Unit 7
Heat Transfer

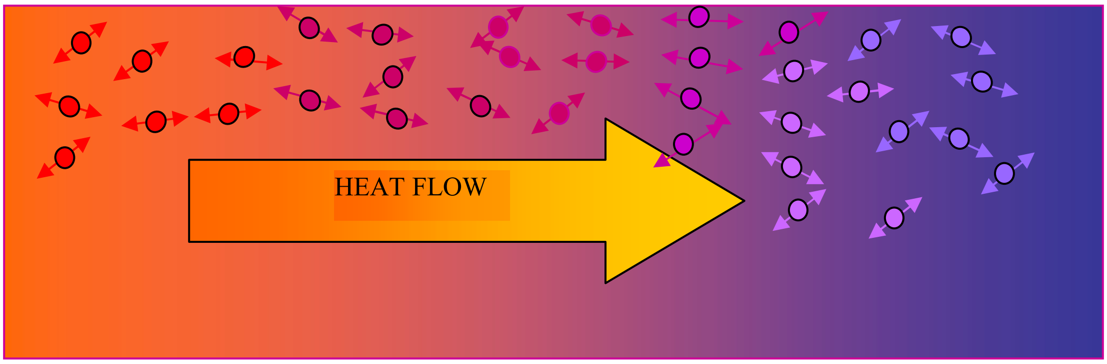
In this section we will examine the theory of one dimensional steady state heat transfer and its application through worked examples and self assessed questions.

7.1 Modes of Heat transfer

Heat energy causes gas and liquid molecules to move around faster and causes particles in solids to vibrate more rapidly.
This extra kinetic energy in the particles is dissipated to the surrounding and shows up as a rise in temperature.
The three main modes of heat transfer are conduction convection and radiation

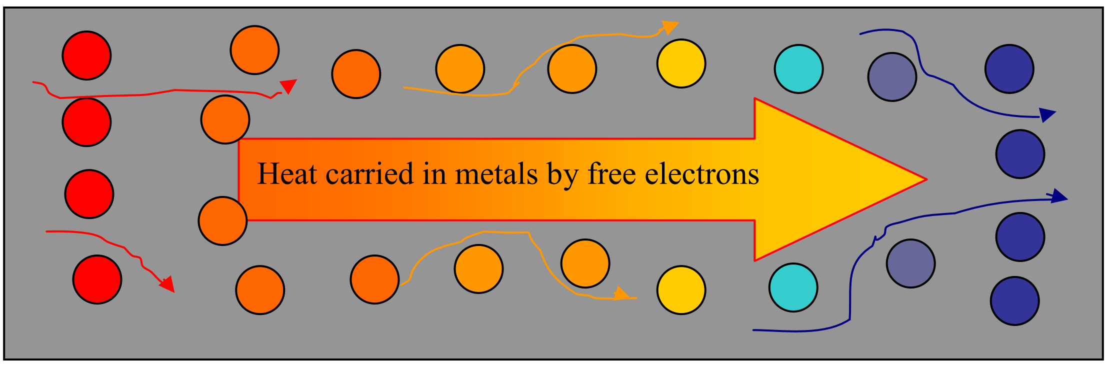
Conduction

This occurs mainly in solids.
It is the process where vibrating particles pass on their extra vibration energy to neighbouring particles.
This process continues throughout the solid and the extra vibrational energy passed through causes a rise in temperature on the other side



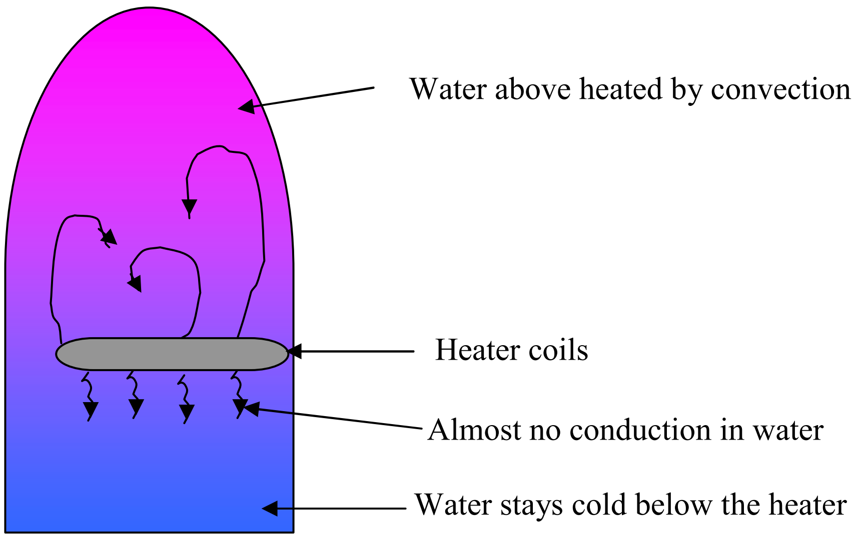
This normal process of conduction as illustrated above is always very slow, but in most non-metal solids it is the only way that heat can pass through.
Non metals such as plastic wood etc. are poor conductors and thus good insulators.

Metals conduct so well because the electrons are free to move inside the metal.
At the hot end the electrons move faster and diffuse more quickly through the metal so the electrons carry their energy quite a long way before giving it up in a collision.
This is a much faster way of transferring the energy through the metal than slowly passing it between jostling neighbouring atoms.



Convection

Gases and liquids are usually free to move so convection takes place when the more energetic particles move from a hot region to a cold region and take their heat energy with them. They then transfer their heat energy by the process of collisions, which warm up the surroundings.



Natural convection currents are caused by changes in density.
Forced convection is carried out mechanically.

Thermal Radiation

Thermal radiation or infra red radiation consists of only one fairly narrow band in the spectrum of electromagnetic waves at a frequency just below that of visible light. They are emitted due to the agitation of molecules within a substance and whenever, these waves are absorbed by matter there is a gain of internal energy.

Thermal Radiation

- Travels in straight lines at the speed of light
- Travels through vacuum
- Can be effectively reflected by a silver surface
- Only travels through transparent media such as air, glass and water.
- Its behaviour is strongly dependent on surface colour and texture.

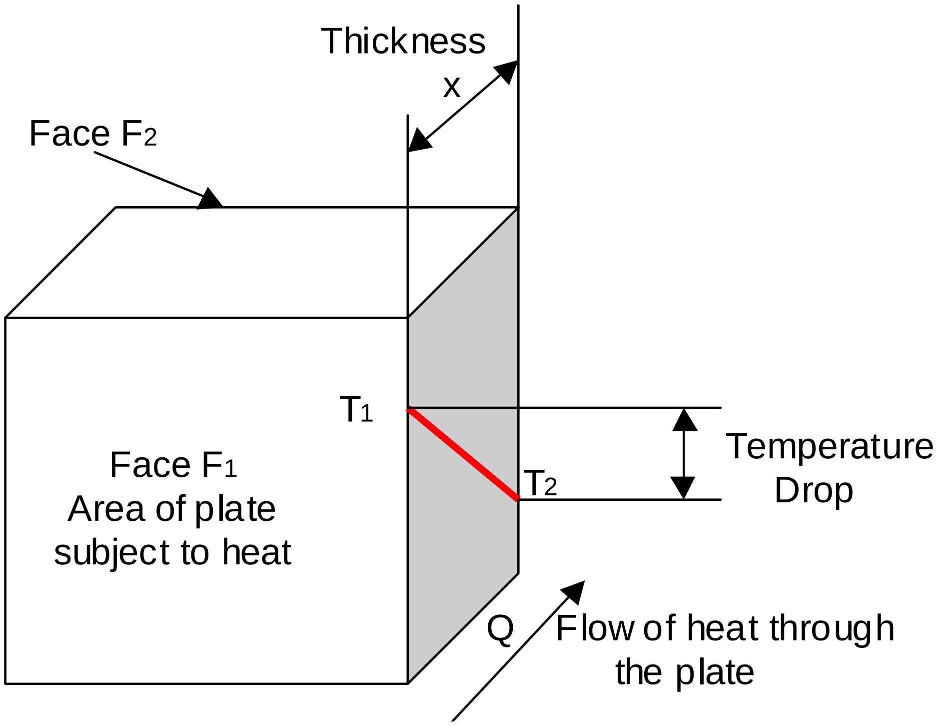
All objects are continually emitting and absorbing heat radiation, the hotter they are the more radiation they emit, cooler objects absorb this radiation.

Dark matt surfaces absorb heat radiation falling on them more strongly than bright glossy surfaces such as gloss white or silver.
Dark matt surfaces also emit more radiation than glossy surfaces.
Silvered surfaces reflect nearly all the thermal radiation falling on them.

7.2 Conduction through single and multilayer flat surfaces.

Fourier's Law

Heat transfer through a single plate



Let us look at a section of flat plate
The thickness is "x" metres
The front face F_1 is at temperature T_1 K
The rear face F_2 at temperature T_2 K

We will assume that temperature T_1 is higher than temperature T_2, heat will then flow in the direction shown.

If the material has uniform conducting properties, the temperature will decrease at a uniform rate as we move from the front to the back of the plate as shown in the sketch above.

The heat transfer "Q" per unit time from face F_1 to face F_2 will vary

1. Directly as the temperature difference. Q is proportional to $(T_1 - T_2)$
2. Directly as the area "A " of the plate. Q is proportional to A
3. Inversely as the thickness "x" of the plate. Q is proportional to $\frac{1}{x}$

These statements can be combined and written as

$$Q \text{ is proportional to } \frac{A(T_1 - T_2)}{x}$$

Fourier's law can now be written as $$Q = \frac{\lambda A(T_1 - T_2)}{x}$$

Where λ is the constant of proportionality

It is called the coefficient of thermal conductivity.

The units are Watts per metre Kelvin (W/mK)

which gives the rate of heat transfer Q in Watts.

Example 7.2-1

A furnace has a wall thickness of 210 mm. The temperature of the inner surface is 935ºC and an outer surface temperature of 16ºC. The thermal conductivity of the wall material is 1.03 W/mK.
Determine the heat flow through the wall.

This is a direct application of Fourier's equation. All we need to do is insert the relevant values taking due care with the units

$$Q = \frac{\lambda A(T_1 - T_2)}{x} \qquad Q = \frac{1.03 \times A(1208 - 289)}{0.21}$$

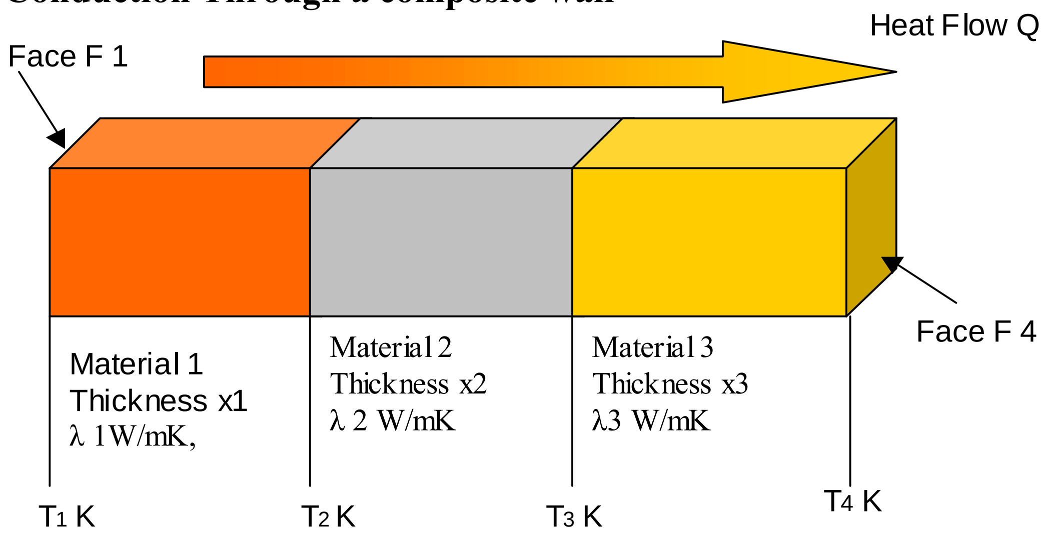
In this case we do not have the area, if we leave this out of the equation then the units of Q will be as follows.

$$Q = \frac{W(K - K)}{mK \times m} = \frac{W}{m^2}$$

In other words the heat flow will be obtained per square meter of wall.

$$Q = 4507.47 \frac{W}{m^2}$$

Conduction Through a composite wall



Instead of a single plate imagine a wall made up of several layers of different materials in this case three.

