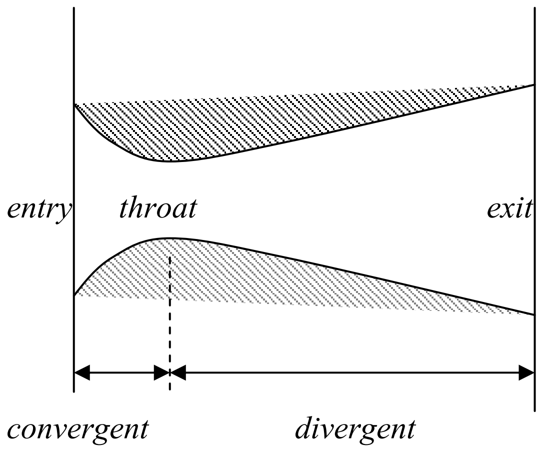
Nozzles and Steam Turbines

Nozzles

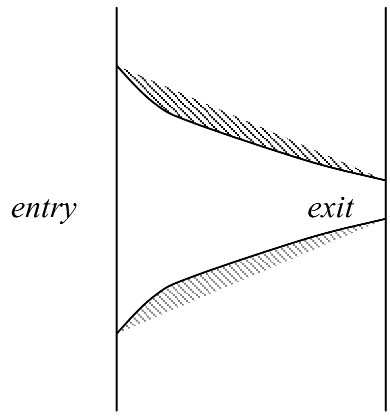
Introduction

A nozzle is a device by which a fluid substance is accelerated to a high velocity by means of a drop in pressure of the substance.

Two of the more common nozzles are shown below, the convergent-divergent nozzle from which it can be seen that the area decreases from entry to a minimum area, called the throat, and then diverges from the throat to the exit area. In the case of the convergent nozzle the cross-section converges to a minimum area at the exit.

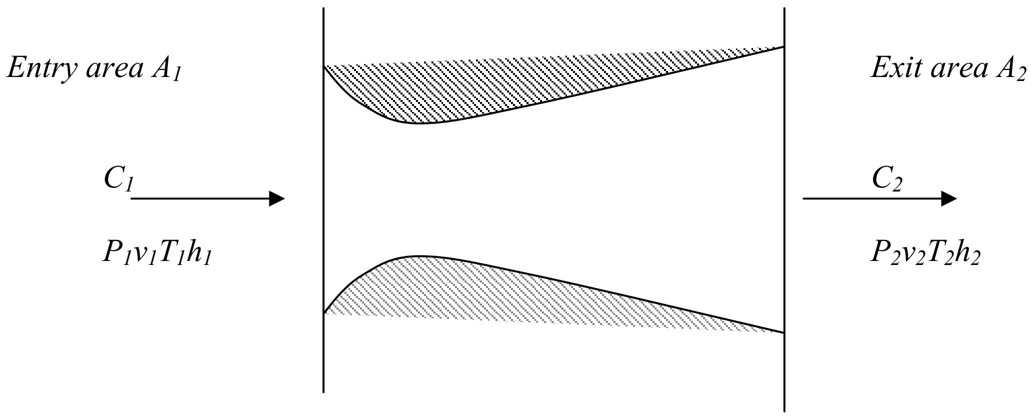


Convergent~Divergent Nozzle



Convergent Nozzle

Consider the diagram below



Neglecting the change in potential energy and putting W = 0, since no work is done in a nozzle, the steady flow energy equation for a nozzle becomes,

$$h_1 + \frac{C_1^{\ 2}}{2} = h_2 + \frac{C_2^{\ 2}}{2} + q$$

all values are quoted per kg of fluid flow

Since the time taken for a substance to pass through the nozzle is very small there is little time for heat exchange to take place between the substance and its surroundings we can assume that q = 0 and the equation becomes.

$$h_1 + \frac{C_1^{\ 2}}{2} = h_2 + \frac{C_2^{\ 2}}{2}$$

In many cases the velocity at entry to the nozzle is small compared to the exit velocity. If we make the assumption that this is the case the term C_1 can be neglected and the equation becomes,

$$h_1 = h_2 + \frac{C_2^{\ 2}}{2}$$

$$\therefore$$

$$\underline{C_2 = \sqrt{2 \times (h_1 - h_2)}}$$

In previous lessons the importance of correct and consistent use of units has been stressed. In the above equation h_1 & h_2 are identified from steam tables and/or H~S chart and are quoted in kJ/kg which is not a fundamental SI unit. To obtain the correct value of velocity the term $(h_1 - h_2)$ must be in J/kg. To overcome this, the equation is modified

$$\underline{C_2 = \sqrt{(2000 \times \Delta h)}}$$

where

$$\Delta h = (h_1 - h_2)\ kJ/kg$$

Isentropic Efficiency
We have come across this term in relation to vapour power cycles. (turbines)
The nozzle theory identified above indicates that the change of enthalpy of the fluid, between entry and exit, results in a change of kinetic energy. In the real world, where friction is always present some of the kinetic energy is re-converted to enthalpy. (think space shuttle re-entering the atmosphere, high temperatures are generated by the friction the high velocity of the craft moving through static air.)

The function of the nozzle is to convert enthalpy to kinetic energy and any conversion back into enthalpy is therefore undesirable. As engineers we need to define how well (or badly) a nozzle is performing. This is described by the *nozzle isentropic efficiency*.

The *nozzle isentropic efficiency* compares the enthalpy drop of a perfect process where friction is absent with the enthalpy drop achieved in practice with friction present. Thus the nozzle efficiency can be defined in terms of the ratio of actual enthalpy drop where there is a change in entropy to the isentropic enthalpy drop where there is no entropy change.

$$\eta = \frac{\Delta h_{actual}}{\Delta h_{isentropic}}$$

Problems covering nozzles often concern the nozzle area required to allow a specific mass of steam to flow through. Obviously, for a given condition, the nozzle area will be greater if the mass flow is greater. The continuity equation shows the relationship between mass flow, specific volume, velocity and area at any section of the nozzle.

$$\dot{m} = \frac{A \times C}{v}$$

where

$\dot{m} = mass\ flowrate$

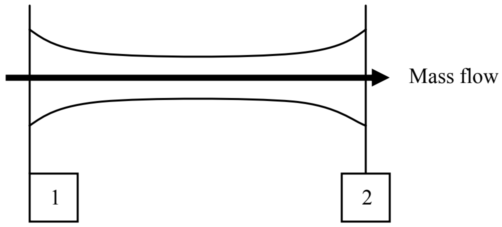
$C = velocity$

$v = specific\ volume$

$A = cross\text{-}sectional\ area\ of\ nozzle\ section$

Example 2.1

A group of nozzles in a turbine is required to expand 10.2 kg/sec of steam from initial conditions of 7 bar, 250^oC to pressure 1 bar. If the isentropic efficiency of the nozzles is 0.925, determine the velocity of the steam and the total area of the nozzles at exit.



h_1 *at 7 bar 250°C* $= 2955 kJ/kg$

$s_1 = s_{2'} = 7.106 kJ/kgK$

but $s_{g\ at\ 1\ bar} = 7.359 kJ/kg$

hence steam is wet after isentropic expansion

from steam tables at 1 bar

$$x = \frac{s_{2'} - s_f}{s_{fg}} = \frac{7.106 - 1.303}{6.056} = 0.954$$

$h_{2'} = h_f + xh_{fg} = 417 + 0.954 \times 2258 = 2570 kJ/kg$

hence isentropic enthalpy drop $\Delta hs = h_1 - h_{2'} = 2955 - 2570 = 385 kJ/kg$

Actual enthalpy drop across the nozzle $= \Delta h = h_1 - h_2 = 385 \times 0.925 = 356.13 kJ/kg$

$C_2 = \sqrt{2000 \times \Delta h} = \sqrt{2000 \times 356.13} = \underline{844 m/s}$

$\dot{m} = \frac{AC}{v} \therefore A = \frac{\dot{m} \times v}{C}$ *where v is the specific volume at the final condition of 1 bar 0.954 dry*

