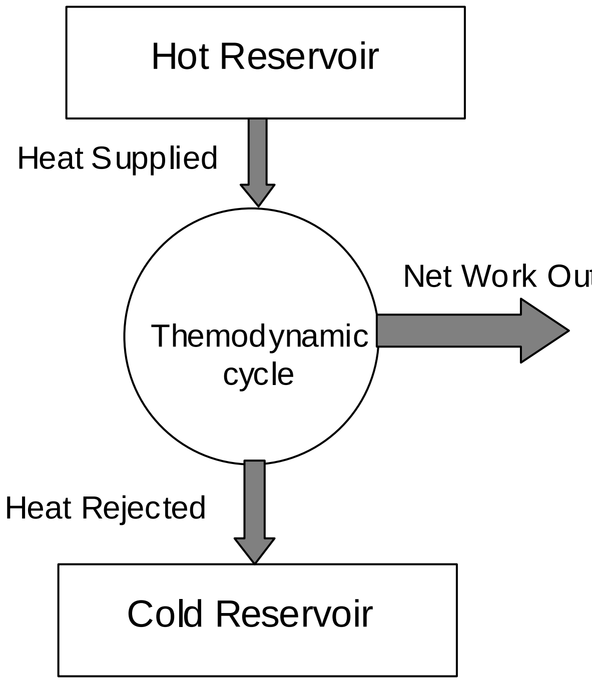
Heat Engine Cycles

The Second Law of Thermodynamics

We have previously seen that the Second Law states that if a system is to undergo a cycle and produce work, it must operate between at least two reservoirs of different temperature, however small this difference may be.
This can be shown in the schematic diagram below.

We can note that

$$\text{Net Work Output} = \text{Heat Supplied} - \text{Heat Rejected}$$



Heat Engine

A heat engine is a device which consists of a series of thermodynamic processes operating in a thermodynamic cycle and producing positive net work.

Reversible Heat Engine

This is a series of reversible thermodynamic processes forming a thermodynamic cycle and producing net work.
If any process is irreversible then the whole cycle is irreversible.
In this section we will only work with reversible cycles.

The Carnot Cycle

Sadi Carnot (1796-1832) expressed the second law in the form of a statement.

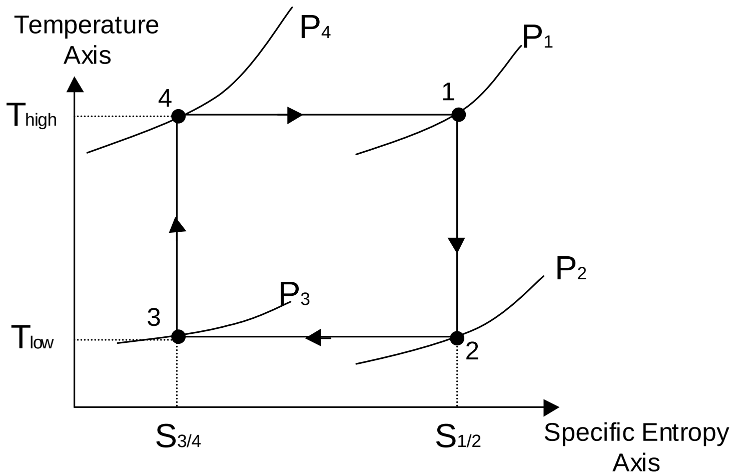
" When ever a temperature difference exists, motive power can be produced".

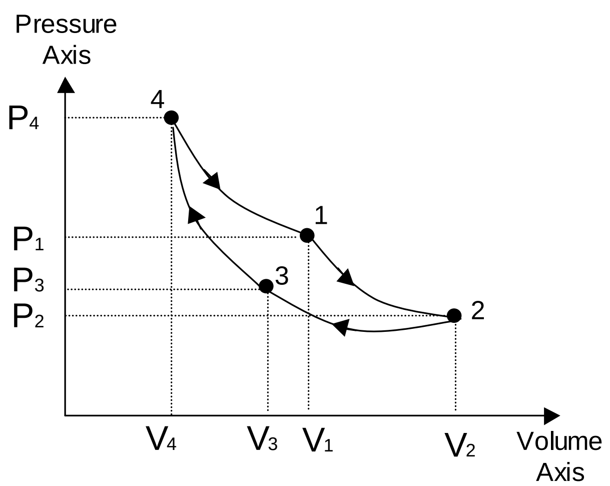
This is saying what the schematic diagram above is showing and leads to a simple reversible cycle comprising of two temperature reservoirs and a transfer processes linking them. From this came the concept that no heat engine can be more efficient than a reversible engine operating between the same temperatures.
This efficiency is only dependent on the temperatures of the reservoirs and is called the Carnot efficiency, it gives the maximum efficiency between any two temperature limits and is thus used for the comparison of other cycles.

The Carnot cycle comprises of four processes

Heat addition at constant temperature (Isothermal Expansion)
Reversible Adiabatic (Isentropic) Expansion
Heat rejection at constant temperature (Isothermal Compression)
Reversible Adiabatic (Isentropic) Compression

These processes for a perfect gas are shown on the following Temperature Specific-entropy and Pressure Volume diagrams.





For a heat engine the cycle efficiency is given by $\eta_{th} = \frac{\text{Net work output}}{\text{Gross heat supply}}$

From the second law the net work output is the difference between the heat supply and heat rejected.

$$\eta_{th} = \frac{\text{Heat supply} - \text{Heat rejected}}{\text{Heat supply}}$$

$$\eta_{th} = 1 - \frac{\text{Heat rejected}}{\text{Heat supply}}$$

All we need to do then is determine the heat supply and heat rejected for the cycle. From earlier work on entropy we know that the area of a T-s diagram is heat.

If we look at the T-s diagram for the Carnot cycle we can see that it is a rectangle.

The heat supply is rectangle 4, 1, $S_{1/2}$ $S_{3/4}$, 4.

The area of this rectangle is therefore the temperature T_{high} multiplied by the change in entropy.

The heat rejected is rectangle 2, 3, $S_{3/4}$, $S_{1/2}$, 2.

The area of this rectangle is the temperature T_{low} multiplied by the change in entropy.

We can substitute these values in the efficiency equation

$$\eta_{th} = 1 - \frac{T_{low} \times \text{change in entropy}}{T_{high} \times \text{change in entropy}}$$

The change in entropy is the same in both cases therefore the efficiency is

$$\eta_{th} = 1 - \frac{T_{low}}{T_{high}}$$

We can also note that the efficiency is independent of the working substance used and depends only on the temperature ratio, it increases as T_{high} increases.

Calculate The Carnot efficiency for cycles operating between the following limits 1500°C and 20°C and 500°C and 20°C.

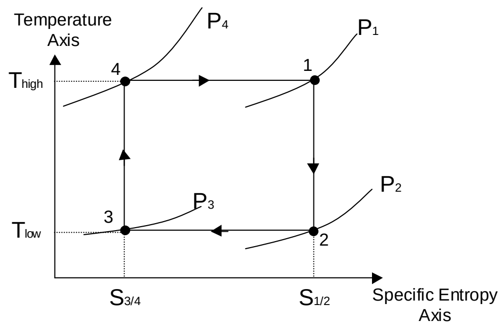
You should have 83.47 % and 62.09 % showing that the efficiency increases with the temperature of the hot reservoir.

