DYNAMICS

So far, we have considered systems that are in equilibrium, where we can say that the sum of the forces in any direction must be zero, and the sum of the moments about any point must equal zero. We must now consider how to solve problems where we are not in equilibrium, that is where there is acceleration. Remember that acceleration may mean speeding up, slowing down, or simply a change in direction.

Let us recall Newton's laws of motion.

First Law
A body will continue in a state of rest or uniform motion in a straight line unless compelled to change that state by the application of a force.

Second Law
The rate of change of momentum is proportional to the applied force and takes place in the direction of the applied force.

Third Law
To every action there is an equal and opposite reaction.

It is Newton's second law that most concerns us here.

Momentum and Inertia.

A body possesses momentum by virtue of being in motion, **and momentum is the product of mass and velocity (m.v)**. Thus even a body moving at slow speed can have considerable momentum by virtue of having a large mass (a slow moving coal wagon), or a light object moving at high speed can still have considerable momentum (a speeding bullet).

The units of momentum are kg.m. **Momentum is not a force**, nor is it a type of energy. A body that is moving will of course posses kinetic energy.

Inertia is defined as **the reluctance to change motion**. Clearly, mass is an important aspect of this. It is much harder to catch a fast moving cricket ball than a tennis ball. We will later see that for rotational inertia, the radius at which the mass acts, known as the radius of gyration, also determines the magnitude of the rotational inertia.

According to Newton's second law, the rate of change of momentum is proportional to the applied force. The change of momentum if a body changes velocity from v_1 to v_2 is given by:

$$\text{Change of Momentum} = mv_2 - mv_1 = m(v_2 - v_1)$$

And the rate of change of momentum $= m\frac{(v_2 - v_1)}{t} = m.a.$

Newton's second law thus implies that Force α mass *x* acceleration, and therefore;

$$\text{Force} = \text{constant } x \text{ acceleration}$$

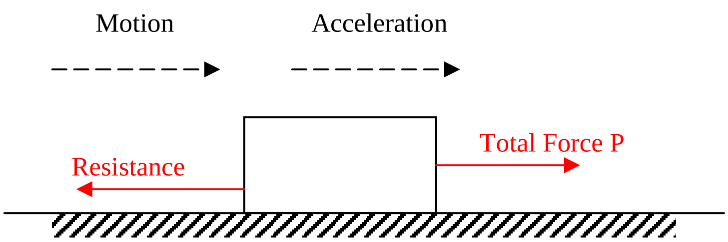
In fact if the force is in Newton, the mass in kg, and the acceleration in m/sec^2, then the constant of proportionality is unity, and we get that **F = <u>m x a</u>**

You will have met this expression before. It means that if there is any imbalance of Force in a system, we will get acceleration.

Example 1.

A car of mass 965 kg is accelerated from 16km/hr to 48 km/hr in 7.75 seconds. If the resistance to motion is 220 N/tonne, calculate:

a) the total propelling force
b) the distance moved during the accelerating period



Solution

Total Force P = Force to overcome resistance F_F plus Force to accelerate, Pa

$$\text{Acceleration } a = \frac{v-u}{t} = \frac{(48-16) \times 1000}{7.75 \times 3600} = 1.147 \text{ m/s}^2$$

Force to accelerate, Pa = m *x* a = 965 x 1.147 = 1107 N

Frictional Force = F_F = 220 *x* 0.965 = 212.3 N

So total Force P = 1107 + 212.3 = **<u>1319 N</u>**

From $s = ut + \frac{1}{2} at^2$,

$s = 4.444 \times 7.75 + \frac{1}{2} \times 1.147 \times 7.75^2$ = **<u>68.89 m</u>**

You should have come across this type of problem and this type of solution before. As problems become more complex however, we need a better method of solving them, and that method will be to use a simple but effective principle known as D'Alemberts principle.