$v = xv_g = 0.954 \times 1.694 = 1.616$

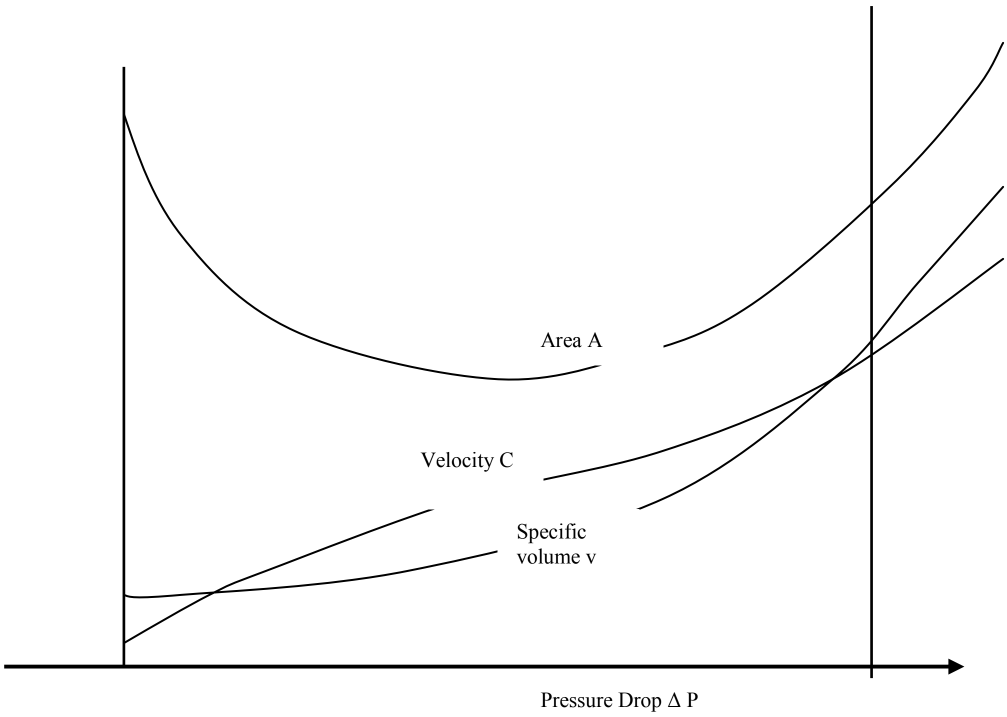
$$A = \frac{10.2 \times 1.616}{844} = \underline{0.0195 m^2}$$

Critical Pressure Ratio

From the continuity equation it can be seen that $A \propto \dfrac{volume}{Velocity}$

As the expansion of steam through a nozzle progresses, both the specific volume and the velocity increase, but initially the velocity increases at a higher rate than the specific volume; the flow area required is therefore a decreasing quantity. It continues until a particular value of pressure is reached, after which the specific volume increases at a higher rate than the velocity; the flow area therefore becomes an increasing quantity. The result is a convergent divergent nozzle, and the pressure at which the changeover takes place is known as the *critical pressure*, and occurs at the throat of the nozzle.

The ratio of the pressure at the throat of the nozzle and the pressure at the inlet to the nozzle is known as the *critical pressure ratio*. The critical pressure ratio for wet steam is about 0.58 and for superheated steam about 0.55, so that:



It can be shown that the velocity at the throat of a nozzle operating at its designed pressure ratio is the velocity of sound at the throat conditions. The velocity up to the throat is ***sub-sonic***; the flow after the throat is ***supersonic***. Sonic or supersonic flow requires a diverging duct to accelerate it.

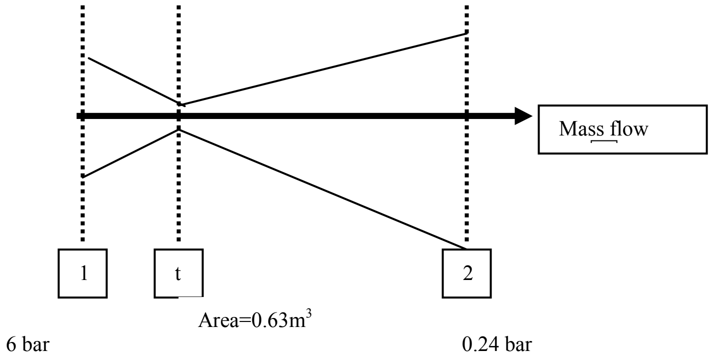
Nozzles designed to handle liquids are always convergent because the specific volume of a liquid is constant over wide pressure range.

Example 2.2

Dry saturated steam at 6 bar is supplied to convergent – divergent nozzle of throat area 0.63 x 10^{-3} m^2. The exit pressure is 0.24 bar. The nozzle isentropic efficiency is 0.88 and the expansion in the convergent portion may be assumed isentropic. Determine:

- *i) the mass flow rate.*
- *ii) the nozzle exit area.*

Take critical pressure ratio to be 0.583.



$$\frac{P_t}{P_1} = 0.583 \therefore P_t = 6 \times 0.583 = 3.5bar$$

$$\dot{m} = \frac{A_t C_t}{v_t} \text{ where suffix t represents conditions at the throat}$$

$$h_1 = 2757kJ/kg$$

$$s_1 = s_t = 6.761kJ/kgK$$

s_g *at 3.5 bar = 6.761 hence the steam at the throat is wet*

$$x = \frac{s_t - s_f}{s_{fg}} = \frac{6.761 - 1.727}{5.214} = 0.9655$$

$$h_t = h_f + xh_{fg} \text{ at 3.5 bar} = 584 + (0.9655 \times 2148) = 2657.9kJ/kg$$

$$C_t = \sqrt{2000 \times \Delta h} = \sqrt{2000 \times (2757 - 2657.9)} = 445.2m/s$$

$$v_t = xv_g = 0.9655 \times 0.5241 = 0.506m^3/kg$$

$$\dot{m} = \frac{A_t C_t}{v_t} = \frac{0.63 \times 445.2}{0.506} = \underline{0.5543kg/s}$$

$$s_2' = s_t = 6.761kJ/kgK$$

s_g *at 0.24 bar is 7.844kJ/kgK hence the steam is now more wet with the isentropic expansion from* $s = sf + xsfg$

$$x = \frac{s_2 - s_f}{s_{fg}} = \frac{6.761 - 0.882}{6.962} = 0.844$$

$$h_2^{/} = h_f + xh_{fg} \text{ at 0.24 bar} = 268 + 0.844(2348)$$

$$\eta_{isen} = 0.88 = \frac{\Delta h}{\Delta h_s} \therefore \Delta h = 0.88(2657.9 - 2250.75) = 358.3 kJ/kg$$

$$C_2 = \sqrt{2000 \times \Delta h} = \sqrt{2000 \times 358.3} = 846.5 m/s$$

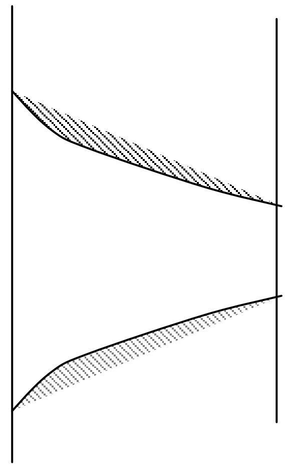
$$v_2 = xv_g \text{ at 0.24 bar} = 0.844 \times 6.445 = 5.44\ m^3/kg$$

$$A_2 = \frac{\dot{m}}{C_2} \times v_2 = \frac{0.5543 \times 5.44}{846.5} = \underline{3.562 \times 10^{-3}}$$

Expansion of a gas through a nozzle.

All of the previous theory of steam expanding through nozzles, in general, holds for gases expanding through nozzles. The difference is in the way the change in enthalpy is calculated.

If the inlet velocity is ignored then the outlet velocity of the nozzle shown can be calculated.



$$\mathbf{c_2} \quad = \quad \sqrt{(2 \times \Delta h)}$$

For a gas $(h_1 - h_2)$, *or* $\Delta h = cp\,(T_1 - T_2)$

$$c_2 \quad = \quad \sqrt{[2 \times c_p\,(T_1 - T_2)\,]}$$

or to ensure units are correct:

$$c_2 \quad = \quad \sqrt{[2000 \times c_p\,(T_1 - T_2)\,]}$$

$$c_2 \quad = \quad 44.72\sqrt{[c_p\,(T_1 - T_2)\,]}$$

if inlet velocity is not neglected then

$$\mathbf{c_2} \quad = \quad \sqrt{\{\,[2000 \times c_p\,(T_1 - T_2)\,] + c^2\}}$$