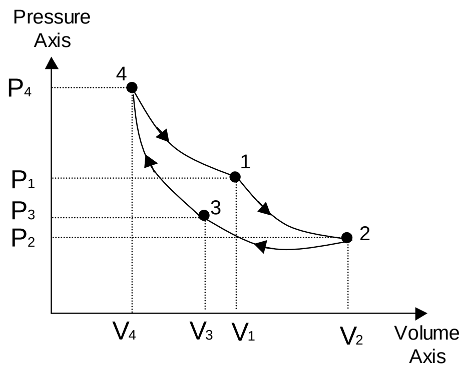
If you did not get these values then have you remembered to use the temperatures in Kelvin.
Remember for all equations use absolute values of temperature and pressure.

Example1

The maximum pressure and temperature in a Carnot cycle are 15 bar 300°C respectively, the minimum temperature is 40°C.
The isothermal compression ratio is 3:1 and the mass of the working fluid in the cycle is 0.3 kg.
For the working fluid c_p = 1005 kJ/kgK and c_v = 718 kJ/kgK
Calculate

a) The heat supply
b) The work done
c) The cycle efficiency
d) The lowest pressure and volume in the cycle.





The diagrams associated with this cycle are shown above.
Since the area of the T-s diagram is heat and the net work is the difference between heat supply and heat rejected all we have to do is calculate the area of this diagram.

We have the temperatures therefore all we need to calculate is the change in entropy which is that due to the isothermal process.

Since we have been given the volume ratio for the isothermal process the entropy change is obtained from

$$S_1 - S_4 = mRLn\frac{V_1}{V_4}$$

R= c_p-c_v =1005 – 718 = 287 J/kgK

$$S_1 - S_4 = 0.3 \times 287 \times Ln\frac{3}{1} = 94.59\frac{J}{K}$$

The entropy change is 94.59 J/K

The heat supply is the area of the rectangle defined by T_{high} x the entropy change.
The heat rejected is the area of the rectangle defined by T_{low} x the entropy change.

Heat supply = (300+273) x 94.59 = 54.2 kJ

Heat rejected = (40+273) x 94.59 = 29.6 kJ

Net work transfer = heat supply – heat rejected

Net work transfer = 54.2 – 29.6 = 24.6 kJ

Cycle efficiency is net work divided by the heat supply

$$\eta_{th} = \frac{Net\ Work}{Heat\ Supply} = \frac{24.6}{54.2} = 0.454$$

The cycle efficiency can also be calculated using the temperatures, the result will be the same

$$\eta_{th} = 1 - \frac{\mathrm{T_{low}}}{\mathrm{T_{high}}} \qquad \eta_{th} = 1 - \frac{313}{573} = 1 - 0.546 = 0.454$$

The thermal efficiency of the Carnot cycle operating at these temperature limits is 45.4%.

The pV diagram show that the lowest volume is at point 4 and the lowest pressure at point 2

Using $pV = mRT$ At point 4 $V_4 = \frac{mRT}{p} = \frac{0.3 \times 287 \times 573}{20 \times 10^5} = 0.0246 m^3$

The lowest volume is therefore 0.0246 m^3

To obtain the lowest pressure we will have to find the values of pV and T at point 1.

We know that T_4=T_1 and that V_1= 3 x V_4= 0.738 m^3 therefore

$$\frac{p_4 V_4}{T_4} = \frac{p_1 V_1}{T_1} \qquad p_1 = \frac{p_4 V_4}{V_1} = 20 \times \frac{0.0246}{0.738} = 6.667 bar$$

The process 1 to 2 is isentropic $pV^\gamma = C$ so we can use the isentropic relationship
Remember γ is obtained from c_p and c_v

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{Rearranging this gives} \quad \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \times p_1 = p_2$$

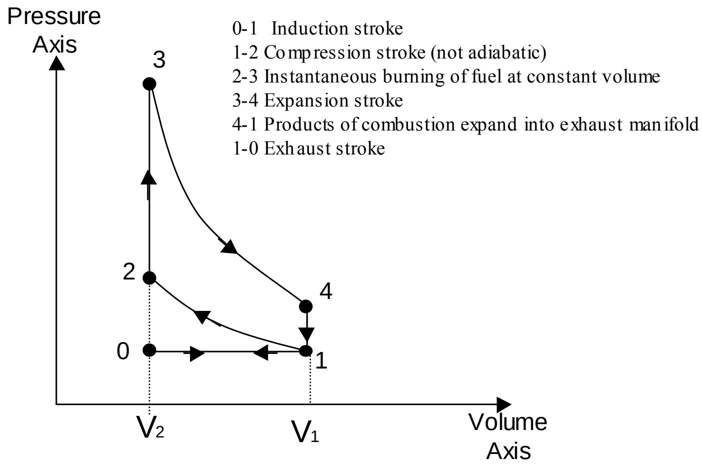
$$\left(\frac{313}{573}\right)^{\frac{1.399}{1.399-1}} \times 6.667 \times 10^5 = 0.8 bar$$

The lowest pressure is therefore 0.8 bar

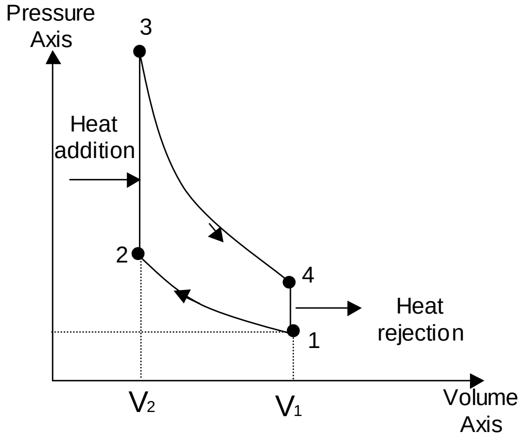
The Carnot cycle is not particularly suited to practical heat engine devices due to its isothermal processes and compression of two phase fluids, however other cycles have been developed which generally fit mechanical devices.
They all consist of non flow process forming a cycle and can be imagined to take place in a cylinder fitted with a reciprocating piston.

Reciprocating Internal Combustion Engines--Ideal Cycles

There are two types of reciprocating IC engines, spark ignition and compression ignition, each can operate on the two or four stroke cycle.
These engines are non cyclic, open-circuit, steady-flow work-producing devices and do not work on a thermodynamic cycle.
However for academic purposes, engineers compare their performance with the thermal efficiency of a corresponding air standard cycle.
This arises from the similarity in appearance of the idealised indicator diagram, (pressure against cylinder volume) obtained from the actual engine and the state diagram (pressure against specific volume) of the hypothetical corresponding cycle.
The shape of an indicator card varies with load, however as only the full load design condition is being considered then the idealised indicator card will be at maximum conditions.
The idealised diagram for a spark ignition and high speed compression ignition four stroke engine is shown below.



To simplify calculations relating to the performance of engines designed for different compression ratios the indicator diagram is compared to the corresponding state diagram giving the corresponding ideal air standard efficiency using the same volumetric compression ratio as the actual engine.
In this diagram, shown below, unit mass of air enclosed in a cylinder goes through a closed reversible thermodynamic cycle.
Unlike the corresponding processes in the engine, compression 1-2 and expansion 3-4 are made to be adiabatic as well as reversible and are called Isentropic processes.
The actual engine processes of sudden pressure rise after ignition and sudden drop after exhaust valve opening are replaced by constant volume heat addition and heat rejection processes.



The Otto Cycle or Constant Volume Cycle

The four non flow process comprising this cycle are;.

