 = 0.2 rad/sec.

$(mg \cos \theta + m\omega^2 r \sin \theta)0.65 - (m\omega^2 r \cos \theta - mg \sin \theta)1.8 = 0$

$1.3/2 (g \cos \theta + 0.2^2\ 150 \sin \theta) - 1.8 (0.2^2\ 150 \cos \theta - g \sin \theta) = 0$

$6.38 \cos \theta + 3.9 \sin \theta - 10.8 \cos \theta + 17.7 \sin \theta = 0$

$-4.4 \cos \theta + 21.6 \sin \theta = 0$

Thus $4.4 \cos \theta = 21.6 \sin \theta$. Dividing by $\cos \theta$, gives $4.4 = 21.6 \tan \theta$,

So $\theta = \tan^{-1} 4.4/21.6 = 11.5^o$ ⟶

STEP FOUR

Now we have found θ, we can use the force equation to find the final answer for the coefficient of friction (μ)

From $\omega^2 r \cos \theta - \mu (g \cos \theta + \omega^2 r \sin \theta) - g \sin \theta = 0$ (canceling the mass value)

$0.2^2\ 150 \cos 11.5 - \mu (g \cos 11.5 + 0.2^2\ 150 \sin 11.5) - g \sin 11.5 = 0$

$5.88 - \mu (9.61 + 1.2) - 1.96 = 0$

$5.88 - 1.96 = \mu (10.81)$

So $\mu = 3.92/10.81 = 0.362$ ⟶

Self Assessed Questions for you to try. (Answers given)

1 A uniform shaft with a mass of 3 tonne is 3.25 m long. It carries an unbalanced flywheel of 1 tonne mass at 1.25 m from one bearing, the bearing being situated at each end of the shaft. If the centre of gravity of the flywheel is 75 mm from the centre of the shaft, find the maximum and minimum reactions on the bearing nearest to the flywheel when the shaft is rotating at 60 rev/min.

Ans: 22.57 kN and 18.93 kN

2 A vehicle travels at a speed of 108 km/h around a bend of 100 m radius. If the angle of banking for the bend is 15^o find the minimum coefficient of friction required between wheels and the road surface if side slip is not to occur.
Ans: 0.53

3 A car is moving round a bend of 80 m radius at a speed of 45 km/h.

(a) If the road surface is horizontal, find minimum value of μ between wheels and road surface so that car will not skid.

(b)What would be the correct angle of banking to eliminate side slip?

(c) If μ were 0.5, determine the maximum permissible speed around the bend when banked if the lateral forces at the road surface are in equilibrium.
Ans: (a) 0.1995 (b) 11.25^o (c) 88.8 km/h

4 Two balancing masses of 10 kg and 20 kg are attached to a shaft at radii of 600 mm and 900 mm respectively, and displaced through 90^o from each other. If the shaft is rotating at 210 rev/min, find the magnitude and position of the mass required to counteract the unbalanced centrifugal force if it is to be fitted at a radius of 750 mm.
Ans: 25.3 kg at 108.4^o ACW from 10 kg mass

5 A circular disc fitted to a spindle carries masses of 2, 3 and 5 kg at radii of 250 mm, 300 mm, and 200 mm respectively from the axis of rotation and displaced from each other by 60^o and then 90^o. Determine the unbalanced centrifugal force when running at 300 rev/min. Find, also, the balancing mass required to be fitted at a radius of 270 mm.
Ans: 1.26 kN 4.71 kg at 117^o CW from 5 kg mass

6 A porter governor has links of equal length each 300 mm long The rotating balls have a mass of 2 kg and the central mass is 25 kg with frictional resistance at the sleeve mounting to 15 N.

(a) If the limiting angles of governor arms are 30^{o} and 45^{o} with the vertical, find the range of speed over which the governor operates.

(b) If the governor has been running at the minimum speed, at what speed will the sleeve begin to move?

Ans: 210 rev/min to 246 rev/min 221 rev/min

7 A centrifugal clutch consists of four shoes, each with a mass of 5 kg, rotating in a casing and connected to the axis of rotation by springs of stiffness 10 kN/m. When at rest, the centre of gravity of each shoe is 120 mm from the axis of rotation and there is a clearance of 20 mm between casing and shoe. The internal diameter of the casing is 320 mm and μ between shoes and housing is 0.25. Calculate:

(a) the speed at which the clutch begins to transmit power

(b) the power transmitted at 600 rev/min.

Ans: (a) 161.5 rev/min (b) 25.72 kW

8 A Hartnell governor has two rotating masses, each being 1.5 kg. The vertical arms are 120 mm long and the horizontal arms 60 mm long. At 300 rev/min, the orbital radius of the masses is 80 mm and at 320 rev/min it is 115 mm. Calculate the spring stiffness, neglecting the angularity of the arms.

Ans: 17.3 N/mm