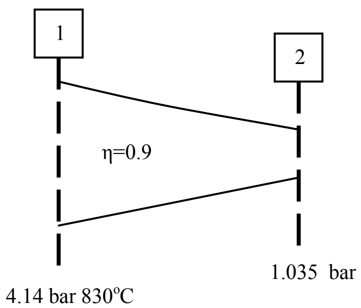
Example 2.3

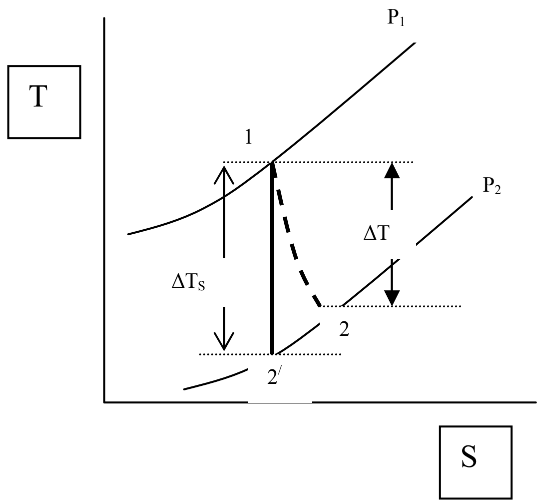
Air at a pressure of 4.14 bar, temperature 837oC, is expanded in a nozzle to a pressure of 1.035 bar. If the isentropic efficiency of the nozzle is 0.9, calculate

a) ***the exit velocity of the jet.***

b) ***the outlet temperature.***

Sketch the process on a temperature ~ entropy diagram





Assume for air Cp = 1005 J/kgK and $\gamma = 1.4$

$T_1 = 830 + 273 = 1103K$

$$\frac{T_2^{/}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2^{/} = 1103 \times \left(\frac{1.035}{4.14}\right)^{\frac{1.4-1}{1.4}} = 742.26K$$

$\Delta Ts = 1103 - 742.26 = 360.74K$

$\therefore$

$\Delta T = 0.9 \times 360.74 = 324.7K$

$C_2 = \sqrt{2 \times Cp \times \Delta T} = \sqrt{2 \times 1005 \times 324.7} = \underline{807.2m/s}$ *(a)*

$T_2 = 1103 - 324.7 = \underline{718.3K = 505.3^o C}$ *(b)*

Critical pressure ratio

When dealing with the expansion of steam through a nozzle we saw that for maximum flow through a convergent/divergent nozzle required that an optimum pressure existed at the throat, known as the *critical pressure ratio*. For steam this ratio is usually given. However, when solving problems involving the expansion of gases the critical pressure ratio must be obtained by using a formula:

$$\frac{p_c}{p_1} = r_{pcrit} = \left[\frac{2}{\gamma + 1}\right]^{\gamma/\gamma-1}$$

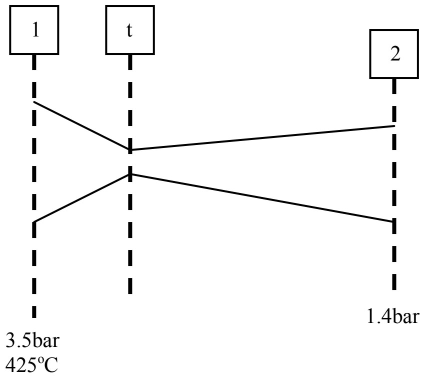
Example 2.4

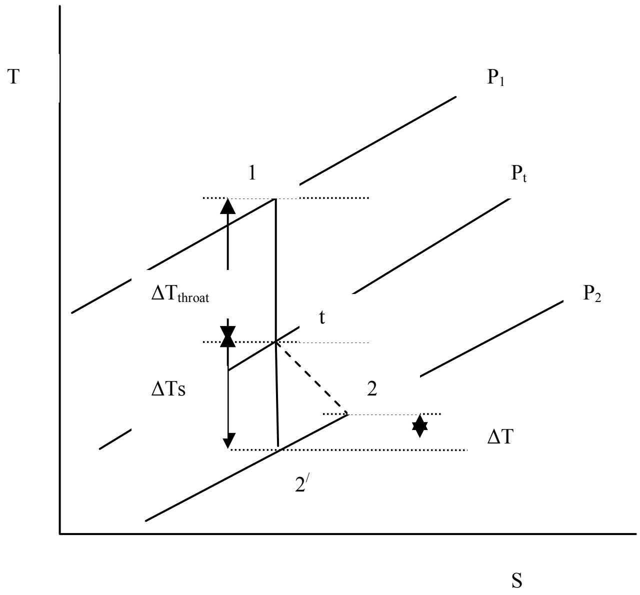
A nozzle is required to pass an airflow of 1.5 kg/s. The inlet conditions are zero velocity, pressure 3.5 bar and temperature 425°C; the air is to be expanded to 1.4 bar. Determine

a) the throat area.

b) the exit area and the exit velocity if the nozzle isentropic efficiency is 0.95.

c) sketch the process on a temperature ~ entropy diagram





Assume that the expansion to the throat is isentropic

$$\frac{p_t}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} = 1.849bar$$

$$\frac{T_t}{T_1} = \left(\frac{p_t}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2}{\gamma+1}\right)$$

$$T_t = 698 \times \frac{2}{2.4} = 581.7K$$

$$\Delta T_{throat} = 689 - 581.7 = 116.3K$$

$$C_t = \sqrt{2 \times Cp \times \Delta T_{throat}} = \sqrt{2 \times 1005 \times 116.3} = 483.56m/s$$

$$\dot{m} = \frac{A \times C}{v} \ \ where$$

$\dot{m}$ = mass flow rate , C = velocity , A = cross sectional area, v = specific volume

now from pv = RT based on 1 kg mass

$$v_t = \frac{RT}{p} = \frac{0.287 \times 581.7}{1.849 \times 10^2} = 0.903m^3/kg$$

$$A_t = \frac{\dot{m} \times v_t}{C_t} = \frac{1.5 \times 0.905}{483.56} = \underline{2.8 \times 10^{-3} m^2} \ \ (a)$$

$$\frac{T_2^{/}}{T_t} = \left(\frac{p_2}{p_t}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2^{/} = 581.7\left(\frac{1.4}{1.84}\right)^{\frac{0.4}{1.4}} = 537.4K$$

$$\Delta Ts = 581.7 - 538 = 43.4K$$

$$\Delta T = 0.95 \times 43.7 = 41.2K$$

$$C_2 = \sqrt{(2 \times Cp \times \Delta T) + C_t^{\,2}} = \sqrt{(2 \times 1005 \times 41.2) + 483.56^2} = 562.76m/s = C_{exit} \ (b)$$

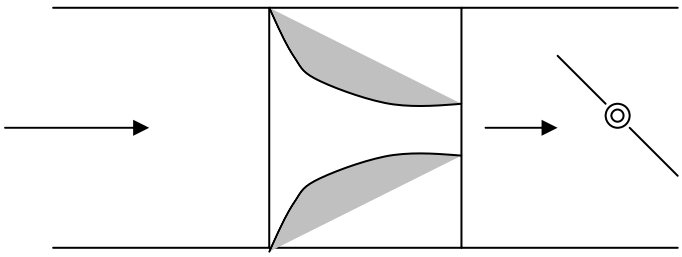
$$T_{exit} = 581.7 - 41.22 = 540.5K$$

$$v_{exit} = \frac{RT_{exit}}{p_{exit}} = \frac{0.287 \times 540.5}{1.4 \times 10^2} = 1.108m^3/kg$$

$$A_{exit} = \frac{\dot{m} \times v_{exit}}{C_{exit}} = \frac{1.5 \times 1.108}{562.76} = \underline{2.95 \times 10^{-3} m^2} \ (b)$$

Maximum mass flow of choked flow.

Consider a convergent nozzle expanding into a space, the pressure of which can be varied while the inlet pressure remains fixed.



When the backpressure p_b is equal to p_1 then no fluid can flow through the nozzle. As p_b is reduced the mass flow through the nozzle increases. Since the enthalpy drop causes and increase in velocity. However, when the back pressure reaches the critical value, it is found that no further reduction in back pressure can affect the mass flow. When the backpressure in exactly equal to the critical pressure, p_c then the velocity at exit is sonic and the mass flow through the nozzle is at a maximum. If the back pressure is reduced below the critical value then the mass flow remains at the maximum value, the exit pressure remains at p_c, and the fluid expands violently outside the nozzle down to the back pressure. It can be seen that the maximum mass flow through a convergent nozzle is obtained when the pressure ratio across the nozzle is the critical pressure ratio. Also, for convergent-divergent nozzle, with sonic velocity at the throat, the cross sectional area of the throat fixes the mass flow through the nozzle for fixed inlet conditions.