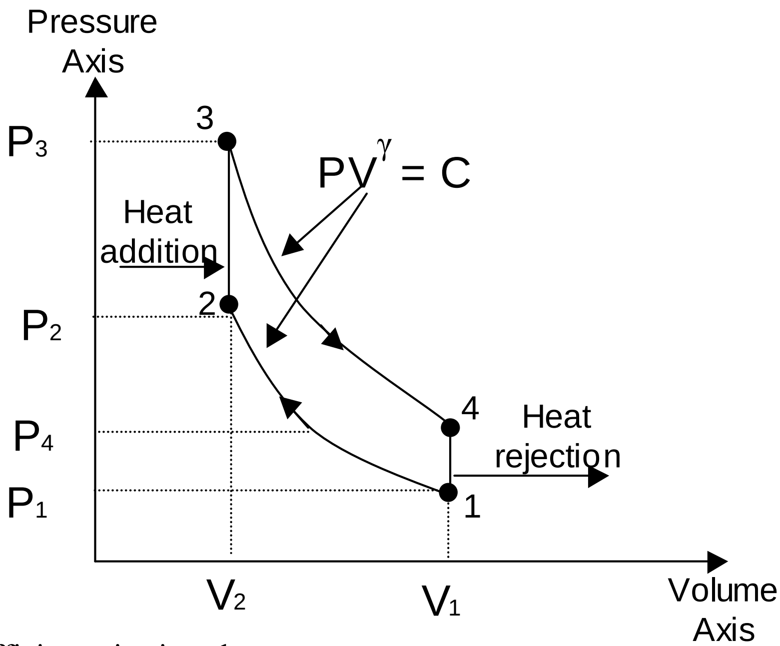
1-2 Isentropic compression through a volume ratio v_1/v_2 known as the compression ratio r_v.

2-3 Heat addition at constant volume until the fluid is at state 3

3-4 Isentropic expansion to the original volume

4-1 Heat rejection at constant volume until the cycle is complete

These are shown on the PV diagram below V_1 at bottom dead centre and V_2 is at top dead centre.



The thermal efficiency is given by

$$\eta_{th} = 1 - \frac{\text{Heat rejected}}{\text{Heat supply}}$$

$$\eta_{th} = 1 - \frac{\mathrm{mc_v}(T_4 - T_1)}{\mathrm{mc_v}(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Using the relationships

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \qquad r_v = \left(\frac{V_1}{V_2}\right)$$

Gives the following values of T_2 and T_3 which can be substituted in the efficiency equation

$$T_2 = T_1(r_v)^{\gamma-1} \qquad\qquad T_3 = T_4(r_v)^{\gamma-1}$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_4 - T_1)r_v^{\gamma-1}}$$

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}$$

It is useful to note that this efficiency is only dependant on the compression ratio and for an actual engine this only depends upon the physical dimensions.
Thus to compare an actual engine efficiency to that of the Otto cycle all we need is the compression ratio.
We can also determine that as the compression ratio increases the efficiency increases thus engines of higher compression ratio will have higher overall efficiencies and lower specific fuel consumptions.
For spark ignition engines compression ratio is limited to around 6 -9 to prevent detonation of the burning charge.
For compression ignition engines the compression ratio must be high to achieve high temperatures for igniting the injected fuel, thus values of around 15 are common.
An upper limit to the allowable compression ratio is set by the mechanical strength of the engine which imposes a limit on the allowable peak pressure.

The Diesel cycle

In compression ignition engines, a combination of high compression ratio and constant volume combustion leads to excessively high peak pressures. These can be reduced by delayed timing and suitably controlled fuel injection, the combustion now occurs more at constant pressure as the piston moves down the cylinder particularly in large engines.
Since the combustion is no longer at constant volume the Otto cycle is no longer suitable for comparison so a corresponding air standard diesel cycle was devised. Ackroyd Stuart was the first to build a successful engine operating on this cycle. Diesel's efforts were originally directed towards building an engine which would have the Carnot cycle as its equivalent cycle.

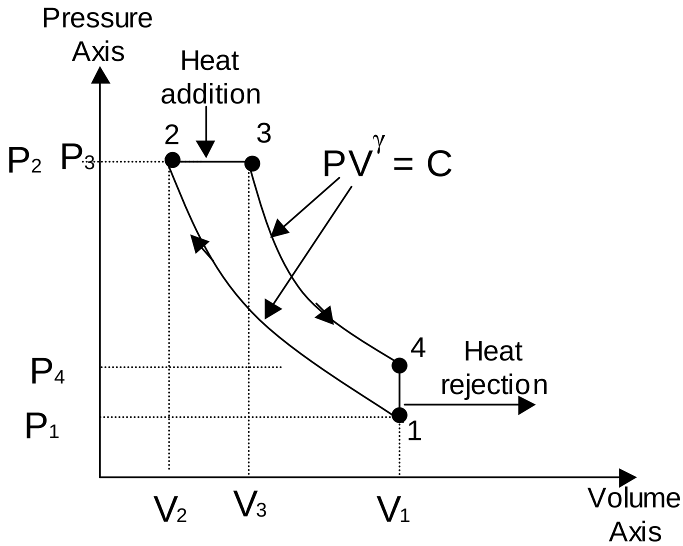
The four non flow process comprising this cycle are;.

1-2 Isentropic compression through the ratio $r_v = v_1/v_2$

2-3 Heat addition at constant pressure to volume v_3.
At state 3 the heat supply is cut off and the volume ratio v_3/v_2 is called the cut off ratio β

3-4 Isentropic expansion to the original volume

4-1 Heat rejection at constant volume until the cycle is complete



The thermal efficiency is given by

$$\eta_{th} = 1 - \frac{\text{Heat rejected}}{\text{Heat supply}}$$

$$\eta_{th} = 1 - \frac{\text{mc}_\text{v}(T_4 - T_1)}{\text{mc}_\text{p}(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)}$$

For the isentropic process 1-2 $$T_2 = T_1 (r_v)^{\gamma-1}$$

For the constant pressure process $\dfrac{T_3}{T_2} = \dfrac{v_3}{v_2} = \beta$ $$T_3 = T_1 \beta (r_v)^{\gamma-1}$$

For the isentropic process 3-4 $\dfrac{T_4}{T_3} = \left(\dfrac{v_3}{v_4}\right)^{\gamma-1} = \left(\dfrac{\beta}{r_v}\right)^{\gamma-1}$ $$T_4 = \beta^{\gamma} T_1$$

By using the above relationships for temperature, the thermal efficiency becomes

$$\eta_{th} = 1 - \frac{(\beta^{\gamma} - 1)}{\gamma\, r_v^{\gamma-1} (\beta - 1)}$$

Thus the efficiency of the diesel engine depends upon the quantity of heat added represented by the cut off ratio β as well as the compression ratio r_v.

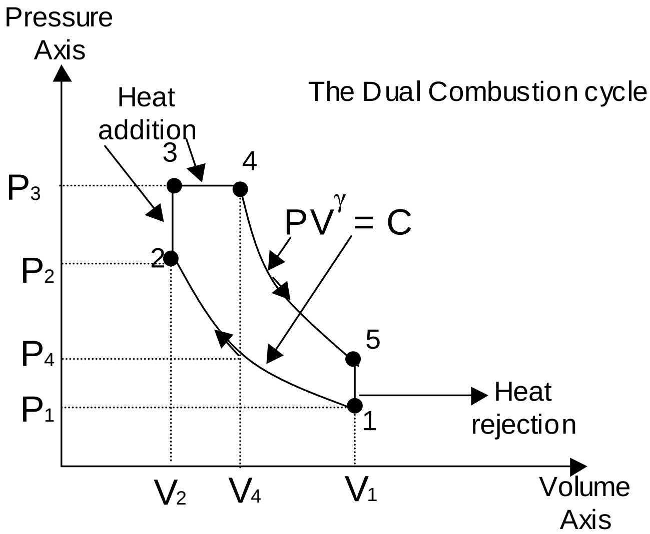
Since the term $\dfrac{(\beta^{\gamma} - 1)}{\gamma(\beta - 1)}$ is always greater than one, the diesel cycle always has

a lower efficiency than the Otto cycle for the same compression ratio.
The diesel cycle is usually taken as the academic basis of a large slow speed marine compression ignition engine, from the above it may be assumed that its performance would be poorer than that of a smaller high speed engine with the same compression ratio since the smaller engine will have less restriction on the peak pressure and thus operate more closely to the Otto cycle.
However practical engines based on a diesel cycle can have higher compression ratios than engines based on an Otto cycle thus the effects of the reducing term can be moderated.