Layer 1
Thickness "x1", Thermal conductivity $\lambda 1$, outer surface temperature T1 and interface temperature T2

Layer 2
Thickness "x2", Thermal conductivity $\lambda 2$ and interface temperatures T2 and T3

Layer 3
Thickness "x3", Thermal conductivity $\lambda 3$, interface temperature T3 and outer surface temperature T4

Any heat transferred from face F1 to face F4 must pass through each layer and therefore as the same quantity of heat energy is transferred across each layer through the same area in the same time then the rate of heat transfer is the same through each layer.

$$Q = \frac{\lambda_1 A(T_1 - T_2)}{x_1} \qquad (T_1 - T_2) = \frac{Qx_1}{A\lambda_1}$$

$$Q = \frac{\lambda_2 A(T_2 - T_3)}{x_2} \qquad (T_2 - T_3) = \frac{Qx_2}{A\lambda_2}$$

$$Q = \frac{\lambda_3 A(T_3 - T_4)}{x_3} \qquad (T_3 - T_4) = \frac{Qx_3}{A\lambda_3}$$

Adding these equations gives

$$\left(T_1 - T_4\right) = \frac{Q}{A}\left(\frac{x_1}{\lambda_1} + \frac{x_2}{\lambda_2} + \frac{x_3}{\lambda_3}\right)$$

$$Q = \frac{A\left(T_1 - T_4\right)}{\sum \frac{x}{\lambda}}$$

The term $\frac{x}{\lambda}$ is the thermal resistance of a single layer of material and

$\sum \frac{x}{\lambda}$ Is the thermal resistance of the composite wall.

Example 7.2-2 Composite wall

A furnace wall is made up of a steel plate 12 mm thick, lined on the inside with silica brick 150 mm thick and on the outside with magnasite brick 150 mm thick.
The temperature on the inside face of the furnace brick is 700˚C and on the outer wall 150˚C.

a) Determine
   (i) The heat transferred through the composite wall
   (ii) The interface temperatures.

b) If the heat transfer is to be reduced to 1kW/m^2 by means of air gap between the steel and the magnasite brick, determine the width of the air gap.

Steel λ = 17.3 W/mK,
Silica brick λ = 1.7 W/mK,
Magnasite brick λ = 5.3 W/mK,
Air λ = 0.035 W/Mk

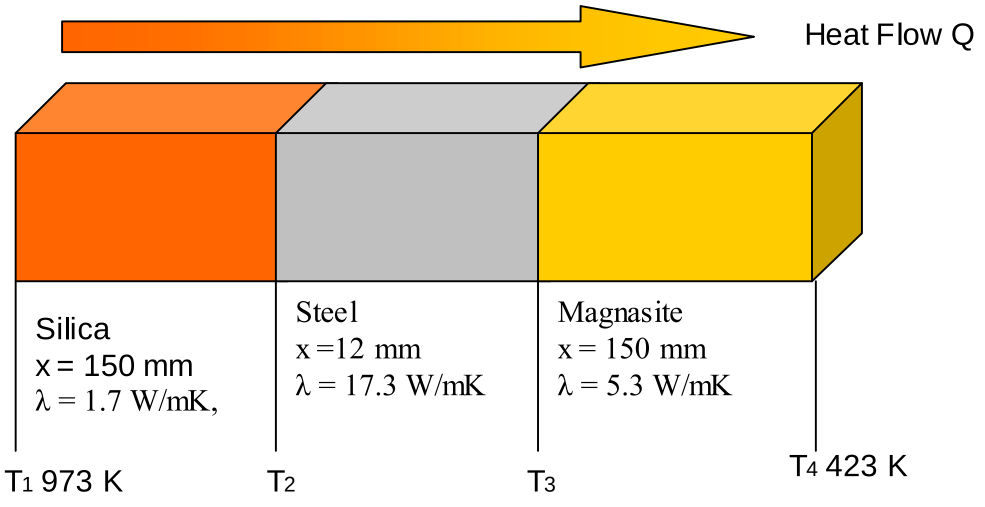
This is a typical composite wall example it requires a diagram to fix the arrangement of the materials, then we can apply Fourier's equation to calculate the required values.

When we come to put the numbers in the equation we must make sure that they are all compatible, millimetre to meters and so on,
However when it comes to the temperature we need the temperature difference so we do not need to add 273 to the temperatures given since the value is cancelled out.

700˚C – 150˚C = 550˚C is the same as 973 – 423 = 550 K

In the following example Kelvin has been used for the temperature.

This is the diagram for the first condition



Fourier's law gives $Q = \dfrac{A(T_1 - T_4)}{\sum \dfrac{x}{\lambda}}$ where $\sum \dfrac{x}{\lambda}$ is the thermal resistance of the wall

$$\sum \frac{x}{\lambda} = \frac{0.15}{1.7} + \frac{0.012}{17.3} + \frac{0.15}{5.3} \frac{m^2 K}{W}$$

$$\sum \frac{x}{\lambda} = 0.0882 + 0.0007 + 0.0283 \frac{m^2 K}{W}$$

$$\sum \frac{x}{\lambda} = 0.1172 \frac{m^2 K}{W}$$

This value can now be placed in the fourier equation

$$Q = \frac{A(T_1 - T_4)}{\sum \frac{x}{\lambda}} \qquad Q = \frac{973 - 423}{0.1172} \frac{W}{m^2 K}$$

$$Q = 4691.59 \frac{W}{m^2 K}$$

Thus the heat flow through the wall is 4.6 kW per m^2

The heat flow through each part of the wall is the same.
The interface temperatures can be found by applying Fourier's law across each section of the wall in turn and rearranging the equation to obtain the temperature.

To find T_2, rearrange $Q = \dfrac{\lambda_1 A(T_1 - T_2)}{x_1}$ to give $(T_1 - T_2) = \dfrac{Qx_1}{A\lambda_1}$

and finally $T_2 = T_1 - \dfrac{Qx_1}{A\lambda_1}$

$$T_2 = 973 - \frac{4691.59 \times 0.15}{1.7} K$$

$$T_2 = 973 - 413.96 K$$

$$T_2 = 559 K$$

The interface temperature T_3 is found in the same way by applying the equation across the steel plate or the magnasite brick, taking care to use the correct temperature

Using the above equation across the steel plate gives

$$T_3 = 559 - \frac{4691.59 \times 0.012}{17.3} K$$

$$T_3 = 559 - 3.25 K$$

$$T_3 = 555.75 K$$

We could also work back from temperature T_4 and obtain the same result

$$(T_3 - T_4) = \frac{Qx_3}{A\lambda_3} \qquad T_3 = \frac{Qx_1}{A\lambda_1} + T_4$$

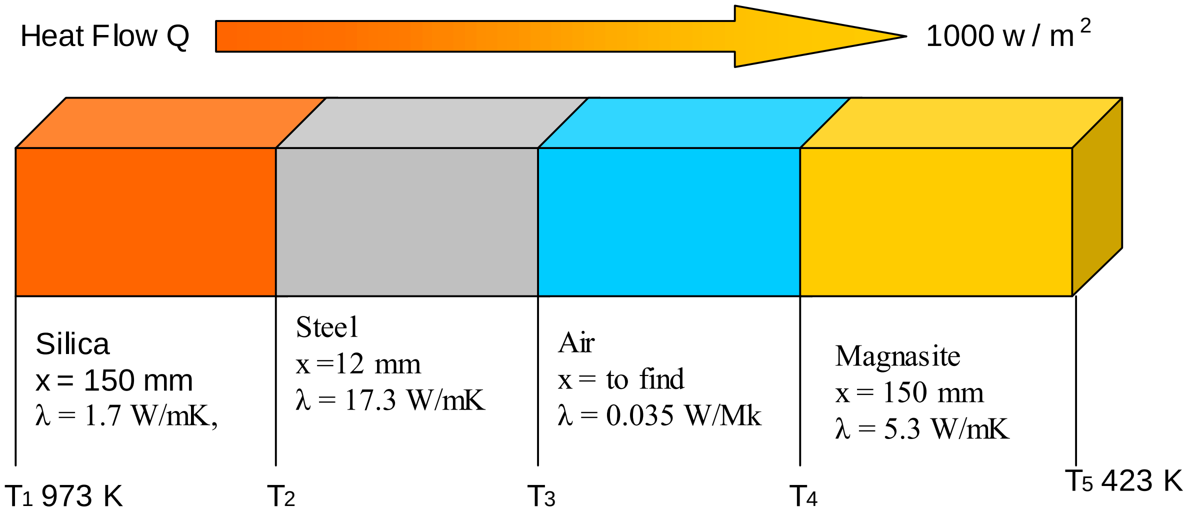
$$T_3 = \frac{4691.59 \times 0.15}{5.3} + 423 K$$

$$T_3 = 132.78 + 423 K$$

$$T_3 = 555.78 K$$

The second part of the problem uses the same procedures but from a different approach.
Another diagram is required to check the new arrangement, to see what we have and perhaps more importantly what we haven't.

This is the new arrangement.



If we apply Fourier's equation the only unknown is the thermal resistance of the wall. If we now calculate a new value for this the difference will be due to the air gap and the size of the air gap can be determined.

$$Q = \frac{A(T_1 - T_4)}{\sum \frac{x}{\lambda}}$$

$$\sum \frac{x}{\lambda} = \frac{A(T_1 - T_4)}{Q}$$

$$\sum \frac{x}{\lambda} = \frac{973 - 423}{1000} = 0.55 \frac{m^2 K}{W}$$

$$\frac{x}{\lambda} = 0.55 - 0.1172 = 0.4328$$

$$\frac{x}{0.035} = 0.4328$$

$$x = 0.4328 \times 0.035$$

$$x = 0.01515 m$$

If we put the units in the equation we have $m = \frac{m^2 K}{W} \times \frac{W}{mK}$

The thickness of the air gap is therefore 15.15 mm

7.3 Heat transfer through boundary layers of fluid

Newton's law of Cooling

We have looked at conduction through a wall from surface to surface, however it is more usual to measure the temperature of the surrounding atmosphere rather than the surface which means we have to take account of the fluid film on the surface over which the heat transfer is taking place.
Conduction in fluids forms a very small part of the heat transfer, convection is the main factor.
Convection is the name given to the motion of the fluid so that fresh fluid is available for heating or cooling, however there are also smaller currents within the bulk of the fluid which also assist in the distribution of the heat energy.
Convection heat transfer can be broadly classified as, natural convection where the heat transfer between a solid and fluid is not disturbed by other effects and forced convection where the motion of the fluid is assisted by an external source.
The analysis of the convection mechanism covers many variables, however Issac Newton (1701) proposed a general equation to described convection heat transfer.

Giving Newton's Law of Cooling as $Q = h\ A\ (T_W - T)$

"A" is the surface area, "T_W" the surface temperature and "T" the Mean temperature of the fluid.

" h" is called the HEAT TRASNSFER COEFFICIENT which is defined as the amount of heat conducted through a film of fluid per unit area of surface in unit time for a unit temperature drop across the thickness of the film.

This can be expressed in equation form as $h = \dfrac{Q}{A(T_W - T)t}$

If we put the in the units we can see that $h = \left(\dfrac{J}{m^2 K \sec}\right) = \dfrac{W}{m^2 K}$

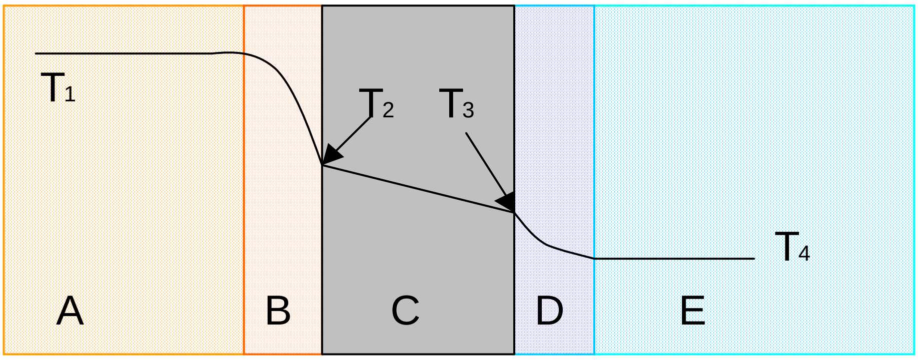
Now the same amount of heat will pass through the solid and the fluid film therefore equating Fourier's equation with Newton's equation gives

$$\frac{\lambda A(T_1 - T_2)}{x} = hA(T_1 - T_2)$$

So in general $\dfrac{\lambda}{x} = h$ where x is the thickness of the stagnant layer of fluid on the surface and λ is the thermal conductivity of the fluid.
The value of "h" depends on the fluid and flow regime typical values are in the region of those shown below.