D'Alemberts Principle

Jean D'Alembert, a French mathematician suggested that Newton's second law relating force and acceleration could be re-written in the form:-

Σ Forces – mass x acceleration = zero

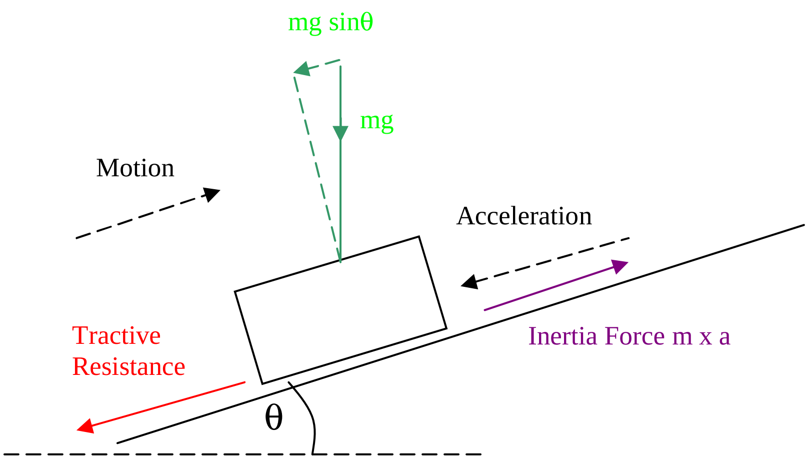
Thus an **imaginary force** (called an inertia force) of magnitude mass *x* acceleration is considered, in the **opposite direction to the acceleration**. This enables us to pretend that the body is in equilibrium, and hence the usual conditions for equilibrium can be applied. Note that, for instance, if there is a surplus of force in a left to right direction, then the body will accelerate in that direction. Our inertia force is considered then to be opposite to the acceleration, that is from right to left. Thus the inertia force is **not** the force **causing** acceleration, it is merely **an imaginary force** that will balance our equations as if the body were in equilibrium, which remember, it cannot be if it is accelerating.

The beauty of D'Alemberts principle lies in it's simplicity. Wherever you see acceleration, put an inertia force "m *x* a" in the opposite direction, and the system appears to be in equilibrium.

Example 2

A truck of mass 5 tonne is travelling at 10m/sec (free wheeling) when it starts to climb an incline of 1 in 50. The tractive resistance is 100N/tonne.
Determine the acceleration of the truck and the distance it climbs the incline before coming to rest.

Solution



First draw a sketch, then consider the forces present. The truck will slow down as it starts to rise up the incline. Remember ***that slowing down in one direction is the same as accelerating in the opposite direction***. Thus our acceleration is down the plane, and our inertia force is opposite this, i.e. up the plane. Since we are given the tractive resistance we do not need to show the normal reaction, Rn, as friction μRn has already been accounted for.

The total tractive resistance F_T will be 5 x 100 = 500 N.

Summing the forces parallel to the plane, upwards positive, the forces will summate to zero if, and only if, we have introduced our inertia force. Including the inertia force, we have;

$$m.a - F_T - mg.\sin\theta = 0$$

$$m.a = F_T + mg.\sin\theta = 500 + 5000 \times 9.81 \times 1/50 = 1481 \text{ N}$$

$$\text{So } a = \frac{1481}{5000} = \underline{\mathbf{0.2962\ m/s^2}} \text{ down the plane}$$

Notice that we will get a positive answer for the acceleration if our assumption about its direction on our forces sketch was correct. We should not expect a negative answer because it is retardation. We can adopt any convention we like when we go on and do the second part of the question. If we now choose that the initial velocity up the plane is positive, then, and only then, will the acceleration down the plane become negative.

From $v^2 = u^2 + 2as$,

Then when the truck comes to rest, $0 = 10^2 + 2 \times (-0.2962) \times s$

Which gives distance travelled s = **168.8 m**.

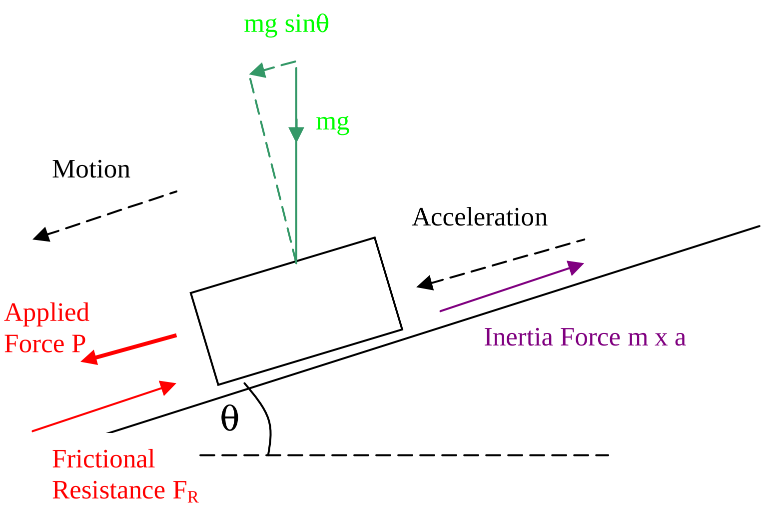
Note that: ***Friction always opposes the tendency to motion***, whilst ***Inertia Force always opposes the acceleration***.