The Dual Cycle

In operation at full load the combustion in spark ignition and high speed diesel engines is followed by a rapid pressure rise at constant volume, while in large slow speed diesels the combustion is followed by an initial expansion at constant pressure. There is therefore the possibility for an engine to operate between these two extremes which gives rise to the dual combustion cycle shown below.
In this cycle part of the heat addition takes place at constant volume and part at constant pressure.

1-2 Isentropic compression through a volume ratio v_1/v_2
2-3 Heat addition at constant volume until state 3
3-4 Heat addition at constant pressure until volume 4
4-5 Isentropic expansion to the original volume
5-1 Heat rejection at constant volume until the cycle is complete



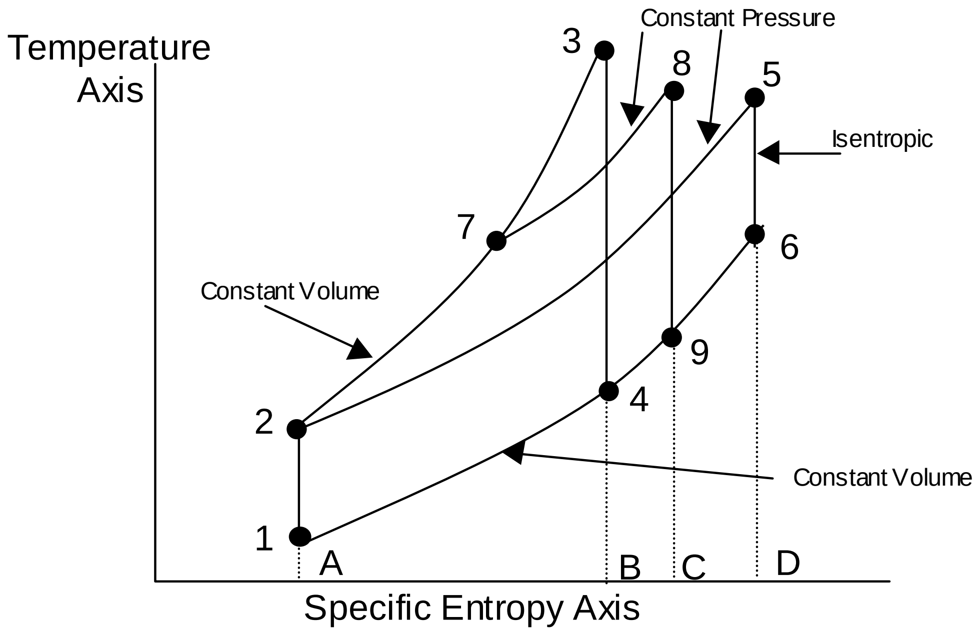
The efficiency of this cycle is also given by $\eta_{th} = 1 - \dfrac{\text{Heat rejected}}{\text{Heat supply}}$

This can be resolved in the same way as the other cycles to give

$$\eta_{th} = 1 - \frac{K\left(\beta^{\gamma} - 1\right)}{r_v^{\gamma-1}\left[(K-1) + \gamma K(\beta - 1)\right]}$$

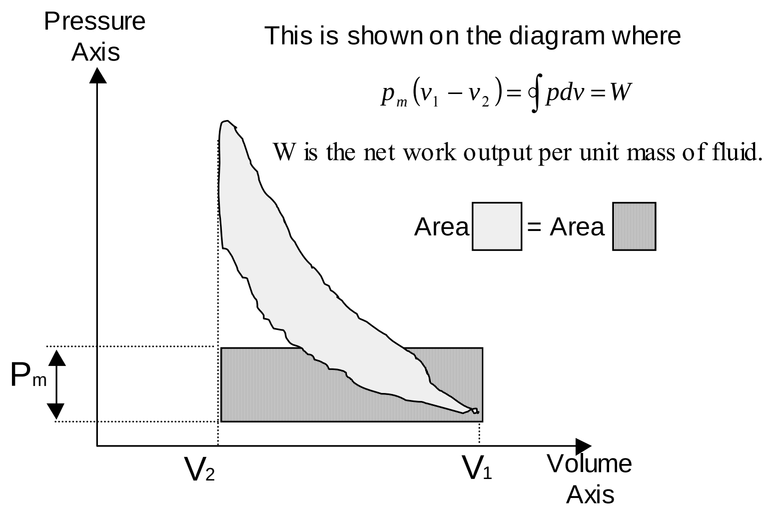
Where $r_v = V_1 / V_2$, $K = P_3 / P_2$, and $\beta = V_4 / V_3$

The diagram below shows the three cycles on a T~ s diagram.
1-2-3-4-1 Otto cycle, 1-2-5-6-1 Diesel cycle, 1-2-7-8-9-1 Dual cycle
They have been drawn for the case where both the compression ratios and heat inputs are the same for each.
The quantity of heat rejected can be seen from the areas below the line of constant volume heat rejection to be least in the Otto cycle (A14A) and greatest in the Diesel cycle A16DA, the Dual cycle (A19CA) being some where between the two extremes. Hence the cycle efficiencies decrease in the order Otto, Dual, Diesel.



Mean effective pressure as a criterion of performance

The ideal cycle efficiency is not the only criterion by which a cycle should be judged. The work ratio which is given by the net work output divided by the gross work output is a useful criterion for steady-flow cycles, since it indicates the extent to which the cycle suffers from irreversibility and also indicates the size of the plant per unit power output.
In a reciprocating engine, it is not so easy to isolate the positive and negative work in the cycle.
For this reason another criterion, called the *mean effective pressure,* is usually preferred to the work ratio when comparing air-standard cycles.
The mean effective pressure p_m is defined as the height of a rectangle on the p-v diagram having the same length and area as the cycle.



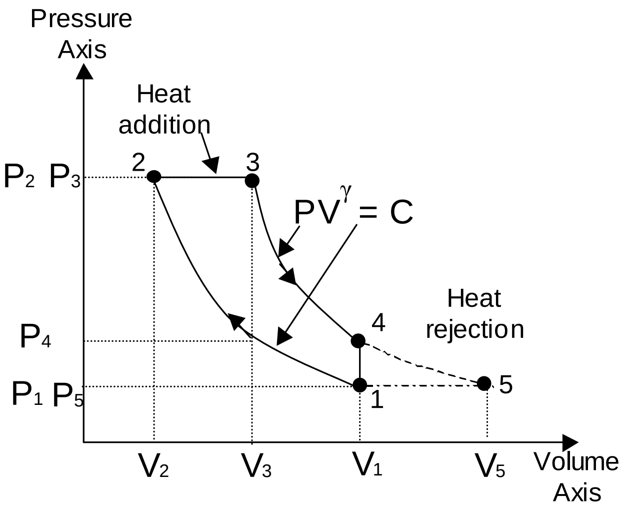
p_m can be regarded as that constant pressure which, by acting on the piston over one stroke, can produce the net work of the cycle.

The mean effective pressure, unlike the work ratio, is not dimensionless, and is usually expressed in bar.