Natural convection	0.004 – 0.05 W/m^2K
Forced convection (air)	0.01 – 0.55 W/m^2K
Forced convection (liquids)	0.1 – 5.5 W/m^2K
Boiling heat transfer (water)	1.0 - 110 W/m^2K
Condensation (steam filmwise)	0.55 – 25.0 W/m^2K

Heat Transfer, Fluid to Fluid, Through a Metal



Consider a gas flowing on one side of a metal plate and water flowing on the other. Adhering to the metal on the gas side is a layer of stagnant gas "B" and then the main gas stream "A".
On the waterside of the plate there will be a layer of very slow moving water "D" and then the main water flow "E".

The heat transferred from the gas to the water tube in three stages
1) By convection from gas "A" to stagnant gas "B"
2) By conduction and convection through the layer of gas "B"
3) By conduction through plate "C"
4) By conduction and convection through the layer of water "D"
5) By convection from water "D" to the main flow of water "E"

In each of these stages, there will be a drop in temperature and since gas is a poor conductor, there will be a considerable drop in temperature through the layer of stagnant gas.

T_1 = temperature of the gas
T_2 = temperature of metal on the hot side of the plate.
T_3 = temperature of metal on the cold side of the plate
T_4 = temperature of the water
λ = thermal conductivity of the metal
x = thickness of the metal
h_1 = film heat transfer coefficient from gas to metal
h_2 = film heat transfer coefficient from metal to water

Consider any area, "A", through which heat transfer is "Q"

Gas to metal $\quad Q = h_1 A(T_1 - T_2) \quad T_1 - T_2 = \dfrac{Q}{h_1 A}$

Conduction through the metal $\quad Q = \dfrac{\lambda A(T_2 - T_3)}{x} \quad T_2 - T_3 = \dfrac{Qx}{\lambda A}$

Metal to water $\quad Q = h_2 A(T_3 - T_4) \quad T_3 - T_4 = \dfrac{Q}{h_2 A}$

Adding these equations gives $$T_1 - T_4 = \frac{Q}{A}\left(\frac{1}{h_1} + \frac{1}{\frac{\lambda}{x}} + \frac{1}{h_2}\right)$$

If we define an overall heat transfer coefficient from gas to water as U the equation for heat transfer becomes

$$Q = UA(T_1 - T_4)$$ Rearranging this gives us $$T_1 - T_4 = \frac{Q}{UA}$$

if we compare the equations for $T_1 - T_4$ then we can see that

$$\frac{1}{U} = \left(\frac{1}{h_1} + \frac{1}{\frac{\lambda}{x}} + \frac{1}{h_2}\right)$$

We now have one general equation $Q = UA(T_{inside} - T_{outside})$ and an equation from which we can evaluate U.
The equation for U can be expanded or contracted to account for any number of layers providing due attention is paid to the temperature drop across the layers included.
Using $(T_{inside} - T_{outside})$ as the temperature difference will give a positive value when the inside temperature is greater than the outside and a negative value when the outside temperature is greater than the inside.
What we must remember is that we require the temperature difference, that heat always flows down the thermal gradient from high to low temperature and in this case the sign is indicating the direction of heat flow.
In the following example $(T_{outside} - T_{inside})$ has been used to give a positive value for the heat flow and due care has been taken in calculating the interface temperatures.

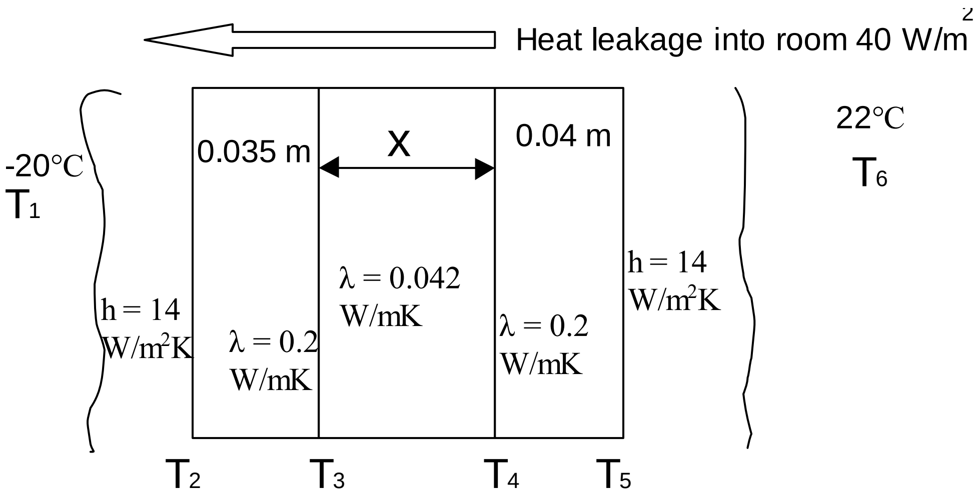
Example 7.3-1

A temporary cold store is to be made from mineral wool sandwiched between two layers of timber.
The inner layer of timber will be 35 mm thick and the outer layer of timber will be 40 mm thick.
The refrigeration equipment available is capable of removing 40 Watts per square metre of wall area.
The cold store is to be maintained at -20°C in an ambient temperature of 22°C
Calculate

- a) The minimum thickness of insulation
- b) i) The inner surface temperature
  - ii) The interface temperature between the inner layer of wood and insulation
  - iii) The interface temperature between the outer layer of wood and insulation
  - iv) The outer surface temperature.

For mineral wool insulation $\lambda = 0.042$ W/mK
For Timber $\lambda = 0.2$ W/mK
Surface heat transfer coefficient for inner and outer surface h = 14 W/m^{2}K

The first thing is to sketch the arrangement and insert all relevant data



Section of insulated wall with surface film coefficients

We need to find the thickness of insulation that will limit the heat flow to 40 W/m^2

The equation we need to use is this $Q = UA(T_{outside} - T_{inside})$

We have the rate of heat transfer Q = 40 W/m^2
We have the temperature change 22 to -20°C
The only unknown is U the overall heat transfer coefficient we can use this to find the thickness from

$$\frac{1}{U} = \left(\frac{1}{h_{inside}} + \frac{1}{\frac{\lambda_{iwood}}{x_{inner}}} + \frac{1}{\frac{\lambda_{insulation}}{x_{insulation}}} + \frac{1}{\frac{\lambda_{iwood}}{x_{outer}}} + \frac{1}{h_{outer}} \right)$$

Now all we need to do is put a few numbers in the above equations

Rearrange $Q = UA(T_6 - T_1)$ to give $U = \frac{Q}{(T_6 - T_1)A}$

And thus $U = \frac{40}{(22 - (-20))} = \frac{40}{42} = 0.952 W/m^2K$

We can now use this to find the thickness of insulation

$$\frac{1}{0.952} = \left(\frac{1}{14} + \frac{1}{\frac{0.2}{0.035}} + \frac{1}{\frac{0.042}{x_{insulation}}} + \frac{1}{\frac{0.2}{0.04}} + \frac{1}{14} \right)$$

$$1.05 = 0.7143 + 0.175 + \frac{1}{\frac{0.042}{x_{insulation}}} + 0.2 + 0.0714$$

$$1.05 = 0.51786 + \frac{x}{0.042}$$

$$x = 0.02235m$$

The minimum thickness of insulation is 22.35 mm.

The task now is to determine the surface and interface temperatures.
We could apply Newton's equation and Fourier's equation in turn, however we have already calculated some values so it is easier to determine a new value for U in each case.

We can rearrange our basic equation to give $$(T_2 - T_1) = \frac{Q}{UA}$$

And $$T_2 = \frac{Q}{UA} + T_1$$

Only the surface coefficient needs to be taken into account thus

$$\frac{1}{U} = \frac{1}{h_{inner}} = \frac{1}{14} = 0.07143$$

therefore U =14 W/m^{2}K, this is just like using Newton's equation

$$T_2 = \frac{40}{14} + (-20) = 2.857 - 20 = -17.142°\text{C}$$

For the next section T_1 to T_3 U includes the surface coefficient and a single layer of timber

$$\frac{1}{U} = \frac{1}{h_{inner}} + \frac{1}{\frac{\lambda_{wood}}{x_{inner}}} = \frac{1}{14} + \frac{1}{\frac{.2}{.035}} = 0.07143 + 0.175$$

Therefore U = 4.011 W/m^{2}K

$$T_3 = \frac{40}{4.011} + (-20) = 9.974 - 20 = -10.1°\text{C}$$

We can repeat this for the next section up to T_4 which includes the insulation all we need to do is insert the values for 1/U from the first part.

$$\frac{1}{U} = 0.07143 + 0.175 + 0.532$$

Therefore U= 1.28 W/m^{2}K

$$T_4 = \frac{40}{1.28} + (-20) = 31.14 - 20 = 11.14°\text{C}$$

For the final temperature $$\frac{1}{U} = 0.07143 + 0.175 + 0.532 + 0.2$$

U= 1.022 W/m^{2}K and

$$T_5 = \frac{40}{1.022} + (-20) = 39.14 - 20 = 19.14°\text{C}$$

7.4 Conduction through thin and thick cylinders including surface boundary layers

Thin Wall

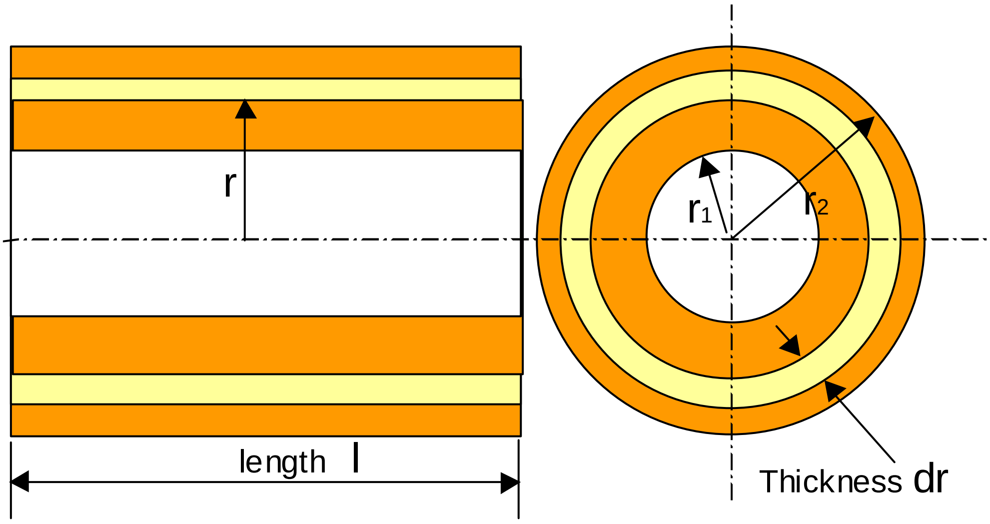
If the thickness of the cylinder is small compared to its radius then we can ignore the fact that the outer surface area is greater than the inner surface area and just use Fourier's equation as previous.

The equation $Q = \frac{\lambda A(T_1 - T_2)}{x}$ can be used with the area $A = 2\pi rl$

Where "r" is the external radius "l" is the length of pipe "x" is the pipe thickness

Thick Wall Cylinder

However in most engineering applications, the pipe wall thickness is not negligible relative to its radius, it is therefore necessary to allow for the difference in areas caused by the different radii of the pipe.



Now consider the thick wall tube shown above, it has a length of "l", internal radius "r_1" and external radius "r_2".

If we take an infinitely thin cylinder of thickness dr at a distance r from the axis of the tube, the temperature drop across the element of material will be dt.