A nozzle operating with the maximum mass flow is said to be ***choked***. A correctly designed convergent-divergent nozzle is always ***choked***

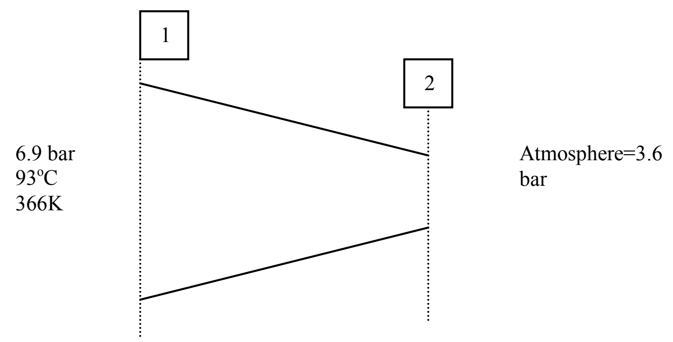
Example 2.5

A fluid at 6.9 bar and 93°C enters a convergent nozzle with negligible velocity and expands isentropically into a space at 3.6 bar. Calculate the mass flow per m2 of exit area.

a) *when the fluid is helium (cp = 5.23 kJ/kgK)*

b) *when the fluid is ethane (cp = 1.66 kJ/kgK)*

Assume that helium and ethane are perfect gases, and take the respective molecular weights to be 4 and 30



In this question we need to determine the values of R and γ for each gas. We must also determine whether the nozzle is running <u>choked</u> since this will determine the maximum flow rate

Helium

$$R = \frac{R_0}{M} \textit{ where } M = \textit{molecular mass and } R_o = \textit{the universal gas constant}$$

$$R = \frac{8.314}{4} = 2.0785 kJ / kgK$$

$$Cv = Cp - R = 5.23 - 2.0785 = 3.1515 kJ / kgK$$

$$\gamma = \frac{Cp}{Cv} = \frac{5.23}{3.1515} = 1.66$$

$$\textit{Critical pressure ratio} = \frac{p_t}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{2}{2.66}\right)^{\frac{1.66}{0.66}} = 0.488$$

$$\therefore \quad p_t = 6.9 \times 0.488 = 3.37 bar$$

The actual surrounding pressure is higher than the critical pressure hence the nozzle is <u>not</u> running choked. We can thus determine the mass flow rate from the surrounding pressure

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = 366 \times \left(\frac{3.6}{6.9}\right)^{\frac{0.66}{1.66}} = 282.6K$$

$$\Delta T \textit{ across the nozzle} = 366 - 282.6 = 83.4K$$

$$C_2 = \sqrt{2 \times Cp \times \Delta T} = \sqrt{2 \times 5.28 \times 10^3 \times 83.4} = 934.11 m/s$$

$$v_2 = \frac{RT_2}{p_2} = \frac{2.0785 \times 282.6}{3.6 \times 10^2} = 1.63 m^3 / kg$$

$$\dot{m} = \frac{A \times C_2}{v_2} = \frac{1 \times 934.1}{1.63} = \underline{572.5 kg / s / m^2}$$

Ethane

$$R = \frac{R_o}{M} = \frac{8.314}{30} = 0.277 kJ / kgK$$

$$Cv = Cp - R = 1.66 - 0.277 = 1.383 kJ / kgK$$

$$\gamma = \frac{Cp}{Cv} = \frac{1.66}{1.383} = 1.2$$

$$\frac{p_t}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{2.2}\right)^{\frac{1.2}{0.2}} = 0.5645$$

$$p_t = 6.9 \times 0.5645 = 3.895 bar$$

The critical pressure is greater than the surrounding pressure hence the nozzle is running choked and the mas flow rate must be determined from the critical pressure conditions

$$T_t = T_1 \times \frac{2}{\gamma+1} = 366 \times \frac{2}{2.2} = 332.72 K$$

$$\Delta T = 366 - 332.72 = 33.28 K$$

$$C_2 = \sqrt{2 \times Cp \times \Delta T} = \sqrt{2 \times 1.66 \times 10^3 \times 33.28} = 332.4 m / s$$

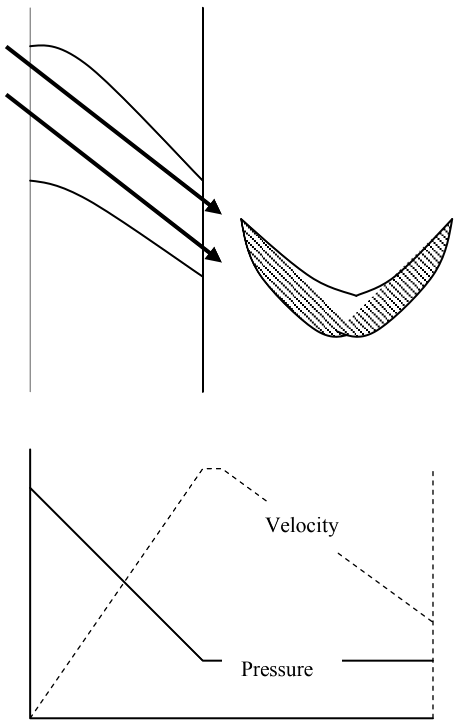
$$v_t = \frac{RT_t}{p_t} = \frac{0.277 \times 332.72}{3.895 \times 10^2} = 0.237 m^3 / kg$$

$$\dot{m} = \frac{A \times C_t}{v_t} = \frac{1 \times 332.4}{0.237} = \underline{1404.66 kg / s / m^2}$$

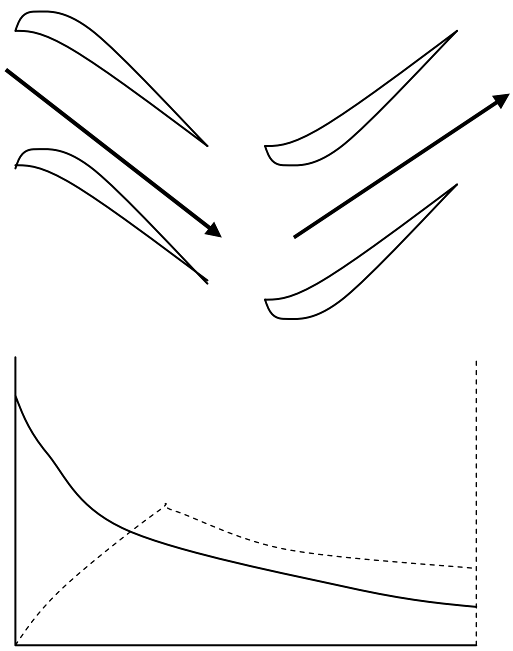
Steam Turbines

In a steam turbine some of the energy of the steam is converted to mechanical work. Steam expands in a stationary element called the nozzle, (sometimes called the guide) where some of the enthalpy is converted to kinetic energy, which is subsequently utilised in a moving element, the blade. The blade is fixed to the periphery of the rotor. Turbines that work on this principle are known as *impulse turbines*.

In other machines, the steam after leaving the stationary elements also expands while flowing through the moving blades. This type of turbine is known as a *reaction turbine*, or more accurately an *impulse-reaction turbine*.



Impulse turbine



Impulse-reaction turbine

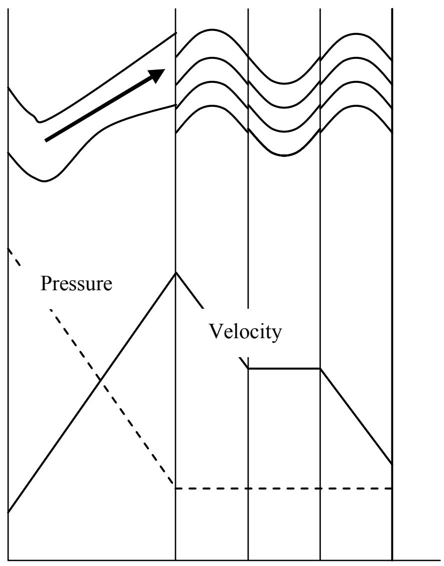
The Impulse Turbine

The impulse turbine has been briefly described above in its simplest terms. Perhaps its main feature is that *the entire pressure drop* occurs in the nozzles; the steam entering the turbine with an extremely high velocity and the pressure remaining constant after the nozzles. However, if the whole pressure drop from boiler to condenser pressure takes place in a single nozzle stage and single row of turbine blades, then the velocity of the steam entering the turbine would be very high. This would lead to excessive rotational speed. Strategies to reduce the velocity of the velocity of the steam must be employed. These are known as *velocity compounding* and *pressure compounding*.

Velocity Compounding.