The mean effective pressure is a useful criterion for the comparison of reciprocating engine cycles in that it indicates relative engine size, and also how far the actual engine efficiency will depart from the cycle efficiency.
A cycle with a large mean effective pressure will produce a large work output per unit swept volume, and hence an engine based on this will be small for a given work output.
Irreversibility in reciprocating engine cycles is generally due to mechanical friction rather than fluid friction.
Mechanical friction is related to engine size and to some extent on the peak and average absolute pressures during the cycle, higher pressures tend to increase bearing losses, particularly with poor lubrication.
Therefore it has been suggested that a better indication of mechanical friction loss can be given by the ratio of mean effective pressure to peak pressure.
This dimensionless criterion is analogous to the work ratio.

Since mechanical friction varies with engine size, a large mean effective pressure giving a smaller engine implies that a smaller fraction of the cycle net work will be used in overcoming friction.

If we modified the diesel cycle so that heat rejection took place at constant pressure which can be achieved by increasing the stroke of the engine as shown below.
The cycle efficiency is improved, because the heat rejection occurs at a lower average temperature T_5 being lower than T_4.
Increasing the stroke would increase the area of the diagram, however it also reduces the mean effective pressure.



The actual engine based upon this cycle would therefore not necessarily have a higher efficiency than one based upon the Diesel cycle, it would also be larger.

This modified cycle is in fact the Joule cycle, which has found acceptance only as a basis for gas turbine plant.

Thermal Efficiency Ratio or Relative Efficiency

This is the relationship between the indicated thermal efficiency of an actual engine under test and the thermal efficiency of the corresponding Ideal cycle.
It indicates how closely the performance of the real engine has approached that of its ideal counterpart.
If the fluid in the actual and ideal cycle are different then the efficiency ratio is termed the Relative efficiency.

$$\text{Relative Efficiency} = \frac{\text{Actual Thermal Efficiency}}{\text{Air standard Efficiency}}$$

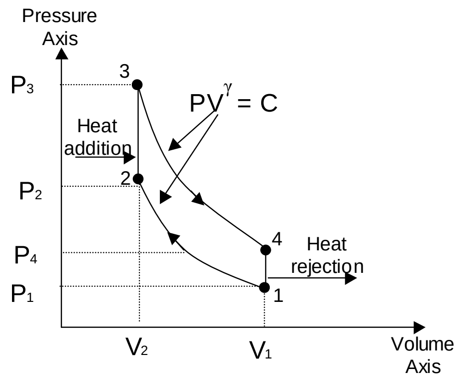
Example 2

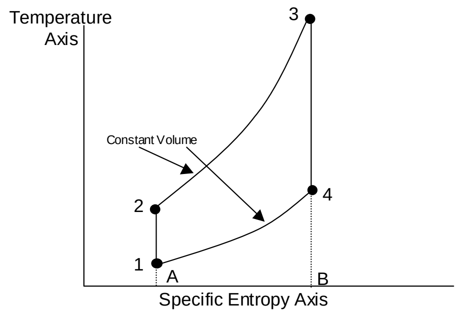
In an ideal Otto cycle conditions at the start of compression are 1.05 bar and 30°C.

The volume compression ratio is 7:1 and maximum pressure is 53 bar.
For air $\gamma = 1.4$, R = 287 J/kgK.

Calculate per kg of working fluid

a) The heat received
b) The heat rejected
c) The cycle efficiency
d) The ratio of maximum to mean pressure





	P(bar)	V	T(K)
1	1.05	$7V_2$	303
2	16	$1V_2$	659.9
3	53 b	$1V_2$	2185.9
4		$7V_2$	1003.7

The table and diagrams, help us to identify the information we have and what we need to find.
In most of these problems it is necessary to calculate the remaining values at all the other points.

Starting at point one we can use the standard relationships to methodically work around the cycle to obtain the remaining values.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad P_1V_1^{\gamma} = P_1V_1^{\gamma} \qquad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 303\left(\frac{7V_2}{1V_2}\right)^{1.4-1} = 659.9K$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1.05\left(\frac{7V_2}{1V_2}\right)^{1.4} = 16bar$$

$$T_3 = T_2\frac{P_3V_3}{P_2V_2} = 659.9\left(\frac{53}{16}\right) = 2185.9K$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \qquad T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{\gamma-1}} = \frac{2185.9}{(7)^{0.4}} = 1003.67K$$

We can now complete the table, however we do not need the pressure at point 4 since heat supplied, rejected and the work transfer can be determined by using the temperatures.

$$\text{Heat supply} = mc_v(T_3 - T_2)$$

$$\text{Heat rejected} = mc_v(T_4 - T_1)$$

For this we need c_v $\qquad \gamma = \frac{c_p}{c_v} \qquad R = c_p - c_v$

If we combine these two equations we get $\quad c_v = \frac{R}{\gamma - 1} = \frac{287}{1.4 - 1} = 717.5J/kgK$

$$\text{Heat supply} = mc_v(T_3 - T_2)$$

$$\text{Heat supply} = 1 \times 717.5(2185.9 - 659.9)$$
$$\text{Heat supply} = 1094.9kJ$$

$$\text{Heat rejected} = mc_v(T_4 - T_1)$$

$$\text{Heat rejected} = 1 \times 717.5(1003.69 - 303)$$

$$\text{Heat rejected} = 502.74kJ$$

From the second law

$$\text{Thermal efficiency} = \frac{\text{Net Work}}{\text{Heat Supply}} = \frac{\text{Heat Supply - Heat Rejected}}{\text{Heat Supply}}$$

$$\text{Thermal efficiency} = 1 - \left(\frac{\text{Heat Rejected}}{\text{Heat Supply}}\right)$$

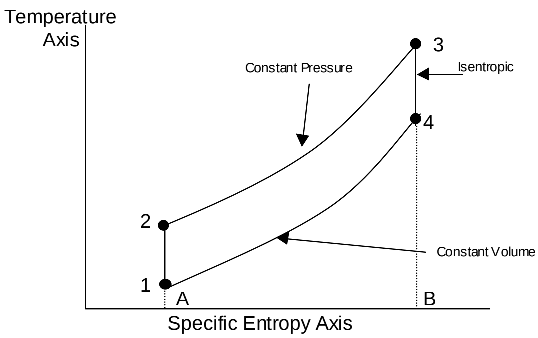
$$\text{Thermal efficiency} = 1 - \left(\frac{502.745}{1094.9}\right)$$

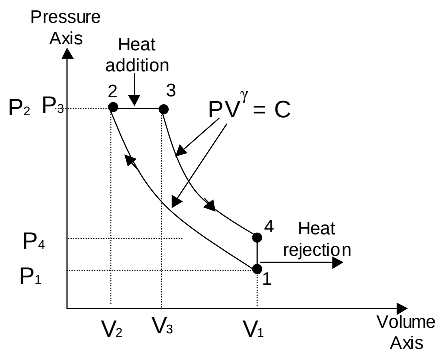
$$\text{Thermal efficiency} = 54\%$$

The alternative is to use the equation previously developed

$$\text{Thermal efficiency} = 1 - \left(\frac{1}{r_v^{\gamma-1}}\right) = 1 - \left(\frac{1}{7^{1.4-1}}\right) = 54\%$$

Example 3
In an ideal diesel cycle the volume compression ratio is 14:1 and fuel is admitted for 10% of the stroke.
The conditions at the beginning of compression are 35°C and 1.03 bar.
Calculate the temperatures and pressures at the cardinal points of the cycle and the ideal thermal efficiency of the cycle.
For air $\gamma = 1.4$, $R = 287$ J/kgK





	P(bar)	V	T(K)
1	1.03	$14V_2$	308
2	41.44	$1V_2$	885.2
3	41.44	$2.3V_2$	2035.7
4	3.3	$14V_2$	988.48

This is all the information given but we do know that $V_3 - V_2$ is 10% of the stroke.