The amount of heat "Q" conducted through the wall of the tube will be the same as that which passes through this element.

So if we apply Fourier equation $Q = \frac{\lambda A(-dt)}{x}$

The negative sign is used because temperature will decreases as the radius increases

$$Q = \frac{\lambda 2\pi rl(-dt)}{dr} \qquad Q\frac{dr}{r} = -\lambda 2\pi l dt \qquad Q\int_{r_1}^{r_2}\frac{dr}{r} = -\lambda 2\pi l\int_{t_1}^{t_2} dt$$

Integrate this equation between the inner and outer surface and putt temperature = t_1 when radius = r_1 and temperature = t_2 when radius = r_2

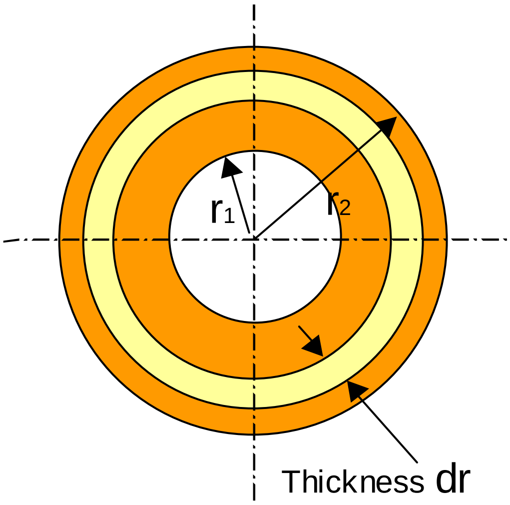
This gives $Q\ln\frac{r_2}{r_1} = -\lambda 2\pi l(t_2 - t_1)$ and $\frac{Q}{l} = \frac{\lambda 2\pi (T_1 - T_2)}{\ln\frac{r_2}{r_1}}$

It is more usual to obtain the heat transfer rate per unit length of pipe so if Q' is the heat transfer per unit length of tube, the equation becomes

$$Q' = \frac{\lambda 2\pi (T_1 - T_2)}{\ln\frac{r_2}{r_1}}$$

Conduction through a Hollow Sphere

If we section a hollow sphere it would look identical to the end view of the thick cylinder shown below, however the heat transfer is only in a radial direction.
The treatment is identical to that for the cylinder only this time the surface area of the sphere $4\pi r^2$ is used.



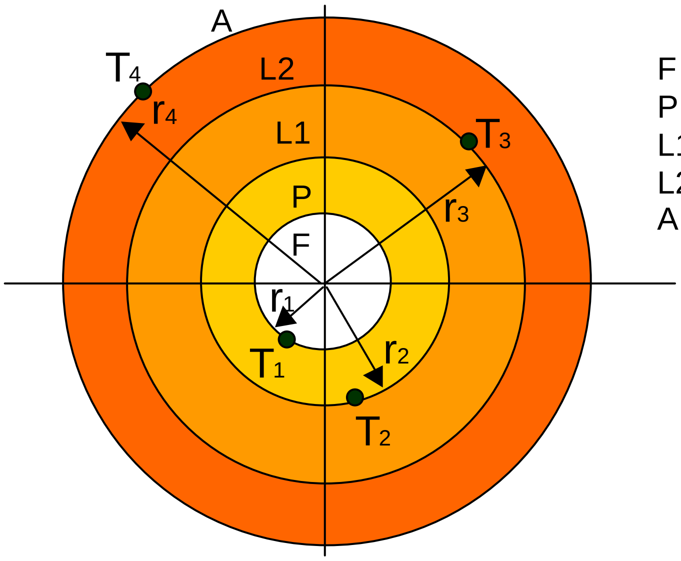
$$Q = \frac{\lambda 4\pi r^2(-dt)}{dr}$$

$$Q\int_{r_1}^{r_2}\frac{dr}{r^2} = -\lambda 4\pi\int_{t_1}^{t_2} dt$$

$$Q = 4\pi\lambda\frac{r_1 r_2}{r_2 - r_1}(T_1 - T_2)$$

Conduction through a Composite Cylindrical wall

This is treated in the same fashion as the flat composite wall.
A typical example is a pipe carrying a hot or cold fluid with layers of insulation, it is modelled as a series of concentric tubes, that is tubes within tubes.
If we take a thick walled pipe with two layers of insulation as shown below and apply Fourier's equation for a pipe in the same way as we did for the flat wall we end up with a general equation for heat flow from the inner to outer surface.



F = Fluid in pipe
P = pipe
L1 = insulation layer 1
L2 = insulation layer 2
A = ambient surroundings

This equation for T_1 - T_4 is obtained by making the temperature drop the subject in the Fourier equation applied to each layer and adding the equations as we did for a composite flat wall.

$$T_1 - T_4 = Q'\sum \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi\lambda_3}$$

This gives the equation $$Q' = \frac{(T_1 - T_{n+1})}{\sum \frac{\ln \frac{r_{n+1}}{r_n}}{2\pi\lambda_n}}$$ Where n is the number of tubes

This equation does not take account of the fluid films on the inner and outer surfaces of the pipe, for this we need to apply Newton's law of cooling to the inner and outer surface.

The heat transfer through the inner surface film will be $Q' = 2\pi r_1 h_{inside}\left(T_{fluid} - T_1\right)$

The heat transfer from the outer surface film will be $Q' = 2\pi r_4 h_{outer}\left(T_4 - T_a\right)$

If we apply the same process as we did for the composite wall and rearrange the equation across each layer making temperature the subject and adding them we can obtain an equation which covers the temperature change from the fluid to the atmosphere.

You may want to do this to confirm that the equation below is the result.
In this equation T_f and T_a are the mean temperatures, of the fluid in the pipe and surrounding atmosphere respectively.

$$T_f - T_a = \frac{Q'}{\left[\dfrac{1}{2\pi r_1 h_f} + \dfrac{1}{\dfrac{2\pi\lambda_1}{\ln\left(\dfrac{r_2}{r_1}\right)}} + \dfrac{1}{\dfrac{2\pi\lambda_2}{\ln\left(\dfrac{r_3}{r_2}\right)}} + \dfrac{1}{\dfrac{2\pi\lambda_3}{\ln\left(\dfrac{r_4}{r_3}\right)}} + \dfrac{1}{2\pi r_4 h_a}\right]}$$

If we let U′ be an overall heat transfer coefficient per unit length of tube from the hot fluid to the cold ambient air, we can have a general equation which would cover heat transfer through an insulated pipe.

Then $Q' = U'\left(T_f - T_a\right)$ which gives $T_f - T_a = \dfrac{Q'}{U'}$

this is identical to the equation above and gives 1/U′ as

$$\frac{1}{U'} = \left[\frac{1}{2\pi r_1 h_f} + \frac{1}{\dfrac{2\pi\lambda_1}{\ln\left(\dfrac{r_2}{r_1}\right)}} + \frac{1}{\dfrac{2\pi\lambda_2}{\ln\left(\dfrac{r_3}{r_2}\right)}} + \frac{1}{\dfrac{2\pi\lambda_3}{\ln\left(\dfrac{r_4}{r_3}\right)}} + \frac{1}{2\pi r_4 h_a}\right]$$

Or if you prefer

$$\frac{1}{U'} = \left[\frac{1}{2\pi r_1 h_f} + \frac{\ln\left(\dfrac{r_2}{r_1}\right)}{2\pi\lambda_1} + \frac{\ln\left(\dfrac{r_3}{r_2}\right)}{2\pi\lambda_2} + \frac{\ln\left(\dfrac{r_4}{r_3}\right)}{2\pi\lambda_3} + \frac{1}{2\pi r_4 h_a}\right]$$

We can use this equation $Q' = U'\left(T_f - T_a\right)$ for all problems involving insulated pipes with or without surface heat transfer coefficients.
All we need to do is obtain a value of U′ for the particular layer or layers we are dealing with and use the temperature change across the arrangement.
Using this approach we only need to remember two equations and how to obtain U′
Not forgetting that the rate of heat transfer is the same through each layer.

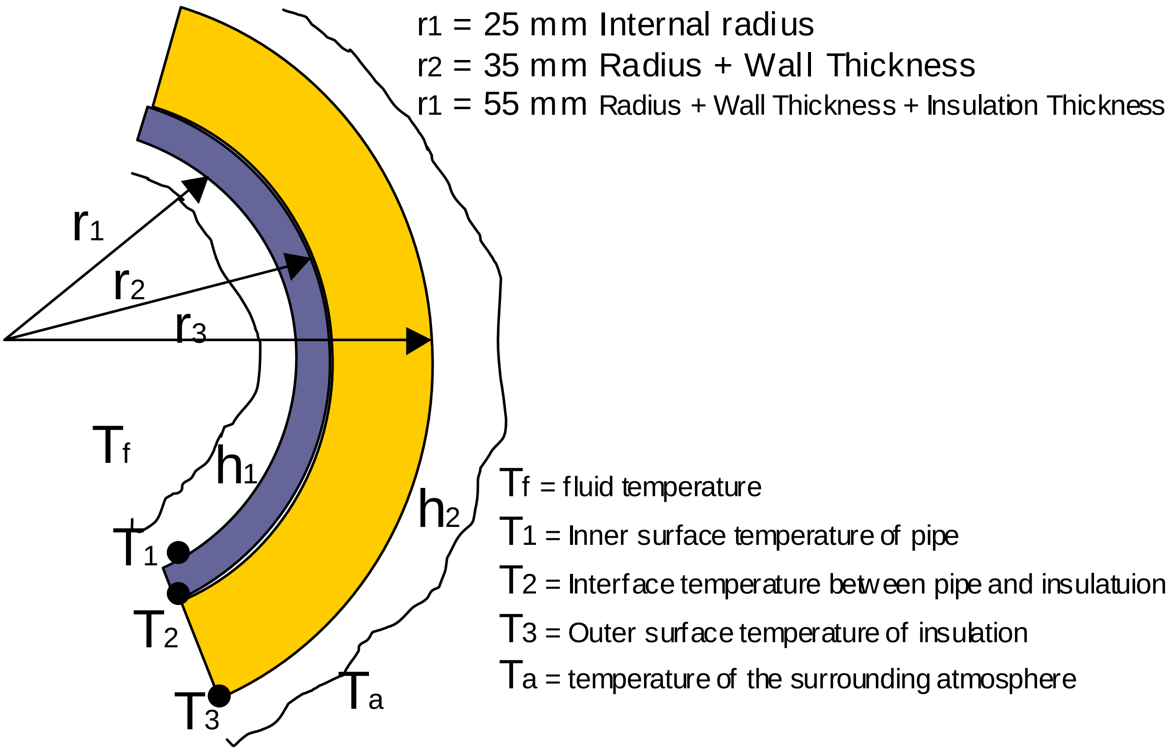
Example 7.4-1

The following data refers to a domestic hot water system.
Pipe;
Bore 50 mm, wall thickness 10 mm, thermal conductivity of material λ = 52 W/mk.
Mean water temperature 60°C.
Inner surface heat transfer coefficient h_1 = 1136 W/m^{2}k.
Insulation:
Thickness 20 mm, λ = 0.17 W/mk, ambient air temperature 20°C.
Outer surface heat transfer coefficient h_2 = 9.7 W/m^{2}k.

Calculate

a) The rate of heat lost per unit length of pipe.
b) The inside surface temperature.
c) The interface temperature between pipe and insulation.
d) The outside surface temperature.

The first thing is to sketch the arrangement of the various layers, taking care to use the correct dimensions and values.



Part (a) asks us to determine Q per unit length of pipe so we need to determine the overall heat transfer coefficient from the fluid temperature to the air temperature.

In this case we have two surface coefficients and two layers the equations are modified to take account of this.