In this type of turbine the steam is expanded in a single nozzle as before, with the steam entering a first row of moving turbine blades where the velocity is only partially reduced. The steam leaves the first row of moving blades and passes to a row of fixed blades (the blades are fixed to the turbine casing). The fixed blades re-direct the steam back to the direction of motion so that it is correct for entry into a second row of moving blades where the velocity is again partially reduced. The important points to note are:

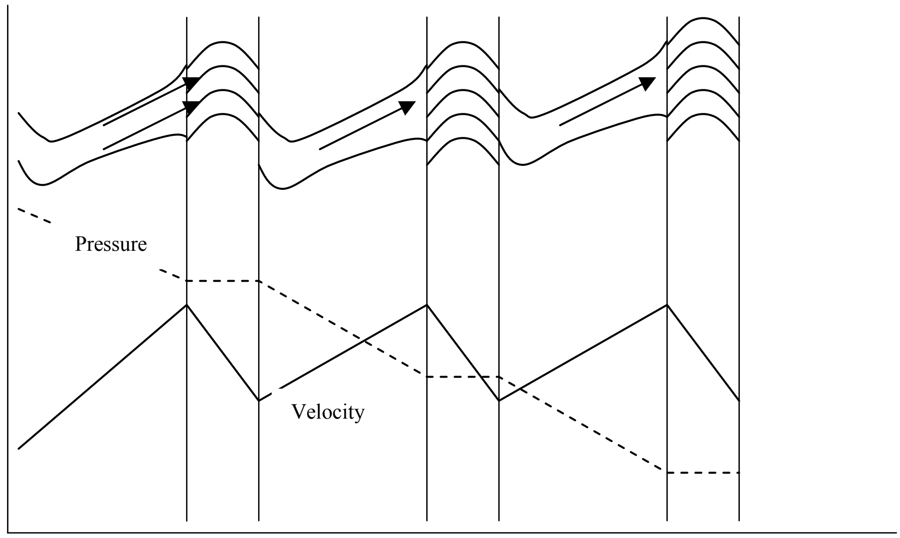
- The entire pressure drop occurs in the nozzles.
- Only part of the velocity of the steam is used up in each row of blades resulting in lower rotational speeds.
- This type of turbine commonly referred to as a *Curtis Turbine*.
- Common in the high-pressure part of a large propulsion turbine.



Pressure compounding.

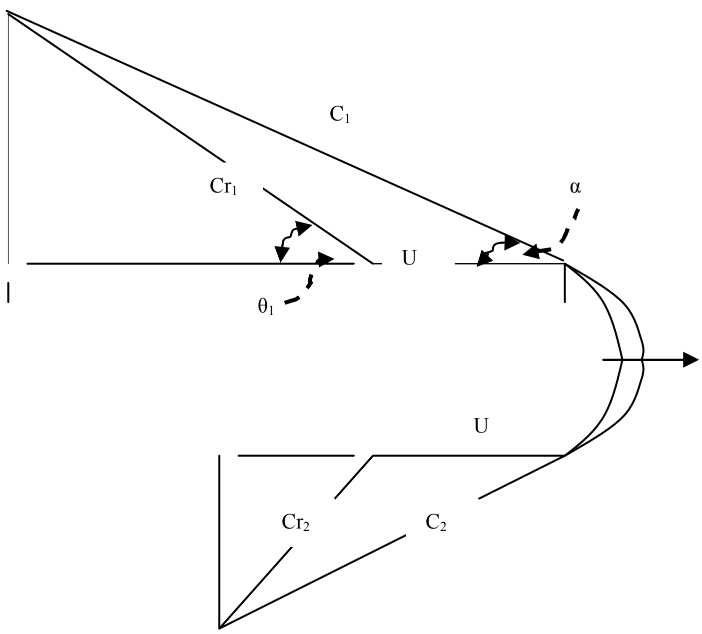
With this type of turbine the steam enters a row of nozzles where its pressure is only partially reduced and velocity increased.. The high velocity steam passes from the nozzles to a row of moving blades where its velocity is reduced. The steam then passes to a second row of nozzles where its pressure is again partially reduced and its velocity again increased. The high velocity steam passes from the nozzles on to a second row of moving blades where its velocity is again reduced. The steam passes to a third row of nozzles and so on. The important points to note are:

- Only part of the pressure drop occurs at each stage so that steam velocity is lower, resulting in lower rotational speeds.
- This type of turbine referred to as a *Rateau Turbine*.



Velocity diagram for impulse turbine.

Problems relating to turbines can be solved using velocity diagrams where the movement of steam and turbine blades is represented by vectors. In essence we are merely trying to identify the velocity of the steam relative to the moving blade. The important consideration when solving turbine problems by diagram is that you must be clear as to notation. The basic velocity diagram for an impulse turbine is shown below.

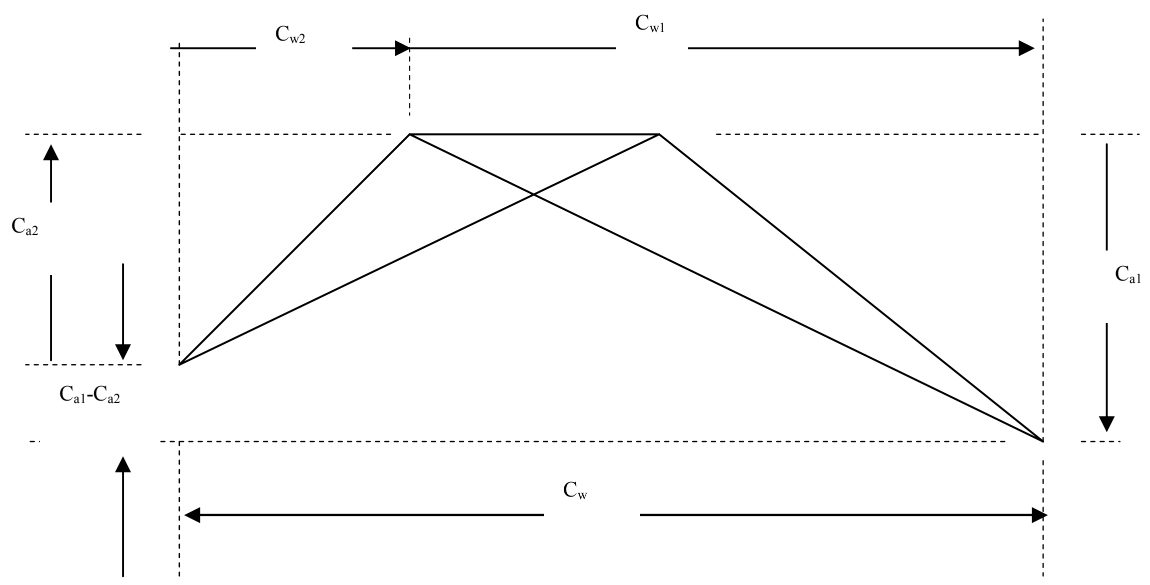


Symbols of velocity diagram.

U	*Mean speed of the blade.*
C_1	*Absolute velocity of the steam entering turbine (or steam leaving nozzle)*
α_1	*Nozzle inlet angle (angle the nozzles are inclined to plane of the wheel)*
C_{r1}	*The velocity with which the steam enters the blade relative to the moving blade at angle θ_1 to the plane of the wheel.*
θ_1	*Moving blade inlet angle.*
C_{r2}	*The velocity with which the steam leaves the moving blade, relative to the moving blade, at angle θ_2 to the plane of the wheel.*
θ_2	*Moving blade outlet angle.*
C_2	*Absolute velocity of the steam leaving the moving blade at angle α_2 to the plane of the wheel.*
C_{w1}	*The velocity of whirl at inlet, that is, the tangential component of C_1(useful component since it provides the impulse force)*
C_{w2}	*The velocity of whirl at exit*
C_w	*Total velocity of whirl*
C_{a1}	*The axial velocity at inlet*
C_{a2}	*The axial velocity at exit*

*The difference between these two axial velocities will influence the magnitude of the axial end thrust and hence the requirement for a thrust bearing on the turbine (**More of this later)***

In the previous diagram the inlet and exit triangles were separate; however it is usual practice to combine them into a single diagram.



Ignoring shock losses at entry for an impulse turbine the blades are usually symmetrical and $\theta_1 = \theta_2$.