Therefore the stoke is $V_1 - V_2 = 14V_2 - 1V_2 = 13V_2$

$$V_3 - V_2 = 0.1 \text{ x } 13V_2 = 1.3V_2$$

$$V_3 = 1V_2 + 1.3V_2 = 2.3V_2$$

Using relationships $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $\quad P_1V_1^{\gamma} = P_1V_1^{\gamma}$ $\quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 308\left(\frac{14V_2}{1V_2}\right)^{1.4-1} = 885.2K$$

$$T_3 = T_2\,\frac{P_3V_3}{P_2V_2} = 885.2\left(\frac{2.33V_2}{1V_2}\right) = 2035.77K$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \qquad T_4 = \frac{T_3}{\left(\frac{V_4}{V_3}\right)^{\gamma-1}} = \frac{2035.77}{\left(\frac{14V_2}{2.3V_2}\right)^{0.4}} = 988.48K$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1.03\left(\frac{14V_2}{1V_2}\right)^{1.4} = 41.44bar$$

$$P_2 = P_3 = 41.44bar$$

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = 41.44\left(\frac{2.3V_2}{14V_2}\right)^{1.4} = 3.3bar$$

We can now complete the table, which is a reasonable way to collate and present the data we are now going to use.

For this we need c_v and c_p $\gamma = \frac{c_p}{c_v}$ $R = c_p - c_v$

If we combine these two equations we get $c_v = \frac{R}{\gamma - 1} = \frac{287}{1.4 - 1} = 717.5 \frac{J}{kgK}$

$$c_p = \gamma\ c_p = 1.4 \times 717.5 = 1004.5 \frac{J}{kgK}$$

We do not have the mass of air for the cycle but this does not matter because we are calculating the efficiency and the unit will cancel out.

$$\text{Heat supply} = mc_p \left(T_3 - T_2\right)$$

$$\text{Heat supply} = m \times 1004.5(2035.77 - 885.2)$$

$$\text{Heat supply} = 1155.74 \frac{kJ}{kg}$$

$$\text{Heat rejected} = mc_v \left(T_4 - T_1\right)$$

$$\text{Heat rejected} = 1 \times 717.5(988.48 - 308)$$

$$\text{Heat rejected} = 488.24 \frac{kJ}{kg}$$

$$\text{Thermal efficiency} = 1 - \left(\frac{\text{Heat Rejected}}{\text{Heat Supply}}\right)$$

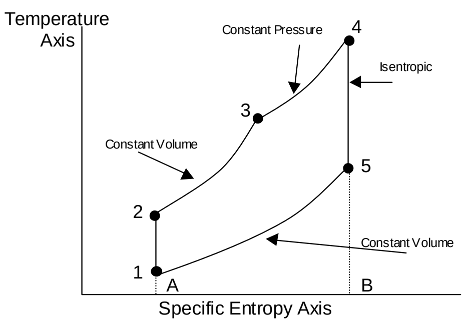
$$\text{Thermal efficiency} = 1 - \left(\frac{488.24}{1155.74}\right)$$

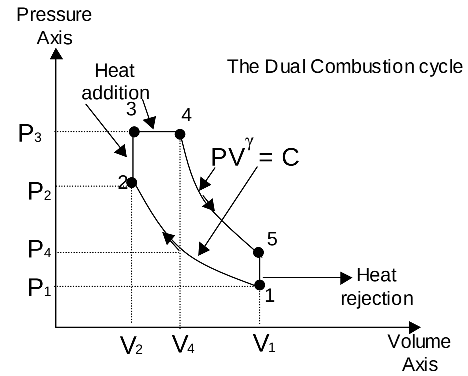
$$\text{Thermal efficiency} = 57.75\%$$

We could have used the equation developed earlier however it is difficult to remember and most questions generally ask us to calculate the temperatures and pressures any way.
You may want to calculate the efficiency using the equation.

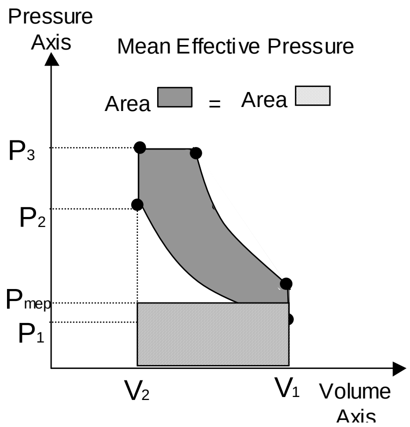
Example 4
In an ideal dual combustion cycle the heat transfer during combustion is equally shared between the constant pressure and constant volume parts of the cycle.
The volume compression ratio is 13:1 and the pressure and temperature at the begging of compression are 1.03 bar and 40°C respectively.
The maximum temperature reached during the cycle is 1950°C.
For air $\gamma = 1.4$
Calculate

The ideal cycle Mean effective pressure.





	P(bar)	V	T(K)
1	1.03	$13V_2$	313
2	37.35	$1V_2$	873.2
3	71	$1V_2$	1660.6
4		$1.34V_2$	2223
5	2.94	$13V_2$	



$$\text{Mean effective pressure} = \frac{\text{Net Area of Diagram}}{\text{Length of diagram}}$$

$$\text{Mean effective pressure} = \frac{\text{Net Work}}{\text{swept Volume}}$$

To obtain this we need to obtain the Temperatures, volumes and pressures at the cardinal points.

No units of volume have been given but this does not matter since the net work can be obtained in pressure volume units and the swept volume will be in volume units thus the mean effective pressure will be in units of pressure.

To find T_2 $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 313\left(\frac{13V_2}{1V_2}\right)^{1.4-1} = 873.2K$$

We can find T_3 from

Heat supply at constant volume = Heat supply at constant pressure

$$mc_v(T_3 - T_2) = mc_p(T_4 - T_3)$$

$$(T_3 - T_2) = \frac{mc_p}{mc_v}(T_4 - T_3)$$

$$(T_3 - T_2) = \gamma(T_4 - T_3)$$

$$T_3 + \gamma T_3 = \gamma T_4 + T_2$$

$$T_3 + 1.4T_3 = 1.4 \times 2223 + 873.2$$

$$2.4T_3 = 3985.4$$

$$T_3 = 1660.6K$$

$$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4} \qquad V_4 = V_3\frac{T_4}{T_3} = V_3\left(\frac{2223}{1660.6}\right) = 1.338V_3$$

$$V_3 = V_2 \quad \text{Therefore} \quad V_4 = 1.338V_2$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1.03\left(\frac{13V_2}{1V_2}\right)^{1.4} = 37.355bar$$

$$\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} \qquad P_3 = P_2\frac{T_3}{T_2} = 37.355\left(\frac{1660,6}{873.2}\right) = 71bar$$

$$P_5 = P_4\left(\frac{V_4}{V_5}\right)^{\gamma} = 71\left(\frac{1.3387V_2}{13V_2}\right)^{1.4} = 2.945bar$$

We now have all the information and you could complete the table if you wish.
We can now calculate the work transfer.