To calculate U′ over the four layers $$\frac{1}{U'}=\left[\frac{1}{2\pi r_1 h_f}+\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1}+\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2}+\frac{1}{2\pi r_3 h_a}\right]$$

$$\frac{1}{U'}=\left[\frac{1}{2\pi 0.025\times 1136}+\frac{\ln\left(\frac{0.035}{0.025}\right)}{2\pi 52}+\frac{\ln\left(\frac{0.055}{0.035}\right)}{2\pi 0.17}+\frac{1}{2\pi 0.055\times 9.7}\right]$$

$$\frac{1}{U'}=\left[5.6\times 10^{-3}+1.0298\times 10^{-3}+0.423+0.298\right]$$

$$\frac{1}{U'}=0.721\frac{mK}{W}$$

$$U'=1.386\frac{W}{mK}$$

Now we have a value for U′ $$Q'=U'(T_f-T_a)$$

$$Q'=1.386(60-20)$$

The rate of heat transfer $$Q'=55.45\frac{W}{m}$$

The remaining parts of the question ask us for temperatures at the film and material boundaries.
Rather than use Newton's law and Fourier's law all we need do is calculate a value for U′ over the layers we are dealing with.
We already have these values partly worked out above.

For the inside surface temperature $Q'=U'(T_f-T_1)$ therefore $T_1=T_f-\frac{Q'}{U'}$

All we need is the value of U′ from $\frac{1}{U'}=\left[\frac{1}{2\pi r_1 h_f}\right]$

This value has already been calculated above $\frac{1}{U'}=\left[5.6\times 10^{-3}\right]$ $U'=\left[178.57\frac{W}{mK}\right]$

We can sub these values into the equation $T_1=T_f-\frac{Q'}{U'}$

$$T_1=60-\frac{55.45}{178.57}\qquad T_1=60-0.31\qquad T_1=59.67°\text{C}$$

This gives the temperature on the inner surface, the remaining temperatures are obtained by repeating the above method.

For the next temperature we can use two layers U′ $$\frac{1}{U'} = \left[\frac{1}{2\pi r_1 h_f} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1} \right]$$

Using previous values $$\frac{1}{U'} = \left[5.6\times10^{-3} + 1.0298\times10^{-3}\right]$$

Hence U′ $$U' = \left[150.83\frac{W}{mK}\right]$$

Put this value into $T_2 = T_f - \frac{Q'}{U'}$ $\qquad T_2 = 60 - \frac{55.45}{150.83}$

$$T_2 = 59.63°\text{C}$$

The value of T_2 is as expected as the pipe wall offers no resistance to heat flow

Repeat this process for the remaining layers.

$$\frac{1}{U'} = \left[\frac{1}{2\pi r_1 h_f} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2} \right] \qquad \frac{1}{U'} = \left[5.6\times10^{-3} + 1.0298\times10^{-3} + 0.423\right]$$

The new value of U′ $$U' = \left[2.3276\frac{W}{mK}\right]$$

Putting this in the equation for temperature $T_3 = T_f - \frac{Q'}{U'}$ $\qquad T_3 = 60 - \frac{55.45}{2.3276}$

$$T_3 = 36.17°\text{C}$$

This is not the only the only way to solve this problem.
You could use Newton's law and Fourier's law across each layer using the calculated values for temperatures at each boundary.
You could also determine the temperatures using the atmosphere as the start and work inwards to the fluid.

Example 7.4-2

A pipe with an outside diameter of 100 mm is to be covered with two 50 mm layers of insulation.

The thermal conductivity of insulation "A" is 0.1 W/mK

The thermal conductivity of insulation "B" is 0.4 W/mK

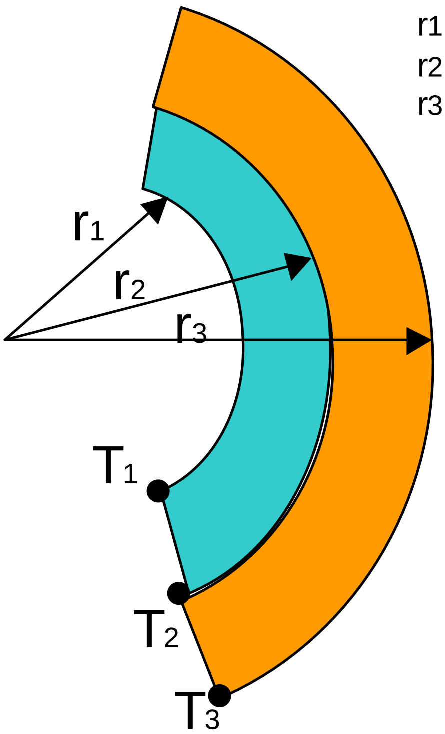
Calculate

The percentage reduction in heat transfer when using the most effective layer combination.

You may assume that the surface heat transfer coefficients remain unchanged regardless of the layer combination.

This question highlights that fact that the combination of insulation is just as important as thickness.

The arrangement is as shown



We do not have any temperatures but this does not matter because they will be the same for both combinations and we have been asked to determine a percentage change so the temperatures should cancel out any way.

Let subscript "AB" be insulation "A "on the inside and "B" on the outside

Let subscript "BA" be insulation "B" on the inside and "A"on the outside

$$\text{Percentage change} = \frac{Q'_{BA} - Q'_{AB}}{Q'_{BA}}$$ Since $Q' = U'(T_1 - T_3)$ we can sub for Q'

$$\text{Percentage change} = \frac{U'_{BA}(T_1 - T_3) - U'_{AB}(T_1 - T_3)}{U'_{BA}(T_1 - T_3)}$$

$$\text{Percentage change} = \frac{U'_{BA} - U'_{AB}}{U'_{BA}}$$

This problem has been reduced to obtaining a value for the thermal resistance.

For combination "A" inside "B" outside

$$\frac{1}{U'_{AB}} = \left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_A} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_B}\right] \qquad \frac{1}{U'_{AB}} = \left[\frac{\ln\left(\frac{100}{50}\right)}{2\pi 0.1} + \frac{\ln\left(\frac{150}{100}\right)}{2\pi 0.4}\right]$$

$$\frac{1}{U'_{AB}} = 1.103 + 0.1611 = 1.264$$

$$U'_{AB} = 0.791\frac{W}{mK}$$

For combination "B" inside and "A" outside

$$\frac{1}{U'_{BA}} = \left[\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_B} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_A}\right] \qquad \frac{1}{U'_{BA}} = \left[\frac{\ln\left(\frac{100}{50}\right)}{2\pi 0.4} + \frac{\ln\left(\frac{150}{100}\right)}{2\pi 0.1}\right]$$

$$\frac{1}{U'_{BA}} = 0.275 + 0.645 = 0.921$$

$$U'_{BA} = 1.085\frac{W}{mK}$$

We can now sub these values into our equation for percentage reduction.

$$\text{Percentage change} = \frac{U'_{BA} - U'_{AB}}{U'_{BA}}$$

$$\text{Percentage change} = \frac{1.085 - 0.791}{1.085}$$

$$\text{Percentage change} = 27.1\%$$

This change is a reduction in heat transfer, the combination AB allowing only 0.791 of the temperature change to escape, while the combination BA allows 1.085 of the temperature drop to escape.

Conduction through a layered Hollow Sphere

Again if we section a hollow sphere made up of several layers it would look identical to the end view of a composite cylinder with the heat transfer is only in a radial direction.
The treatment is identical to that for the cylinder and yields the following equation for U.

$$\frac{1}{U}=\left[\frac{1}{4\pi r_1^2 h_{inner}}+\frac{r_2-r_1}{4\pi\lambda_1 r_1 r_2}+\frac{r_3-r_2}{4\pi\lambda_2 r_2 r_3}+\frac{1}{4\pi r_3^2 h_{outer}}\right]$$

This can be placed in the equation $Q=U\left(T_{inner}-T_{outer}\right)$ and the overall heat transfer for the sphere can be determined.
The heat transfer for a hemi-sphere would simply be half of this.

Example 7.4-3

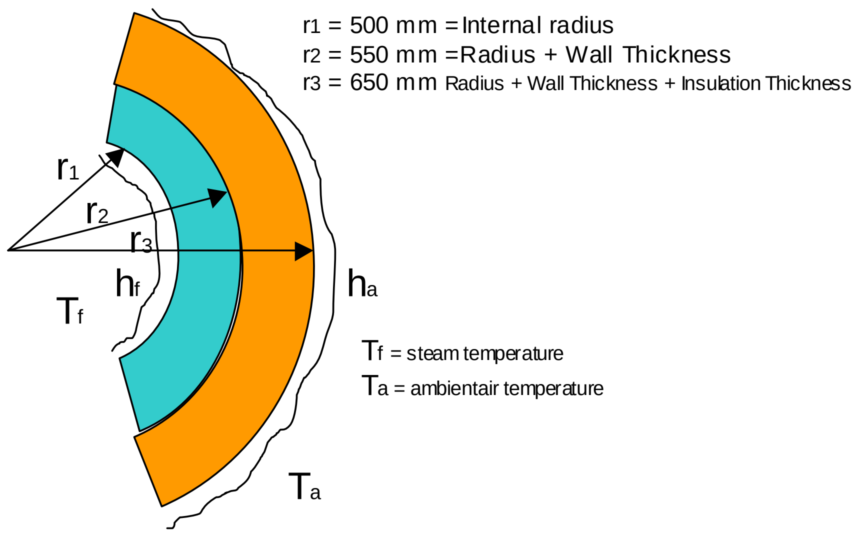
A steam- steam generator produces saturated steam at 20 bar in an ambient temperature of 30°C.
The shell is 50 mm thick and is covered in 100 mm of insulation.
The centre section is in the form of a cylinder 1 m internal diameter and 3 m in length.
The ends are hemispherical with a 1 m internal diameter.
Thermal conductivity of the shell λ_1 = 52 W/mk.
Inner surface heat transfer coefficient h_f = 5.2 W/m^{2}k.
Thermal conductivity of insulation λ_2 = 0.04 W/mk.
Outer surface heat transfer coefficient h_a = 1.7 W/m^{2}k.

Ignore any heat loss through fittings and mountings.

Calculate The total rate of heat loss.

This problem can be broken down into a cylinder and sphere.

For the cylinder



$$Q' = U'(T_{inner} - T_{outer})$$

$$\frac{1}{U'} = \left[\frac{1}{2\pi r_1 h_f} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi\lambda_2} + \frac{1}{2\pi r_3 h_a} \right]$$

If we now put some values into this equation

$$\frac{1}{U'} = \left[\frac{1}{2\pi 0.5 \times 5.2} + \frac{\ln\left(\frac{550}{500}\right)}{2\pi \times 52} + \frac{\ln\left(\frac{650}{550}\right)}{2\pi \times 0.04} + \frac{1}{2\pi 0.65 \times 1.7} \right]$$

$$\frac{1}{U'} = 0.0612 + 2.9 \times 10^{-4} + 0.6646 + 0.144$$

$$\frac{1}{U'} = 0.87$$

$$U' = 1.149 \frac{W}{mK}$$

we can put this value in the equation $Q' = U'(T_{inner} - T_{outer})$ but remember this is the heat transfer per unit length so we must multiply this value by the length of the cylinder.

The inner temperature is the saturation temperature corresponding to the steam pressure which is obtained from the steam tables.

$$Q' = 1.149(212.4 - 30)$$

$$Q' = 209.61 \frac{W}{m}$$

The total heat transfer Q from the cylindrical section is 209.61 x 3 = **628.84 W.**

The hemispherical ends together make a sphere.
The diagram is exactly as that for the cylindrical section, the method is also the same.

For the sphere. $Q = U(T_{inner} - T_{outer})$

$$\frac{1}{U} = \left[\frac{1}{4\pi r_1^2 h_{inner}} + \frac{r_2 - r_1}{4\pi \lambda_1 r_1 r_2} + \frac{r_3 - r_2}{4\pi \lambda_2 r_2 r_3} + \frac{1}{4\pi r_3^2 h_{outer}} \right]$$

$$\frac{1}{U} = \left[\frac{1}{4\pi 0.5^2 \times 5.2} + \frac{0.55 - 0.5}{4\pi 52(0.55 \times 0.5)} + \frac{0.65 - 0.55}{4\pi 0.04(0.65 \times 0.55)} + \frac{1}{4\pi 0.65^2 \times 1.7} \right]$$

$$\frac{1}{U} = \left[0.0612 + 2.78 \times 10^{-4} + 0.5565 + 0.11 \right]$$

$$\frac{1}{U} = 0.7287 \frac{K}{W}$$

$$U = 1.372 \frac{W}{K}$$

Heat loss from the sphere is therefore $Q = 1.372(212.4 - 30)$

$$Q = 250.28W$$

We must add this to the heat lost from the cylinder to obtain the total heat loss.