Due to friction as the steam moves across the blades the relative velocity of the steam is reduced, i.e. $Cr_2 < Cr_1$ and $C_{r2} = kC_{r1}$ where k is referred to as the blade friction factor with a value less than 1

Tangential steam force on blades = F

Rate of change of momentum in the direction of the moving blades:

$$\begin{aligned} \Delta\, momentum &= m \times \text{change of tangential velocity} \\ &= m \times (C_1 \cos\alpha_1 + C_2 \cos\alpha_2) \\ &= m \times (C_{w1} + C_{w2}) \\ &= m\ C_w \end{aligned}$$

$$\begin{aligned} \text{Power developed} &= \text{Force} \times \text{Speed} \\ &= m\ C_w\ U \end{aligned}$$

$$\begin{aligned} \text{Blade or Diagram } \eta &= \frac{\text{useful work produced}}{\text{energy available at inlet.}} \\ &= \frac{C_w U}{C_1^2/2} \\ &= 2\frac{C_w U}{C_1^2} \qquad \text{per unit of mass.} \end{aligned}$$

The end thrust can be determined by utilising the change in axial velocity

$From\ \ Force = rate\ of\ change\ of\ momentum$

$End\ thrust = mass\ flow\ rate \times change\ in\ axial\ velocity$

$\underline{End\ Thrust = \dot{m} \times (C_{a2} - C_{a1})}$

Depending upon the magnitude of each axial velocity then the end thrust will have either a positve sign or a negative sign. This determines the direction of the foce i.e. a negative value means that the force is in the direction of flow.

<u>Frictional Effects</u>

As the steam flows across the blades the friction will occur between the steam and blades such that the relative velocity at exit will be smaller than the relative velocity at inlet. This reduction in relative velocity is expressed in terms of a friction factor k such that:

$\underline{C_{r2} = k \times C_{r1}}$

Problems are generally solved by drawing vector diagrams to scale.

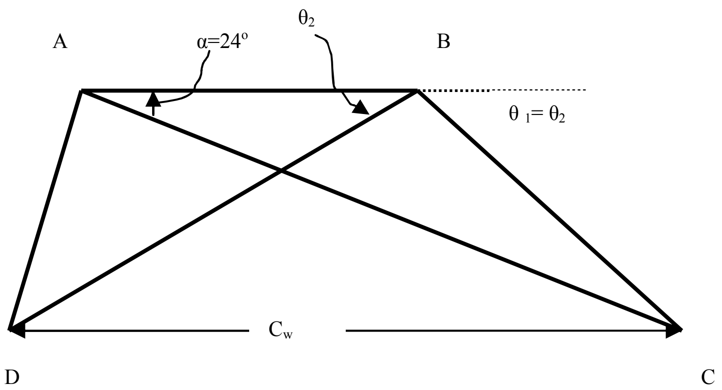
Example 2.6

Steam leaves the nozzles of a single stage impulse turbine with a velocity of 1000 m/s. The nozzles are inclined at 24^o to the direction of motion of the turbine blades. The mean blade speeds is 400 m/s and the blade inlet and out let angles are equal. The steam enters the blade without shock and the flow over the blades can be considered frictionless. Determine:

i) *the inlet angle of the blades.*

ii) *The force exerted on the blades in the direction of their motion.*

iii) *The power developed when the steam flow rate is 4000 kg/h.*



NOT TO SCALE

1. FROM "**A**" DRAW TO SOME SUITABLE SCALE A HORIZONTAL LINE REPRESENTING THE BLADE SPEED "U" i.e. 400m/s TO POINT "**B**"

2. FROM "**A**" AT AN ANGLE OF 24^o (clockwise from U) DRAW A LINE TO SCALE TO REPRESENT THE NOZZLE EXIT VELOCITY C_1 = 1000 m/s TO POINT "**C**"

3. JOIN POINT "**C**" TO POINT "**B**". THIS LINE **BC** REPRESENTS THE RELATIVE VELOCITY AT IN LET TO THE BLADE i.e. $C_{r1.}$

4. SINCE THERE IS NO SHOCK THEN $\boldsymbol{\theta_1 = \theta_2}$ AND SINCE THERE IS NO FRICTION THEN $\mathbf{C_{r1} = C_{r2}}$. (The angle θ_1 measured from the vector diagram is approx 39^o don't worry if yours is slightly different) AT AN ANGLE OF 39^o AT THE POINT "**B**" MEASURED COUNTER CLOCKWISE FROM U DRAW A LINE OF LENGTH **BD** EQUAL IN MAGNITUDE TO **BC** i.e. **BC** = **BD** SINCE $\mathbf{C_{r1} = C_{r2}}$

5. JOIN "**A**" TO "**D**". (This line represents the exit velocity C_2)

6. THE HORIZONTAL LINE FROM **D** to **C** REPRESENTS THE VELOCITY OF WHIRL C_w =1003 m/s approx

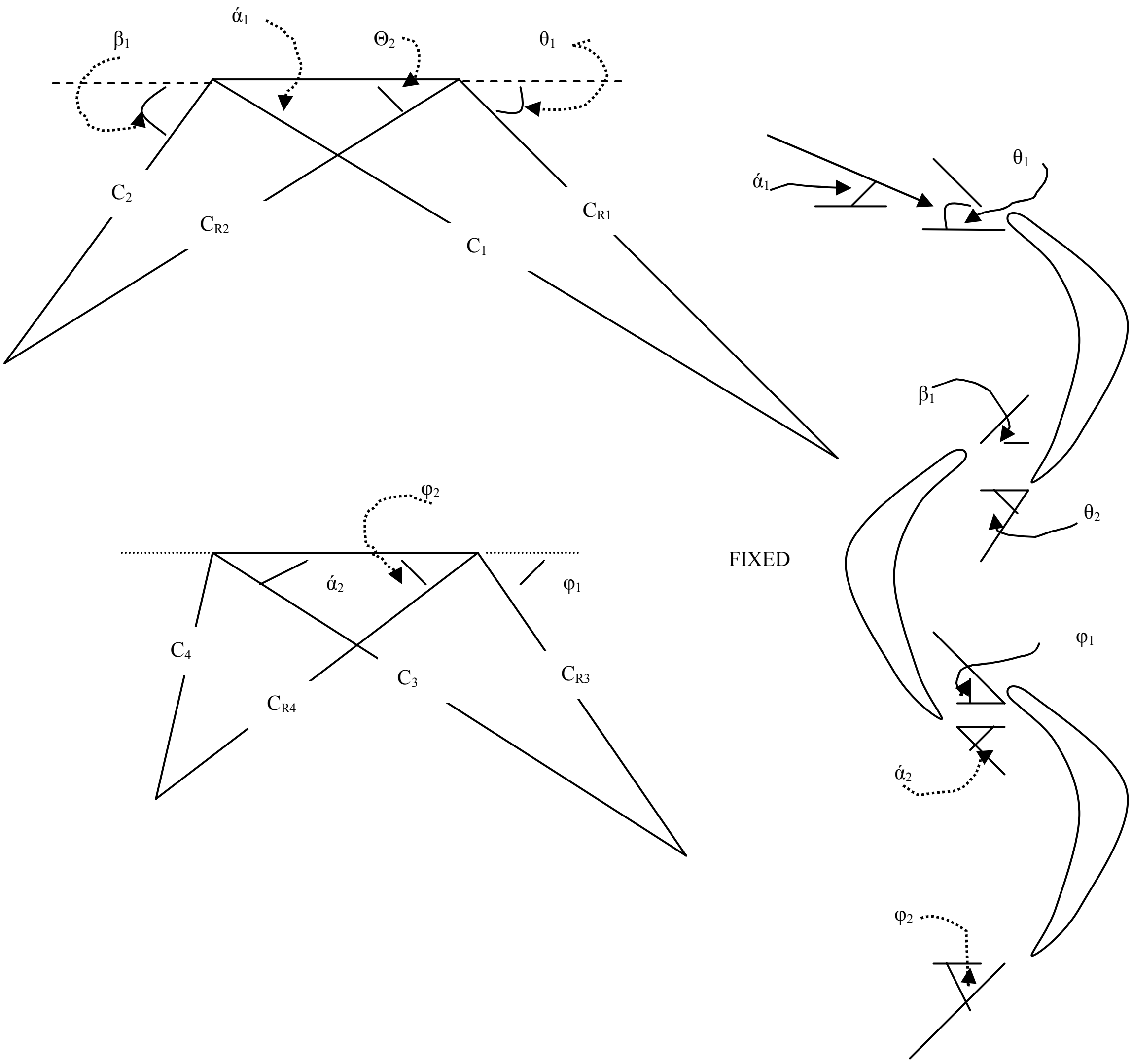
$$Force = rate\ of\ change\ of\ momentum\ in\ the\ plane\ of\ rotation$$

$$Force\ on\ the\ blades = \dot{m} \times C_w = \frac{4000}{3600} \times 1002 = \underline{1114 N}$$

$$Power = \dot{m} \times C_w \times U = 1114 \times 1000 = 1114 \times 10^3\ watts = \underline{1114 kW}$$

Velocity diagrams for two rows of blades.