Expansion work is that due to constant pressure 3 to 4 plus that due to adiabatic expansion 4 to 5

$$\text{Expansion work} = \mathrm{P}_4(V_4 - V_3) + \frac{P_4V_4 - P_5V_5}{\gamma - 1}$$

$$\text{Expansion work} = 71(1.338V_2 - 1V_2) + \frac{71 \times 1.338V_2 - 2.94 \times 13V_2}{1.4 - 1}$$

$$\text{Expansion work} = 24\mathrm{V}_2 + 142V_2$$

$$\text{Expansion work} = 166V_2 bar$$

$$\text{Compression Work} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$\text{Compression Work} = \frac{1.03 \times 13V_2 - 37.355 \times 1V_2}{1.4 - 1}$$

$$\text{Compression work} = \text{-}59.91V_2 bar$$

Net Work = Expansion work + Compressive work

$$\text{Net work} = \text{Expansion work} + \text{compression Work}$$

$$\text{Net work} = 166V_2 \text{ - } 59.91V_2 bar$$

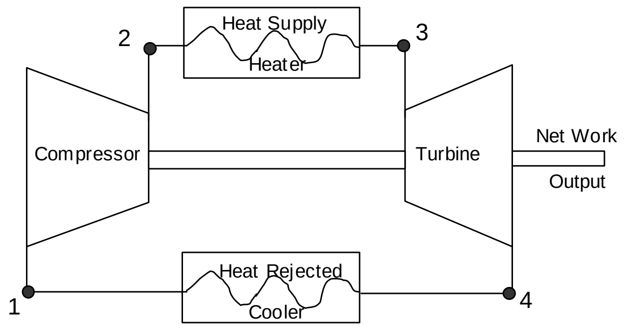
$$\text{Swept volume} = 13V_2 \text{ - } 1V_2 = 12V_2$$

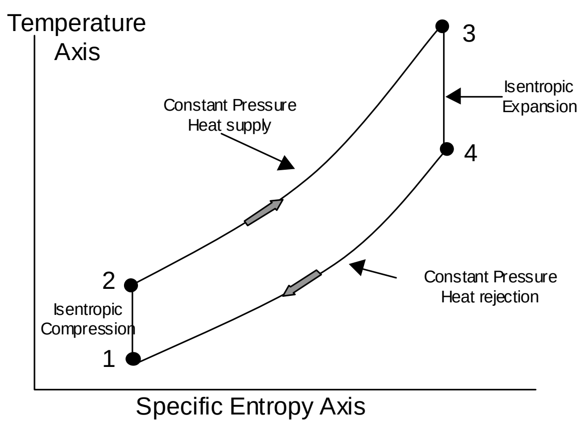
$$\text{mep} = \frac{166V_2 - 59.91V_2}{12V_2} = 8.84 bar$$

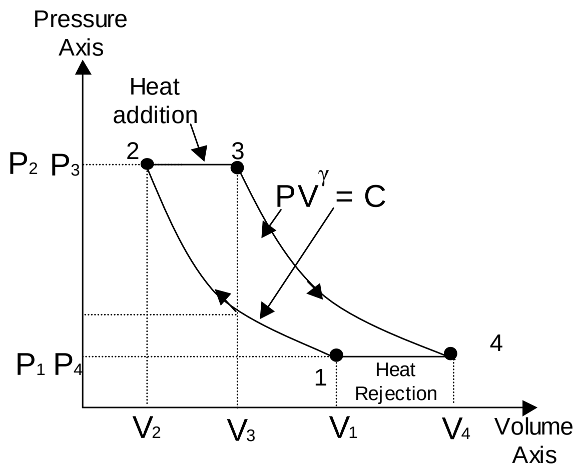
Gas Turbine Cycles

The Constant Pressure Cycle

In this cycle the heat supply and heat rejection processes occur reversibly at constant pressure.
The expansion and compression processes are isentropic,
This cycle was at one time used as the ideal basis for a hot-air reciprocating engine, and the cycle was known as the Joule or Brayton cycle.
However the cycle is now the ideal for the closed cycle gas turbine unit as shown below.
The cycle is also shown on T-s and p-v diagrams.







The working substance is air which moves in steady flow round the cycle.
Neglecting velocity changes, and applying the steady-flow energy equation to each part of the cycle

$$\text{Work input to the compressor } = \mathrm{H_2 - H_1 = mc_p}(T_2 - T_1)$$

$$\text{Work output from the Turbine} = \mathrm{H_3 - H_4 = mc_p}(T_3 - T_4)$$

$$\text{Heat supplied in heater} = \mathrm{H_3 - H_2 = mc_p}(T_3 - T_2)$$

$$\text{Heat rejected in cooler} = \mathrm{H_4 - H_1 = mc_p}(T_4 - T_1)$$

$$\text{Thermal Efficiency } \eta = \frac{\text{Net Work}}{\text{Heat supply}}$$

$$\text{Thermal Efficiency } \eta = 1 - \frac{\text{Heat Rejected}}{\text{Heat supply}}$$

$$\text{Thermal Efficiency } \eta = 1 - \frac{\text{mc}_\text{p}(T_4 - T_1)}{\text{mc}_\text{p}(T_3 - T_2)}$$

Since processes 1 to 2 and 3 to 4 are isentropic between the same pressures p_2 and p_1

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = r_p^{\frac{\gamma-1}{\gamma}}$$

where r_p is the pressure ratio, P_2/P_1

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1 r_p^{\frac{\gamma-1}{\gamma}} \qquad T_3 = T_4\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = T_4 r_p^{\frac{\gamma-1}{\gamma}}$$

Hence, substituting for T_2 and T_3 in the expression for the efficiency in the same way as we did for the Otto cycle give an value based on pressure ratio.

$$\text{Thermal Efficiency } \eta = 1 - \frac{1}{\text{r}_\text{p}^{\frac{\gamma-1}{\gamma}}}$$

Thus for the constant pressure cycle the cycle efficiency depends only on the pressure ratio.

In the ideal case the value of γ for air is constant.

In practice, due to the eddying of the air as it flows through the compressor and turbine, which are both rotary machines, the actual cycle efficiency is greatly reduced compared to that given above.

$$\text{Work Ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$\text{Work Ratio} = \frac{\mathrm{mc_p}(T_3 - T_4) - \mathrm{mc_p}(T_2 - T_1)}{\mathrm{mc_p}(T_3 - T_4)}$$

$$\text{Work Ratio} = 1 - \frac{\mathrm{mc_p}(T_2 - T_1)}{\mathrm{mc_p}(T_3 - T_4)}$$

If we substitute T_2 and T_3 in this expression in the same way as we did above, the expression becomes

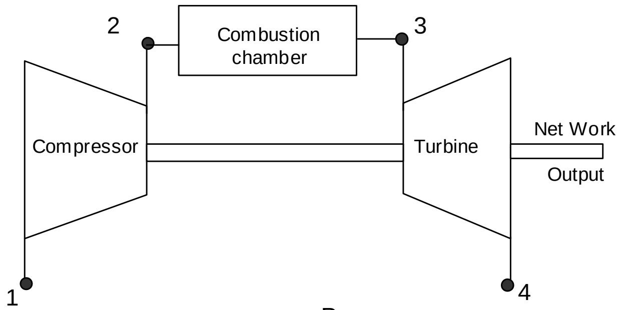
$$\text{Work Ratio} = 1 - \frac{T_1}{\mathrm{T_3}}\left(r_p^{\frac{\gamma-1}{\gamma}} \right)$$

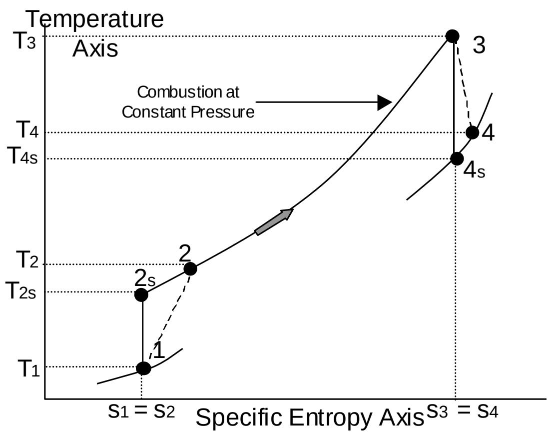
This equation shows that the work ratio depends not only on the pressure ratio but also on the ratio of the minimum and maximum temperatures.
For a given inlet temperature, T_l, the maximum temperature, T_3, must be made as high as possible for a high work ratio.
For an open-cycle gas turbine unit the actual cycle is not such a good approximation to the ideal constant pressure cycle, since fuel is burned with the air and a fresh charge is continuously induced into the compressor.