$Q_{total} = 250.28 + 628.84 = 879.12$ W

7.5 Heat transfer by Thermal Radiation

Thermal radiation consists of only one fairly narrow band of a spectrum of electromagnetic waves, which are emitted due to the agitation of molecules within a substance and whenever matter absorbs these waves it gains internal energy.
The waves are similar to light waves in that they are propagated in straight lines at the speed of light and they require no medium for propagation.
Radiation striking a body can be absorbed by the body, reflected from the body, or transmitted through the body.
The fractions of the radiation absorbed, reflected, and transmitted are called the absorptivity, "α", the reflectivity, "ρ", and the transmissivity, "τ" , respectively.

Thus $$\alpha + \rho + \tau = 1$$

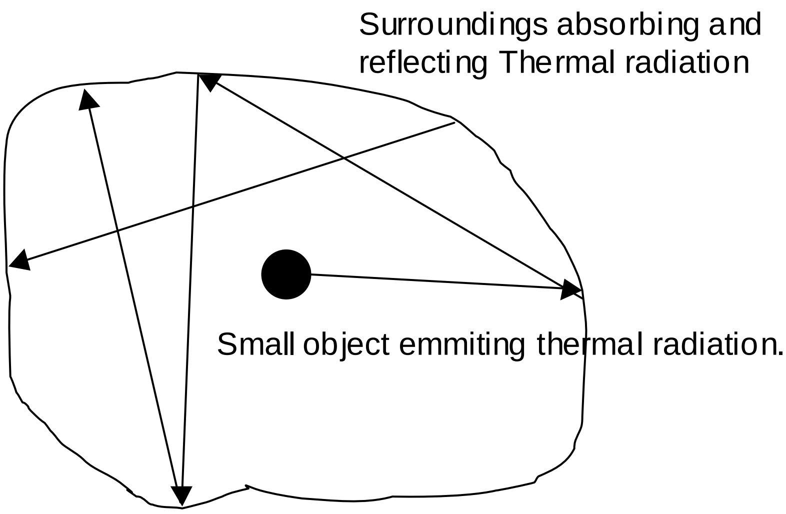
For most solids and liquids encountered in engineering the amount of radiation transmitted through the substance is negligible and it is possible to write

$$\alpha + \rho = 1$$

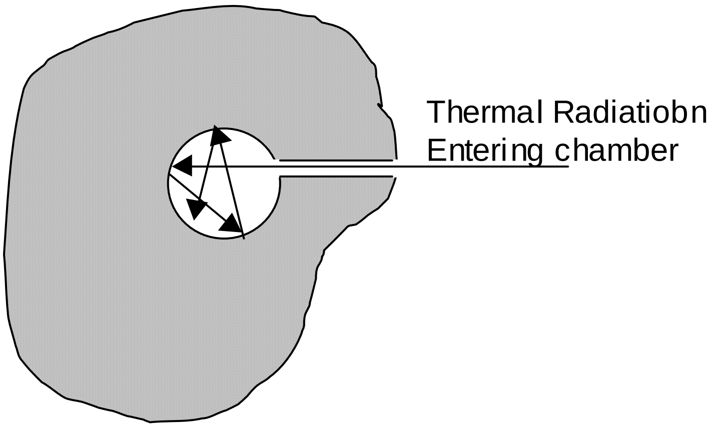
It is useful to define an ideal body, which absorbs all the radiation falling upon it as a black body.
For a black body $\alpha = 1$ and $\rho = 0$.

The term 'black' in this context does not necessarily imply black to the eye.

A surface which is black to the eye is one which absorbs all the light incident upon it, but a surface can absorb all the thermal radiation incident upon it without necessarily absorbing all the light.
For example, snow is almost 'black' to thermal radiation and has an absorptivity of $\alpha = 0.985$.
Although no totally black body exists in practice, many surfaces approximate to the definition.



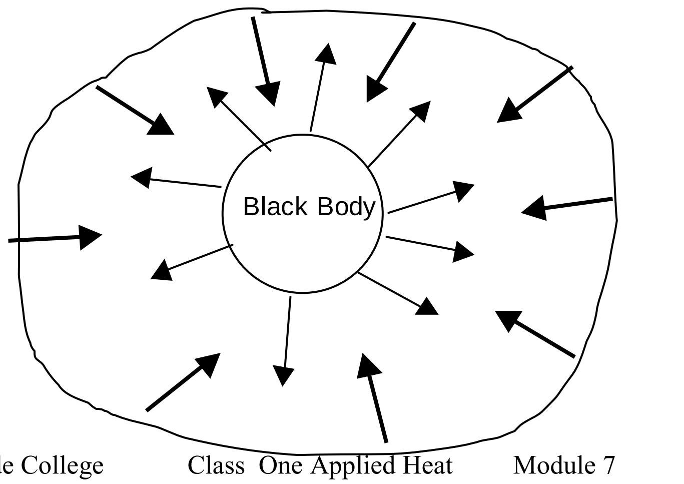
If we consider a small object radiating energy in a large space, as shown above.
Then the energy striking the surface surrounding the body is reflected and absorbed many times by the surface, and the fraction of energy reflected back and absorbed by the body is very small.
Therefore, as far as the object is concerned, the surroundings are approximately black to thermal radiation.



The closest practical approximation to a black body, would be the inside surfaces of a chamber inside an object lined with a material of high absorptivity such as lampblack.
The walls of the chamber absorb rays of thermal radiation entering the hole, so that only a negligible amount of radiation leaves.
Thus the hole acts as a black body.

The energy radiated from a body per unit area per unit time, is called the emissive power, $\dot{E}$.

It can be shown that a black body, as well as being the best possible absorber of radiation, is also the best possible emitter.
If we put a black body be placed in a space as shown, it will emit and absorb radiation
Now if the body is at the same temperature as the space they both must emit and absorb the same amount of radiation



Let the emissive power of the black body be $\dot{E}_B$

The rate at which energy impinges on a unit surface of the black body is also $\dot{E}_B$
If we replace the black body with any other body at the same temperature, and of the same shape and size.
This body must receive exactly the same amount of energy from the space as the black body received when it was in the same position in the space.
However, this body is not black and hence will only absorb a fraction of the energy it receives,
Rate of energy absorption = $\alpha \dot{E}_B$ where α is the absorptivity of the body.

Now as before the energy absorbed must be equal to the energy emitted, therefore if the body has an emissive power of E, we have

$$\dot{E} = \alpha \dot{E}_B \qquad \text{or} \qquad \alpha = \frac{\dot{E}}{\dot{E}_B}$$

Since $\alpha \langle 1 \; then \; \dot{E} \langle \dot{E}_B$ hence the black body is the best possible emitter of radiation.
The ratio of the emissive power of a body, to the emissive power of a black body, is called the emissivity, ε.
When two bodies are at the same temperature, then the absorptivity, α equals the emissivity, ε.
This is known as **Kirchhoff s law**, which may be stated as follows:
The emissivity of a body radiating energy at a temperature, T, is equal to the absorptivity of the body when receiving energy from a source at the same temperature.

THE GREY BODY

The energy emitted by thermal radiation is not the same for all wavelengths of the radiation.
To simplify calculations, surfaces in practice are very often assumed to have a constant emissivity over all wavelengths and for all temperatures.
Such an ideal surface is called a **grey body**.

Then, for a grey body, ε = α at all temperatures, where α and ε are the total absorptivity and the total emissivity over all wavelengths.

Stefan-Boltzmann Law

The total energy radiated by a body is proportional to the fourth power of its absolute temperature.
This was discovered experimentally by Stefan and later deduced by Boltzmann and is more explicitly stated thus;

If a body of surface area A (m^2), is at an absolute temperature T_1 (K) and ε is the emissivity, then the quantity of heat Q (kJ/s) radiated to the surroundings at absolute temperature T_2 (K) is given by the expression

$$Q = \sigma_{SB} \varepsilon A \left(T_1^4 - T_2^4\right)$$

The constant of proportionality σ_{SB} is known as the Stefan-Boltzmann constant

$$\sigma_{SB} = 56.7 \text{ x } 10^{-12} \text{ kW/m}^2\text{K}^4$$

Example7.5-1
The temperature of the flame in a furnace is 1277°C and the temperature of its surroundings is 277°C.
Calculate the maximum theoretical quantity of heat energy radiated per minute per square meter to the surrounding surface area..

In this case the maximum possible heat transfer due to thermal radiation is if we treat the furnace as a black body.
In this case the emissivity is 1.
The Stefan-Boltzmann constant is in kW therefore we need to multiply the equation by 60 to bring the answer to minutes.

$$Q = \sigma_{SB} \varepsilon A \left(T_1^4 - T_2^4\right)$$

$$Q = 56.7 \times 10^{-12} \times 1 \times 1 \left(1550^4 - 550_2^4\right) \times 60$$

$$Q = 19.33 \frac{MJ}{\min}$$

Example7.5-2
An electric heater with a surface area of 0.15 m^2,is in a large room with a wall temperature of 15°C. the emissivity of the heater is 0.8 with a temperature of 700°C. Calculate the heat transfer rate from heater to room.

The heater may be regarded as a grey body in large surroundings which may be considered to be black.

$$Q = \sigma_{SB} \varepsilon A \left(T_2^4 - T_1^4\right)$$

$$Q = 56.7 \times 10^{-12} \times 0.8 \times 0.15 \left(288^4 - 973^4\right)$$

$$Q = -6.045 kW$$

Example7.5-3

A hot fuel pipe of 100 mm outer diameter has a surface temperature of 80°C with a surrounding atmospheric temperature of 15°C. Calculate the emissivity of the pipe if the heat lost by radiation is 10 W per metre length of pipe.

$$Q = \sigma_{SB} \varepsilon A \left(T_1^4 - T_2^4\right)$$

$$10 = 56.7 \times 10^{-9} \times \varepsilon \times \pi \times 0.1 \left(353^4 - 288^4\right)$$

$$\varepsilon = 0.065$$

7.6 Heat transfer in shell and tube heat exchangers

One of the most important processes in engineering is the exchange of heat between flowing fluids.
Practical examples in which this occurs are air intercoolers and preheaters, condensers and boilers in steam plant, condensers and evaporators in refrigeration units.

There are three main types of heat exchanger:

1. **Recuperator** in which the flowing fluids exchanging heat are on either side of a dividing wall;
2. **Regenerator** in which the hot and cold fluids pass alternately through a space containing a matrix of material that provides alternately a sink and a source for heat flow;
3. **Evaporative** in which a liquid is cooled evaporatively and continuously in the same space as is the coolant as in a cooling tower.

We are only going examine the recuperator type and more specifically ones in which the flow of the two fluids are on the same axis.
In this case the two fluids may flow in the same or opposite directions giving rise to the names parallel and counter flow.
This in-line heat exchanger may consist simply of two concentric tubes, one fluid flowing in the inner tube and the other in the annulus or there may be a number of tubes within a large tube or shell and to increase heat transfer the shell fluid is made to flow partly across the tubes by means of baffles.

The mechanisms involved are therefore convection to or from the solid surface and conduction through the wall.
The wall may be corrugated or finned to increase turbulence and the heat transfer area, this analysis can very complex, however at this level we will only be dealing with overall heat transfer coefficients.

Most of the basic conduction and convection theory is applied in this heat exchanger, however the temperature of each fluid changes as it passes through the exchanger, and consequently the temperature of the dividing wall between the fluids also changes along its length.
This means that our previous equation $Q = U\left(T_{inner} - T_{outer}\right)$ must be modified since the temperatures previously where fixed but now they are continuely changing.

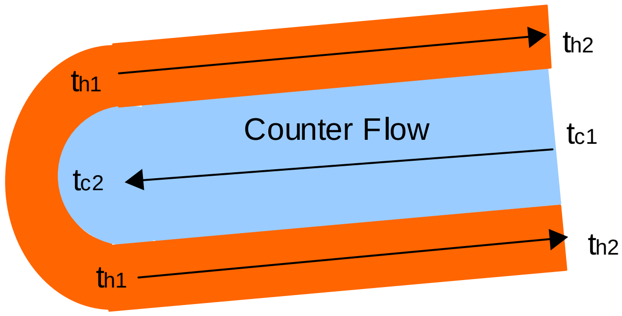
So which one do we use? Inlet, outlet, some average value?