But this does **not** mean that inertia force and friction force are in the same direction, since, as was the case here, motion and acceleration **may** be in opposite directions.

Example 3

Determine, using the conditions of static equilibrium, the force required to accelerate a car at 0.1 m/s^2 down an incline of 1 in 100. The car has a mass of 1.5 tonne, and the resistance to motion is constant at 200 N.

Solution



First draw a sketch, then consider the forces present. The car is accelerating down the plane so our inertia force is opposite this, i.e. up the plane. Since we are given the tractive resistance we do not need to show the normal reaction, Rn, as friction μRn has already been accounted for.

Summing the forces parallel to the plane, upwards positive, the forces will summate to zero if, and only if, we have introduced our inertia force. Including the inertia force, we have;

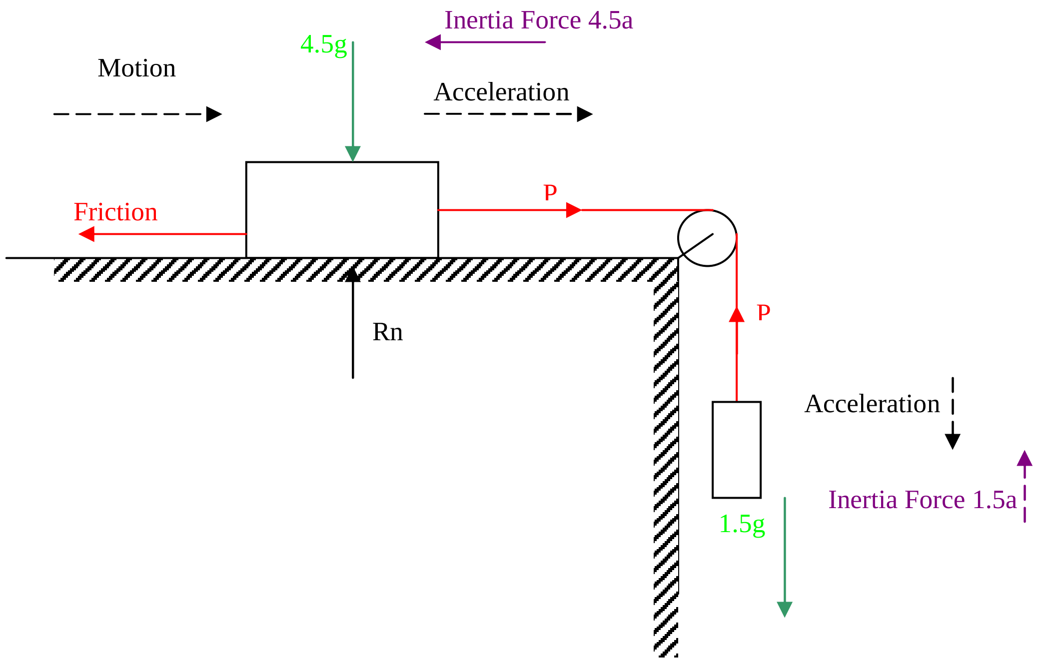
$m.a + F_R - P - mg.\sin\theta = 0$

Which gives P = 200 + 1500 *x* 9.81 *x* 1/100 + 1500 *x* 0.1 = **<u>202.8 N</u>**

Example 4

A mass of 4.5 kg is pulled along a level track by a 1.5 kg mass that hangs vertically. The masses are attached by a light cord that passes over a light smooth pulley. Calculate the acceleration of the 4.5 kg mass and the distance travelled by it in 2 seconds, given that the coefficient of friction between the 4.5kg mass and the track is 0.2.