The diagram below shows the velocity diagram for a two row Curtis type impulse turbine. Both rows have the same blade speed U. Although the diagrams can be superimposed this tends to clutter the diagram and make it confusing. You may feel that you prefer to deal with each stage in turn.



Symbols of velocity diagrams.

U	*Mean speed of the blade.(This is the same for both diagrams)*
C_1	*Absolute velocity of the steam entering turbine (or steam leaving nozzle) This is the first row of moving blades*
α_1	*Nozzle inlet angle (angle the nozzles are inclined to plane of the wheel)*
C_{r1}	*The velocity with which the steam enters the first row blade relative to the moving blade at angle θ_1 to the plane of the wheel.*
θ_1	*First row of moving blades inlet angle.*
C_{r2}	*The velocity with which the steam leaves the first row moving blade, relative to the moving blade, at angle θ_2 to the plane of the wheel.*
θ_2	*First row moving blade outlet angle.*
β_1	*Inlet angle of the fixed blade.*
$\acute{\alpha}_2$	*Exit angle for the fixed blade, for a symmetrical blade this would be equal in magnitude to angle β_1*
φ_1	*Second row of moving blades inlet angle.*
φ_2	*Second row of moving blades outlet angle*
C_3	*Absolute velocity of the steam entering the second row of moving blades*
C_4	*Absolute velocity of the steam leaving the second row of moving blades*
C_{R3}	*The velocity with which the steam enters the second row blade relative to the moving blade at angle φ_1 to the plane of the wheel.*
C_{R4}	*The velocity with which the steam leaves the second row blade relative to the moving blade at angle φ_2 to the plane of the wheel.*

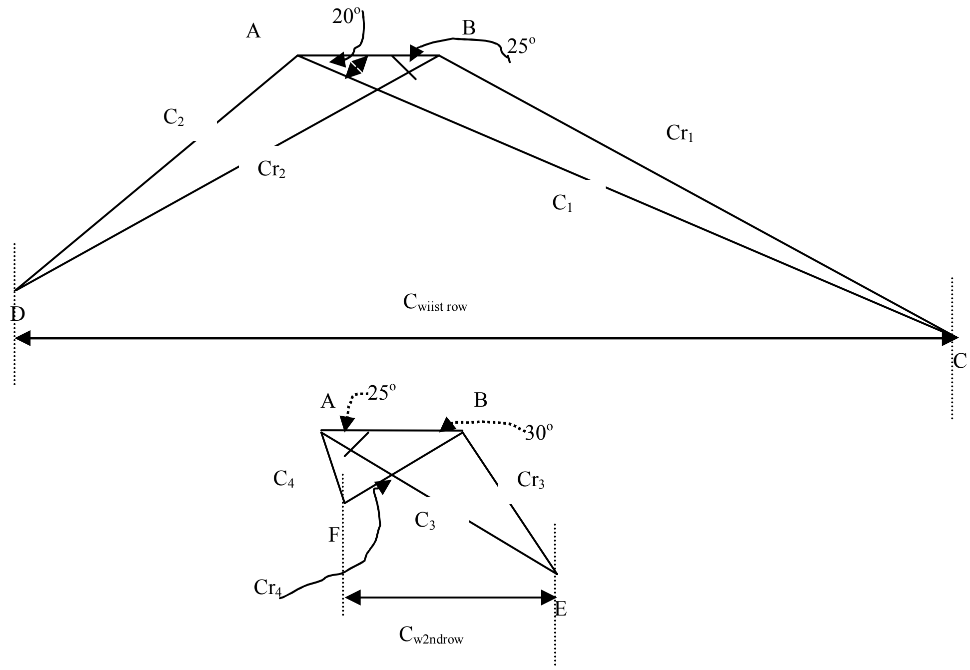
Possible the most confusing aspect of this type of problem is not getting mixed up with the notation of the inlet and outlet angles.

Example. 2.7

Two rows of a Curtis type velocity compounded impulse turbine have a mean blade speed of 150 m/s and a nozzle velocity of 675 m/s. The nozzle angle is 20^o. The exit angle of the first moving row, fixed and second row of moving blades are 25^o, 25^o and 30^o respectively. There is a 10% loss of velocity due to friction in all blades. The steam flow is 4.5 kg/s. Draw velocity diagram to a suitable scale and determine:

i) *the output power of the turbine.*

ii) The diagram efficiency.



NOT TO SCALE

1. To some suitable scale draw a horizontal line **AB** to represent the blade speed of 145 m/s.
2. From **A** at an angle of 20^o clockwise to the horizontal draw a line **AC**, to the same scale, to represent the nozzle velocity of 675m/s.
3. Join point **C** to **B**. This represents the relative velocity Cr_1 (Cr_1=525m/s)
4. The frictional effects of the steam on the blade mean that the velocity Cr_2 will be 90% of Cr_1 hence Cr_2=0.9 X 525 =473 m/s. Measure line **BD** to scale to represent this value at an angle of 25^o anticlockwise to the horizontal.
5. Join **D** & **A** . This represents the exit velocity from the first row of moving blades. (340m/s)
6. The magnitude of the exit velocity is reduced in the fixed blade by 10% hence the absolute velocity at entry to the 2[nd] row of moving blades is C_3 and C_3=0.9x C_2 = 0.9x340=306m/s

7. Utilizing a new vector diagram draw a horizontal line to scale to represent the blade speed =145m/s i.e. **AB.**
8. From **A** at an angle of 25^o clockwise to the horizontal draw a line **CE** to represent the exit velocity from the fixed blade. (306m/s)
9. Join **B** to **E** and measure. This line represents the relative velocity at inlet to the 2^{nd} row of moving blades. (Cr_3=140m/s)
10. Cr_4= 0.9xCr_3=0.9x140=126m/s. Draw a line **BF** at an angle of 30^o anticlockwise to the horizontal to represent the velocity Cr_4 = 126m/s.
11. Join **A** to **F.** This is the absolute velocity at exit for the 2^{nd} row of moving blades
12. Measure the velocity of whirl for each row i.e. **DC** and **EF** (900m/s and 225m/s)

$$Power = \dot{m} \times Cw_{total} \times U$$

$$= 4 \times (900 + 225) \times 150 = 777.6 \times 10^3 \, watts = \underline{777.6kW}$$

$$\eta_{Diagran} = \frac{2 \times Cw \times U}{C_1^{\ 2}} = \frac{2 \times (900 + 225) \times 150}{675^2} = 0.7585 = \underline{75.85\%}$$

These answers are probably on the low side and will depend upon the accuracy to which the vector diagram is drawn. Work it out yourself, use as big a scale as possible and use the page in landscape

.

Blade Speed Ratio

Study of the turbine velocity diagram will show that the output of the turbine increases as the nozzle outlet angle α_1 is reduced. The evidence for this is that the velocity of whirl is increased. Reduction of the nozzle outlet angle will reduce the axial velocity of the steam, which, in turn, requires that there is an increase in annulus area for the same mass flow of steam. This may lead to larger nozzle friction losses. Turbine designers must choose the correct steam and blade velocities to ensure that the turbine gives optimum efficiency. For practical considerations the designer will choose a nozzle outlet angle between 15^o and 30^o. Calculations can then be completed to show the relationship between the ratio of blade speed and the speed of the steam and the nozzle outlet angle, which will yield the maximum diagram efficiency.