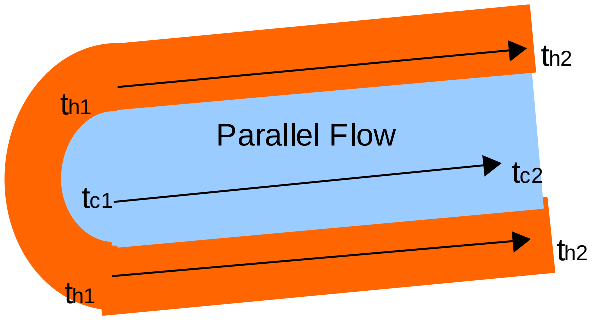
What we actually use is a value called the

LOGARITHMIC MEAN TEMPERATURE DIFFERENCE (LMTD)

We shall use an equation to obtain the LMTD over the next few pages but first we must look at the various configurations of this type of heat excahnger.

The diagrams below show what we mean by parallel flow where both fluids travel in the same direction, and counter flow where the fluids travel in opposite directions. The diagrams are of a concentric tube heat exchanger, the hot fluid is in the annulus and the cold fluid in the centre tube.



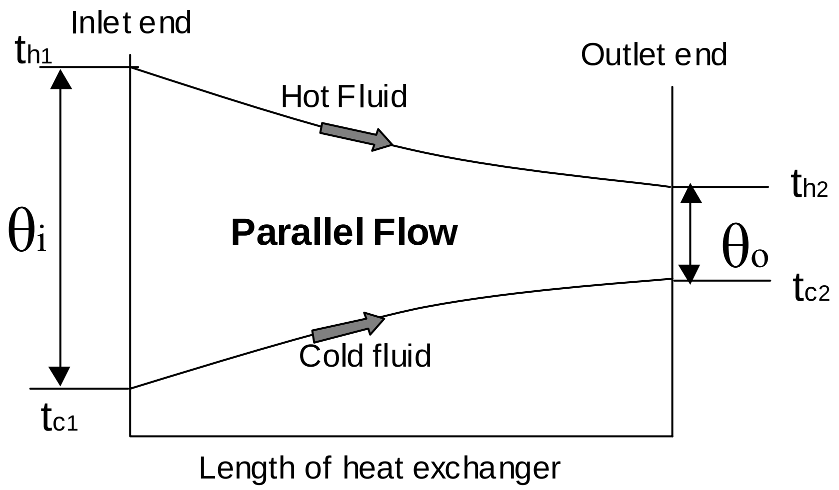


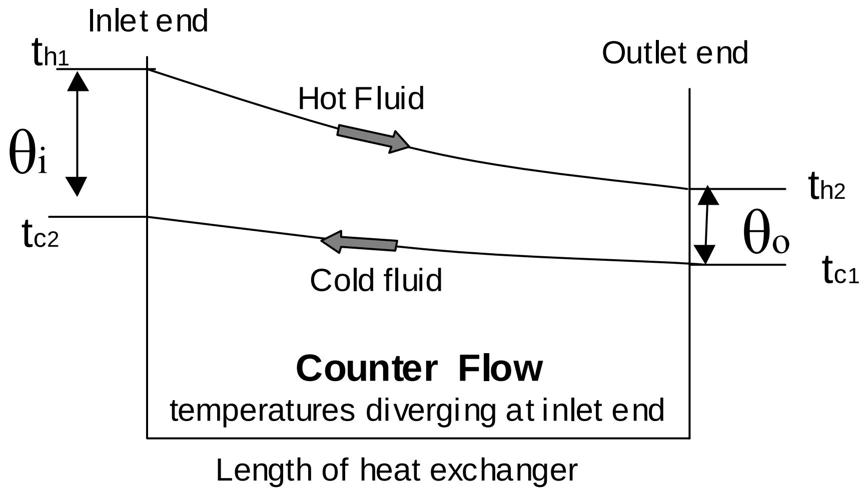
Definitions

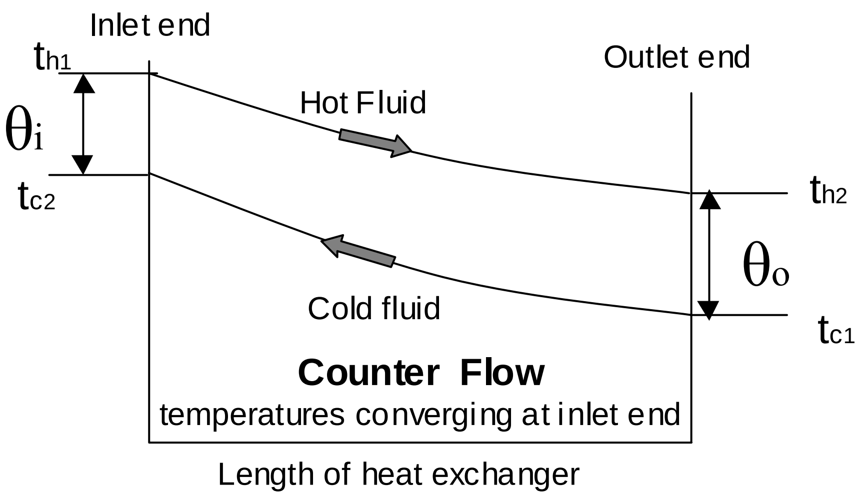
The temperature variations of the fluids in parallel and counter flow are shown below. Temperatures are plotted against length or area of the heat exchanger surface.
The inlet end, where length or area is zero is at the hot fluid inlet.

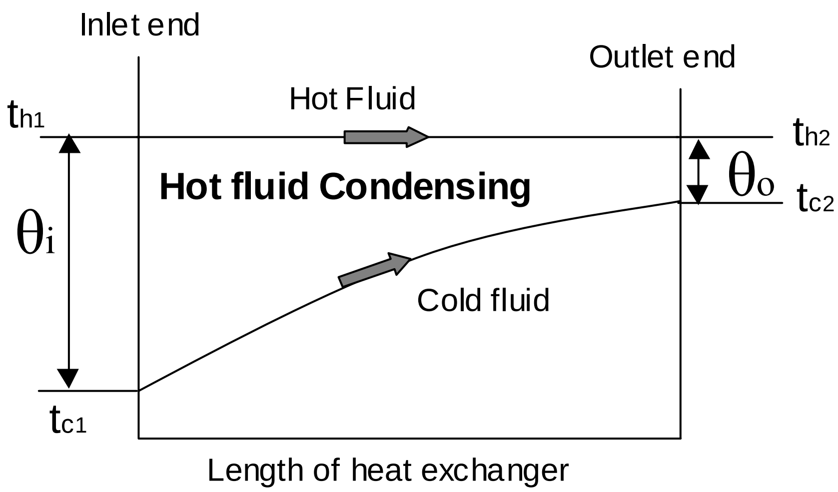
- t_{h1} is the inlet temperature of the hot fluid
- t_{h2} is the outlet temperature of the hot fluid
- t_{c1} is the inlet temperature of the cold fluid
- t_{c2} is the outlet temperature of the cold fluid
- θ_i is the temperature difference between the fluids at the inlet end of the heat exchanger
- θ_o is the temperature difference between fluids at the outlet end of the heat exchanger

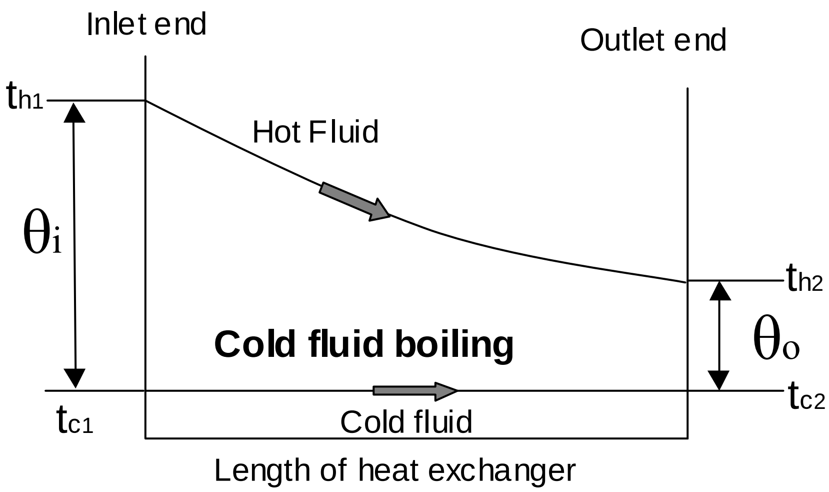
Heat exchanger configurations showing variation in temperatures of the hot and cold fluid











CHANGE OF PHASE

Temperature distributions with a change of phase are also shown above.

Only the phase change takes place in the exchanger, so the temperature of the boiling or condensing fluid does not change.

The temperature distributions are the same for both parallel and counter flow.

EFFECTIVENESS ε

Effectiveness is the ratio of energy actually transferred to the maximum theoretically possible.
The definition depends on the relative thermal capacities of the streams that is the mass x specific heat capacity, for the hot fluid this is $m_h c_{ph}$ and for the cold fluid $m_c c_{pc}$

In parallel flow t_{c2} will approach t_{h2} for an infinitely long heat exchanger, but can never exceed t_{h2}.
In counter flow it is quite normal for t_{c2} to exceed t_{h2} and, consequently, the counter flow exchanger is the more 'effective'.

The maximum theoretical transfer will take place in counter flow in an exchanger of infinite length and, in such a case,

$t_{c2} \rightarrow t_{h1}$ when $m_h c_{ph} > m_c c_{pc}$ and $t_{h2} \rightarrow t_{c1}$ when $m_c c_{pc} > m_h c_{ph}$

Thus the maximum transfer is $(mc_p)_{min}(t_{h1} - t_{c1})$ and in the two cases are:

$$m_c c_{pc}(t_{h1} - t_{c1}) \qquad \text{when } m_h c_{ph} > m_c c_{pc}$$

and

$$m_h c_{ph}(t_{h1} - t_{c1}) \qquad \text{when } m_c c_{pc} > m_h c_{ph}$$

The actual transfers in the two cases are, $m_c c_{pc}(t_{c2} - t_{c1})$ and $m_h c_{ph}(t_{h1} - t_{h2})$ hence ε, the effectiveness, becomes

$$\varepsilon = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} \text{ when } m_h c_{ph} > m_c c_{pc}$$

$$\varepsilon = \frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}} \text{ when } m_c c_{pc} > m_h c_{ph}$$

These definitions may be used in either counter or parallel flow, but the value of ε will be lower in parallel flow.

Determination of Heat Exchanger Performance

The primary purpose of a heat exchanger is to achieve the required transfer rate using the smallest possible transfer area and fluid pressure drop. A large exchanger can mean unnecessary capital outlay and high pressure drop means a reduced efficiency of the plant considered overall.
Generally, a smaller exchanger can be produced by, finning surfaces to increase the overall heat transfer coefficient. However, this leads to a higher fluid pressure drop, and the best design is often a compromise between the conflicting requirements, in fact, a number of different designs for a given duty may be acceptable.

The heat transfer requirement, Q, can be expressed in several ways:

$$Q = U_A A \theta_M$$
$$Q = U_L L \theta_M$$
$$Q = m_c c_{pc}\left(t_{c2} - t_{c1}\right)$$
$$Q = m_h c_{ph}\left(t_{h1} - t_{h2}\right)$$

θ_m is a mean temperature difference between the fluids, and U_A and U_L are mean coefficients, in W/m^{2}K and W/mK respectively applicable over the entire area "A" or length "L "of the exchanger and are determined in the usual way shown previously.

Counter and Parallel Flow Log Mean Temperature Difference (LMTD)

If the mass flow rates and inlet and outlet temperatures are known, the heat transfer will be known from $Q = m_c c_{pc}\left(t_{c2} - t_{c1}\right)$ or $Q = m_h c_{ph}\left(t_{h1} - t_{h2}\right)$
but further details of the exchanger cannot be specified until θ_m is known.