Solution



First draw a sketch, then consider the forces present. The 4.5 kg mass is accelerating from left to right, so our inertia force here is opposite this. We must not forget the 1.5 kg mass. This will also accelerate at the same rate as the 4.5 kg mass- they are joined together by a cord!

The significance of the pulley being light is that we can neglect it's angular inertia (covered in a later section), and as it is described as ***smooth*** we can take friction at the pulley as negligible. There will of course be friction at the 4.5 kg mass, in the direction opposite to motion.

It is always a good idea to mark the direction of the forces in the cord. The cord is a tie, hence the arrows point inwards. Since the pulley is light and frictionless, the force in the cord at the 1.5 kg mass will be equal to the force in the cord at the 4.5 kg mass.

It is best with a question of this nature to divide the problem up into manageable chunks, and we can do this here by considering the concurrent forces, firstly at the 1.5 kg mass, then at the 4.5 kg mass.

Summating forces at the 1.5 kg mass, upwards positive,

$P + 1.5a - 1.5g = 0$

So $P = 1.5g - 1.5a$ …1

Summating forces at the 4.5 kg mass,

Firstly, $Rn = 4.5g$ since these are the only two vertical forces here.

Then, considering horizontal forces, left to right positive,

$P - \mu Rn - 4.5a = 0$

$P = 0.2 \times 4.5 \times 9.81 + 4.5a = 8.829 + 4.5a$ …..2

And combining equations 1 and 2, which both equal P, then

$8.829 + 4.5a = 1.5g - 1.5a$

$a = \frac{1.5g - 8.829}{6} = \mathbf{\underline{0.981\ m/s^2}}$

From $s = ut + \frac{1}{2}at^2$, then

$S = 0 + \frac{1}{2} \times 0.981 \times 2^2 = \mathbf{\underline{1.962\ m}}$

Self Assessed Questions for you to try. (Answers given)

1 A body with a mass of 50 kg is moving due North at a speed of 18 km/h. It now undergoes a change in course so that after 15 s, it is moving E 30^o N at a speed of 27 km/h. Determine the magnitude of the average accelerating force exerted during this period.

Ans: 22 N

2 A winch lifts a load of 12 kN through a distance of 8 m in 16 s. Starting from rest, the load is accelerated for 4 s, raised uniformly for 10 s and finally brought to rest in 2 s. Find the acceleration, maximum speed and retardation, also the tension in the rope in each case.

Ans: 0.154 m/s^2 0.615 m/s 0.3075 m/s^2
12.1885 kN 12 kN 11.625 kN

3 A light cord passing over a smooth pulley has masses of 25 kg and 40 kg respectively attached to its end. The system is held at 10 m above ground level and then released. Determine the time taken to reach ground level and the time the cord remains slack if the 40 kg mass does not bounce.

Ans: 2.975 s and 1.37 s

4 A mass of 10 kg is pulled up a plane inclined at 15^o to the horizontal by a force of 45 N acting parallel to the plane, the coefficient of friction between mass and plane being 0.2. If it starts from rest find the acceleration of the mass, its velocity after 5 s and the distance travelled in that time.

Ans: 0.066 m/s^2, 0.33 m/s, 0.825 m

5 An electrically driven capstan at the top of an incline hauls a truck with a mass of 5 tonne up a track with an inclination of 1 in 20. The total tractive resistance remains constant at 400 N and the truck is accelerated uniformly from rest to a speed of 27 km/h in 30 s. Find the tension in the cable and the power being exerted when this speed is reached. If upon reaching a speed of 45 km/h the cable were to break, how far would the truck travel up the incline.

Ans: 4.1 kN, 30.75 kW, 137 m

6 A train with a mass of 450 tonne moves up an incline of 1 in 150. The tractive resistance to motion in newton per tonne mass is given by $27 + 0.0128\ V^2$ where V is the speed in km/h. If it attains a speed of 78 km/h in 200 s from rest, calculate the power exerted at the instant its speed is 48 km/h.

Ans: 1.381 MW

7 Two bodies directly in line and 20 m apart are held at rest on an incline of 20^o. Coefficient of friction between body and plane are 0.15 and 0.2 for the upper and lower bodies respectively. If both are set in motion at the same instant, determine the time taken for contact to be made and the distance travelled by the upper body.

Ans: 9.32 s and 85.6 m