$$\frac{U}{C_1} = \frac{\cos \alpha_1}{2}$$

U/C_1 is known as the ***blade speed ratio.***

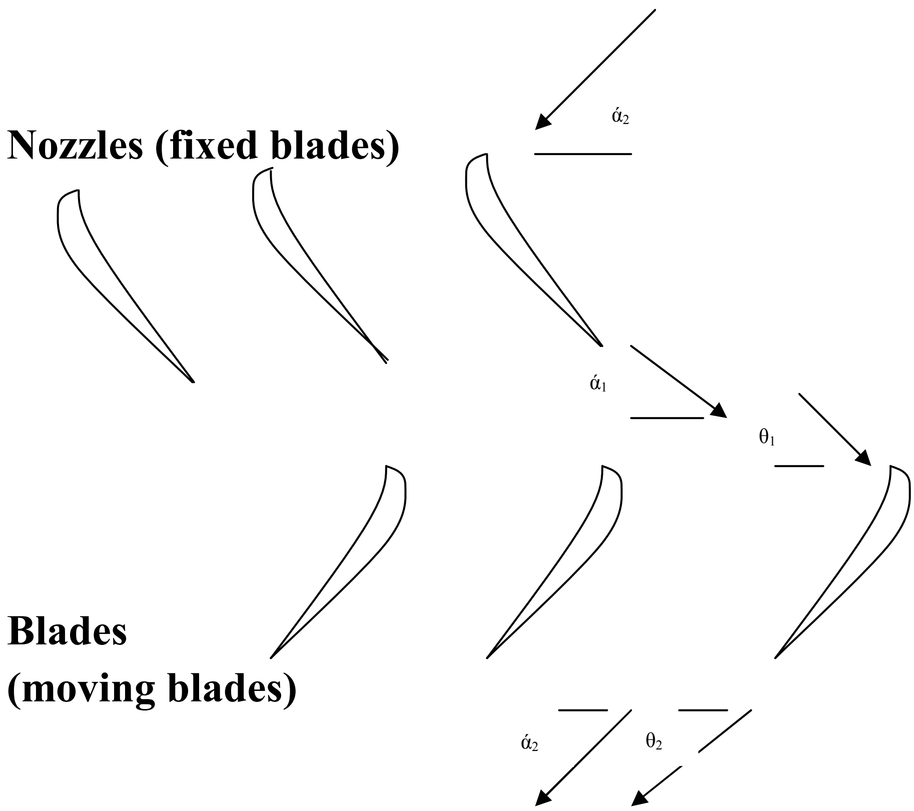
Reaction Turbines

The characteristic that distinguishes the *reaction turbine* from the *impulse turbine* is that that in the reaction type a pressure drop takes place in both the stationary and moving rows. In pure impulse turbines the pressure drop occurs only in the stationary row (i.e. In the nozzles). The stationary row in both types, though called different names, and differing slightly in geometry, have identical functions.

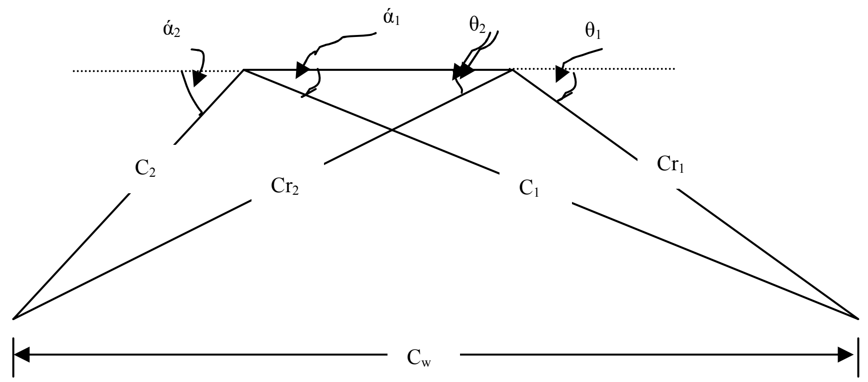
In the reaction type a further pressure drop occurs in the moving row, and as in the stationary row, causes an increase in the velocity of the steam between inlet and outlet.

The velocity diagram for a reaction turbine is shown below. Since there is a pressure drop in the moving row, and a consequent increase in velocity, the steam does not slow as in the impulse type. It can be seen that $C_{r2}>C_{r1}$.

One stationary row and one moving row are termed a pair; the pressure drop in a pair is much smaller than in a stage of an impulse turbine. Consequently, in reaction turbines, there are a large number of pairs in series, and groupings of blade pairs are known as stages. There may be several stages in one machine.



Reaction Turbine velocity Diagram



Symbols for reaction turbine velocity diagram.

U	mean blade speed
C_1	Absolute velocity of steam entering turbine.
α_1	Nozzle exit angle.
θ_1	Moving blade inlet angle
C_{r1}	The velocity with which the steam enters the blade relative to the moving blade at angle θ_1 to the plane of the wheel.
C_{r2}	The velocity with which the steam leaves the moving blade relative to the moving blade at angle θ_2 to the plane of the wheel.
θ_2	The moving blade exit angle.
C_2	Absolute velocity of steam leaving moving blade.
α_2	Inlet angle of steam entering next pair

Degree of Reaction

In a blade pair, the pressure drop and hence the enthalpy drop may be distributed in any proportion between the fixed and moving row. The *degree of reaction* is defined as the ratio:

$$\textit{degree of reaction} = \frac{\textit{change of enthalpy in moving blades}}{\textit{change of enthalpy in the pair}}$$

$$\textit{Power} = mU(\textit{change in velocity of whirl})$$

$$= mUC_w$$

$$\textit{End thrust} = m(\textit{change in velocity of flow})$$

$$\text{Stage efficiency} = \frac{\text{work done in stage}}{\text{Enthalpy drop in stage}}$$

$$= \frac{mUC_w}{mh}$$

$$= \frac{UC_w}{h}$$

$$\text{Diagram Efficiency } (\eta_D) = \frac{CwU}{\frac{C_1^2 - C_2^2}{2}}$$

$$\Delta h \text{ in moving blade} = \frac{Cr_e^2 - Cr_1^2}{2}$$

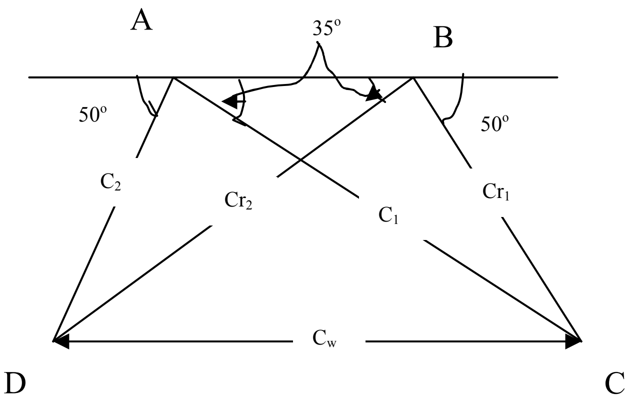
In a 50% reaction turbine the blades are identical hence $ά_1=\theta_2$ and $ά_2=\theta_1$ thus $Cr_1=C_2$ and $Cr_2=C_1$ i.e. the diagrams are symmetrical

Example 2.8

At a stage in a reaction turbine, the mean blade ring diameter is 1 m and the turbine runs at a speed of 54 rev/sec. The blades are designed for 50% reaction with exit angles of 35^o and inlet angles of 50^o. The turbine is supplied with steam at the rate of 167 kg/s and the stage efficiency is 85%. Determine.

a) *the power output of the stage*
b) *the specific enthalpy drop in the stage in kJ/kg.*
c) *the percentage increase in relative velocity in the moving blades due to expansion in these blades.*

$$Mean\ blade\ speed = \pi dn = \pi \times 1 \times 54 = 169.7\ m/s$$



NOT TO SCALE

DIAGRAM CONSTRUCTION

1. Chose a suitable scale and draw the horizontal line AB to represent the blade speed. U =169.7m/s

2. From point A draw a line at an angle of 35^o clockwise to the horizontal AB the length of which is as yet unknown.

3. From point B draw a line at 50^o clockwise to the horizontal until it intersects the line drawn from A This locates the point C and C1 andCr_1 can be determined.

4. The point D can be located in a similar manner and it will be noted that the triangles are symmetrical and that $C_1 = Cr_2$ and $C_2 = Cr_1$

5. Measure these values of velocity from the diagram

The values are as follows;

$C_1 = Cr_2 = 502m/s$

$C_1 = Cr_1 = 376m/s$

$Cw_{total} = 652m/s$

$$Power = \dot{m} \times Cw_{total} \times U$$
$$= 167 \times 652 \times 169.7 = \underline{18.5MW}$$

$$\eta_{dstage} = \frac{work\ done\ per\ stage\ (W)}{enthalpy\ drop\ per\ stage\ (\Delta h)}$$
$$\Delta h = \frac{W}{\eta_{dstage}} = \frac{Cw_{total} \times U}{\eta_{dstage}} = \frac{652 \times 169.7}{0.85} = \underline{130kJ}$$

$$\%\ increase\ in\ relative\ velocity = \frac{Cr_2 - Cr_1}{Cr_1} = \frac{502 - 376}{376} = \underline{33.5\%}$$

Blade height for reaction turbine

Reaction turbine problems sometimes include the requirement to calculate blade height. The following equation is useful to solve these problems.

$$H = \frac{m \text{ x } v}{\pi \text{ x } d \text{ x } C_{a1}}$$

where H *blade height, m*
m *mass flow of steam, kg/s*
v *specific volume,* m^3/kg
d *mean blade diameter, m*
C_{a1} *Velocity of flow (axial velocity)*