The derivation of θ_m can be found in any standard text on heat transfer.
The result and how to use it is all that concerns us here.
The required logarithmic mean temperature difference is .

$$\theta_m = \frac{\theta_0 - \theta_i}{Ln\dfrac{\theta_o}{\theta_i}}$$

It is the same for counter and parallel flow, though θ_o and θ_i in terms of values of t_h and t_c are different

Example 7.6-1

Water passes through the core of a 4 m long concentric tube heat exchanger in the opposite direction to air flowing in the annular space.
The core diameter is 30 mm while that of the outer shell is 80 mm.
The wall thickness of both tubes is 5 mm.
The air enters with a velocity of 30 m/s and temperature of 130°C.
The water enters with a velocity of 0.75 m/s and temperature of 6°C.
The heat exchanger has a thermal ratio of 0.88.
For air c_p = 1.012 kJ/kgK, density 1.009 kg/m^3
For water c_p = 4.19 kJ/kgK, density 1025 kg/m^3

Calculate

a) The outlet temperature of the air and water.
b) The overall heat transfer coefficient per unit length between the fluids.

In this example we know the heat lost by the air has been gained by the water

$$Q_{air} = Q_{water}$$

$$m_{air} \times c_{p(air)}\left(t_{h1} - t_{h2}\right) = m_{water} \times c_{p(water)}\left(t_{c2} - t_{c1}\right)$$

The mass can easily be determined using volume and density but in this case we do not have all the temperatures.
However we do have the thermal ratio which is akin to the effectiveness in that it is the ratio of the actual heat transferred to the maximum possible.
Since the minimum mc_p is that for the air then this becomes the ratio of the air temperature change and the maximum temperature change that is possible.

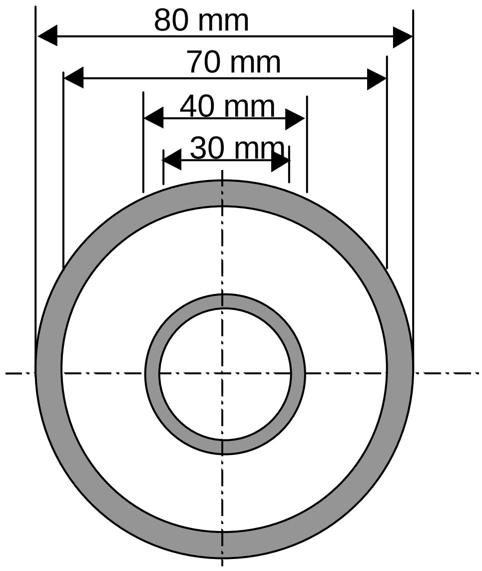
$$thermal\ \ ratio = \frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}}$$

$$0.88 = \frac{130 - t_{h2}}{130 - 6}$$

$$t_{h2} = 130 - 0.88(130 - 6)$$

$$t_{h2} = 20.88°\mathrm{C}$$

This gives us the temperature of the air leaving the cooler, if we calculate the mass flow of air and water we can obtain the water exit temperature.



This is the section of the cooler, the air passes through the annulus and the water through the core.
We must take the wall thickness of the tube into account when calculating the flow areas.

Mass flow rate of air = annular area x air velocity x density

$$\dot{m}_{air} = \frac{\pi\left(d_{outer}^{2} - d_{inner}^{2}\right)}{4} \times c \times \rho$$

$$\dot{m}_{air} = \frac{\pi\left(0.07^{2} - 0.04^{2}\right)}{4} \times 30 \times 1.009$$

$$\dot{m}_{air} = 0.07845\ \frac{kg}{s}$$

Mass flow rate of water = area of core x water velocity x density

$$\dot{m}_{water} = \frac{\pi 0.03^{2}}{4} \times 0.75 \times 1025$$

$$\dot{m}_{water} = 0.5434\ \frac{kg}{s}$$

The water outlet temperature can now be calculated from

$$m_{air} \times c_{p(air)}\left(t_{h1} - t_{h2}\right) = m_{water} \times c_{p(water)}\left(t_{c2} - t_{c1}\right)$$

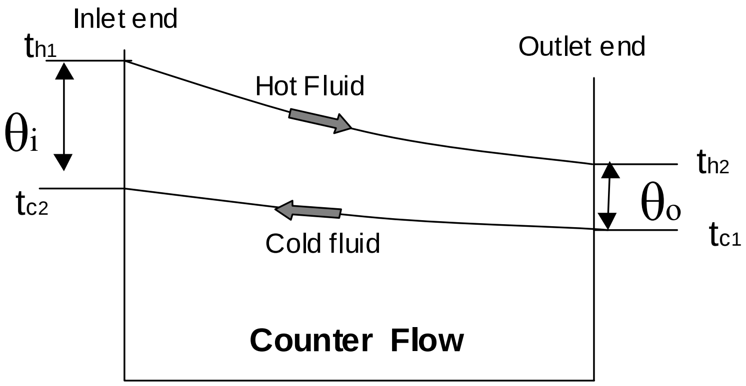
$$0.07845 \times 1.012(130 - 20.88) = 0.5434 \times 4.19\left(t_{c2} - 6\right)$$

$$3.8 = \left(t_{c2} - 6\right)$$

$$t_{c2} = 9.8°\mathrm{C}$$

Now we have both temperatures for both fluids we can look at obtaining the overall heat transfer coefficient for the cooler from $Q = U_L L \theta_M$

For this equation we need to calculate the θ_M which is the log mean temperature difference.



$$\theta_m = \frac{\theta_0 - \theta_i}{Ln\dfrac{\theta_o}{\theta_i}}$$

$\theta_i = t_{h1} - t_{c2}$ $\qquad \theta_i = 130 - 9.8$ $\qquad \theta_i = 120.2K$

$\theta_o = t_{h2} - t_{c1}$ $\qquad \theta_o = 20.88 - 6$ $\qquad \theta_o = 14.88K$

$$\theta_m = \frac{14.88 - 120.2}{Ln\dfrac{14.88}{120.2}}$$

$$\theta_m = 50.413K$$

We also need to calculate the rate of heat transfer.

$$Q = m_{air} \times c_{p(air)}(t_{h1} - t_{h2})$$

$$Q = 0.07845 \times 1.012(130 - 20.88)$$

$$Q = 8.663kW$$

We can find the overall heat transfer coefficient U_L from

$\dfrac{Q}{L\theta_M} = U_L$ $\qquad \dfrac{8663}{4 \times 50.413} = U_L$

$$U_L = 42.9\frac{W}{mK}$$

Example 7.6-2
A multi tubular single pass heat exchanger is used as a waste heat recovery unit in a diesel generator exhaust system.
The diesel engine develops 600 kW with a bsfc of 0.25 kg/kWhour at an air fuel ratio of 25:1.
The gas passes through 30 mm diameter tubes at a velocity of 12 m/s and pressure of 1.2 bar.
The gas inlet temperature is 360°C and the outlet temperature is 180°C.
Water enters the shell at 15°C and leaves at 90°C.
The theoretical overall heat transfer coefficient for the heat exchanger was estimated as
50 W/m^{2}K, however a fouling allowance of 0.6 m^{2}K/kW must be assumed to allow for in-service conditions.

For the gas c_p = 1.11 kJ/kgK, R=0.29 kJ/kgK
For water c_p = 4.19 kJ/kgK,

Calculate
a) The mass flow rate of water
b) The number of tubes required
c) The length of tube for a Parallel flow arrangement
d) The length of tube for a Contra flow arrangement.

This is a typical example where we must look at two aspects.
To obtain the mass flow of water we need to know the heat taken from the gas.
The number of tubes must allow the mass flow rate of the gas to pass through them so we need the volume flow rate of the gas.
The tube length is obtained from the surface area of the cooling tubes which in turn is obtained using the LMTD.

The mass flow of water is obtained from

$$m_{gas} \times c_{p(gas)}(t_{h1} - t_{h2}) = m_{water} \times c_{p(water)}(t_{c2} - t_{c1})$$

The mass of gas comes from the fuel consumption and the air fuel ratio.

$$bsfc = \frac{mass\text{ flow of fuel}}{\text{engine brake power}} \qquad Air\text{ / fuel ratio} = \frac{mass\text{ flow of air}}{\text{mass flow of fuel}}$$

$$\text{mass flow of air} = air\text{ fuel ratio x bsfc x brake power}$$

$$\text{mass flow of air} = 25 \text{ x } 0.25 \text{ x } 600$$

$$\text{mass flow of air} = 3750 \frac{\text{kg}}{\text{hour}}$$

$$\text{mass flow of gas} = \text{mass flow of air} + \text{mass flow of fuel}$$

$$\text{mass flow of gas} = 3750 + 150$$

$$\text{mass flow of gas} = 3900 \frac{\text{kg}}{\text{hour}}$$

using the equation $$m_{gas}\times c_{p(gas)}\left(t_{h1}-t_{h2}\right)=m_{water}\times c_{p(water)}\left(t_{c2}-t_{c1}\right)$$

$$\frac{3900}{3600}\times 1.11(360-180)=m_{water}\times 4.19(90-15)$$
$$\text{mass flow of water}=0.6887\,\frac{\text{kg}}{\text{second}}$$
$$\text{mass flow of water}=2.479\,\frac{\text{tonne}}{\text{hour}}$$

The number of tubes is obtained from the area required to pass the mass flow of gas divided by the area of one tube.

From $\quad \text{PV}=\text{mRT} \quad\quad \text{volume of gas}=\dfrac{\text{mRT}}{\text{P}}$

$$\text{volume of gas}=\frac{3900\times 290\times 633}{3600\times 1.2\times 10^{5}}$$

$$\text{volume of gas}=1.657\,\frac{\text{m}^{3}}{\text{second}}$$

$$\text{area of gas flow}=\frac{\text{volume flow}}{\text{gas velocity}}=\frac{1.657}{12}$$

$$\text{area of gas flow}=0.1381\text{m}^{2}$$

$$\text{area of gas}=\text{area of one tube}\times\text{number of tubes}$$

$$\text{number of tubes}=\frac{\text{area of gas}}{\text{area of tube}}=\frac{4\times 0.1381}{\pi\times 0.03^{2}}$$

$$\text{number of tubes}=196$$

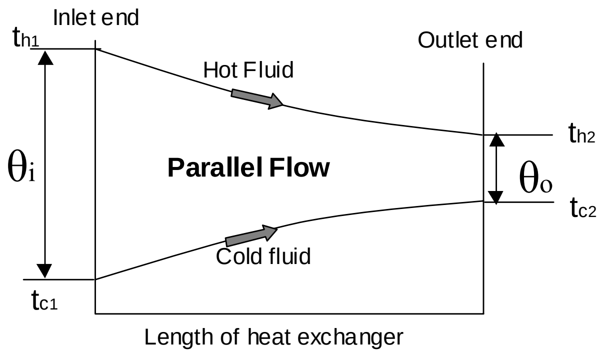
The length of each tube is obtained from the total surface area required to exchange all the heat.
The surface area is obtained from $\quad Q=U_{A}A\theta_{M}$

The value for U can be obtained from the information given $\quad \dfrac{1}{U_{A}}=\dfrac{1}{50}+\dfrac{0.6}{1000}$

$$U_{A}=48.54\frac{W}{m^{2}K}$$

This is the same for parallel and contra flow configurations as is the heat flow, however the temperature difference will change so this must be calculated for both arrangements.

Parallel Flow



$$\theta_i = t_{h1} - t_{c1} \qquad \theta_i = 360 - 15 = 345K$$

$$\theta_o = t_{h2} - t_{c2} \qquad \theta_o = 180 - 90 = 90K$$

$$\theta_m = \frac{\theta_0 - \theta_i}{Ln\dfrac{\theta_o}{\theta_i}}$$

$$\theta_m = \frac{90 - 345}{Ln\dfrac{90}{345}} = \frac{-255}{-1.3437} = 189.76K$$

$$\text{surface area of tubes required} = \frac{Q}{U_A\theta_m} = \frac{216450}{48.54 \times 189.76} = 23.498m^2$$

$$\text{surface area of tubes required} = \text{surface area available}$$

$$\text{surface area of tubes available} = \text{circumference of tube} \times \text{length} \times \text{number of tubes}$$

$$23.498 = \pi \times 0.03 \times \text{length} \times 196$$

$$\text{length of tube} = 1.272\text{ m}$$

This is the length of tube required for the parallel flow.

The length required for contra flow is obtained in exactly the same way in fact the only change is to the temperature difference obtained for the LMTD.

You should attempt this yourself the value for the LMTD should be 213.2K

This gives a required surface area of 20.92m^2 and a length of 1.132m.

This highlights the fact that a contra-flow heat exchanger is more effective.