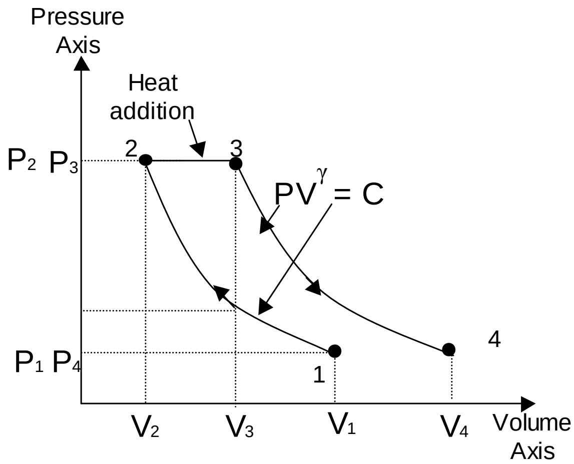
The ideal cycle, provides nevertheless a good basis for comparison, and in many calculations for an ideal open-cycle gas turbine the effects of the mass of fuel and the change in the working fluid are neglected.

Open Cycle Plant

This is the most basic gas turbine unit in which a rotary compressor and turbine unit, are mounted on a common shaft. Air flows through the compressor to the combustion chamber where it mixes with fuel and is burnt, the resulting gasses are then expanded in a turbine which must develop more power than that required to drive the compressor and overcome mechanical losses.
The compressor is either a centrifugal or axial flow type, the compression is therefore irreversible but approximately adiabatic as is the expansion process through the turbine.







A simplified approach may be taken if the combustion process is assumed to be the equivalent of a heat transfer at constant pressure to a working fluid with a constant mean specific heat.
This allows the actual process to be compared with the ideal and can be shown on a T~s diagram (above).
The pressure loss in the combustion chamber has also been ignored.

Line 1-2 represents irreversible adiabatic compression between P_1 and P_2
Line 2-3 represents constant pressure heat addition in the combustion chamber
Line 3-4 represents irreversible adiabatic expansion between P_2 and P_1
Line 1-2_s represents ideal isentropic compression between P_1 and P_2
Line 3-4_s represents ideal isentropic expansion between P_2 and P_1

Apply the SFEE to each part of the cycle and assume kinetic energy changes are small compared to the enthalpy changes.

$$\text{Work input to the compressor} = \mathrm{H_2 - H_1 = m_{air} c_{p_{air}}}(T_2 - T_1)$$

$$\text{Heat input in combustion chamber} = \mathrm{H_3 - H_2 = m_{air} c_{p_{air}}}(T_3 - T_2)$$

$$\text{Work output from the Turbine} = \mathrm{H_3 - H_4} = \mathrm{m_{gas} c_{p\,gas}} \left(T_3 - T_4\right)$$

$$\text{Thermal efficiency} = \frac{\text{Net workoutput}}{\text{Heat supplied}}$$

$$\text{Thermal efficiency} = \frac{\text{Turbine work - Compressor work}}{\text{Heat supplied}}$$

The compressor isentropic efficiency is defined as the ratio between the work input for isentropic compression between P_1 and P_2 to the actual work required. Neglecting kinetic energy

$$\text{Isentropic efficiency of the compressor} = \frac{\text{Ideal Work}}{\text{Actual Work}} = \frac{\mathrm{H_{2s}} - H_1}{H_2 - H_1}$$

$$\text{Isentropic efficiency of the compressor} = \frac{m_{air} \times c_{p(air)}\left(T_{2s} - T_1\right)}{m_{air} \times c_{p(air)}\left(T_2 - T_1\right)}$$

$$\text{Isentropic efficiency of the compressor} = \frac{\left(T_{2s} - T_1\right)}{\left(T_2 - T_1\right)}$$

The turbine isentropic efficiency is defined as the ratio of the actual work output to the isentropic work output between pressure P_2 and P_1. Neglecting kinetic energy

$$\text{Isentropic efficiency of the compressor} = \frac{\text{Actual Work}}{\text{Ideal Work}} = \frac{\mathrm{H_3} - H_4}{H_3 - H_{4s}}$$

$$\text{Isentropic efficiency of the Turbine} = \frac{\mathrm{m_{gas}} c_{p(gas)}\left(T_3 - T_4\right)}{\mathrm{m_{gas}} c_{p(gas)}\left(T_3 - T_{4s}\right)}$$

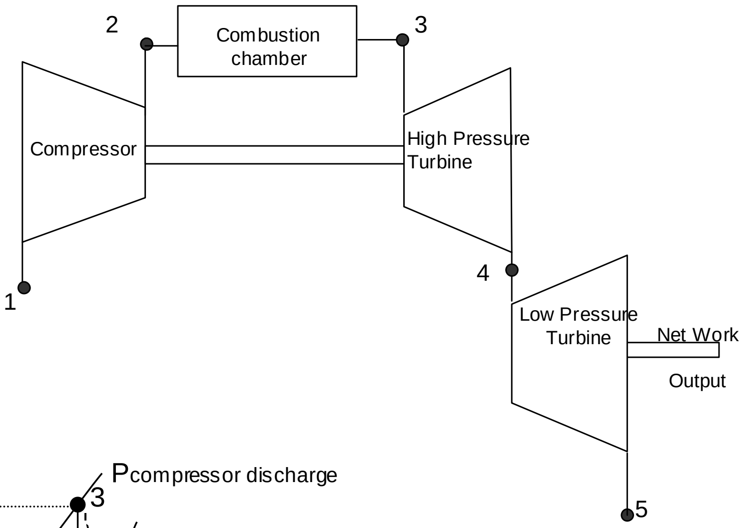
$$\text{Isentropic efficiency of the Turbine} = \frac{\left(T_3 - T_4\right)}{\left(T_3 - T_{4s}\right)}$$

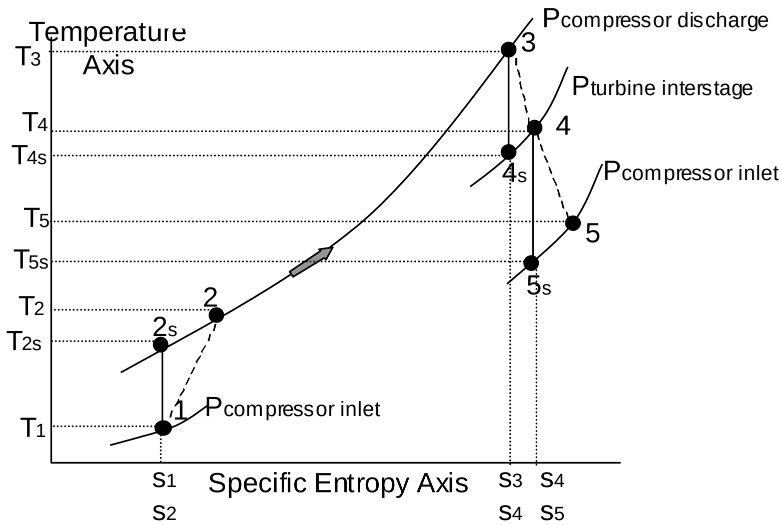
It is usual to assume a fixed mean value of Cp and γ for the compression process and another mean value of Cp and γ for the expansion process.
The mass flow through the turbine is greater than that through the compressor due to the addition of fuel, if air is bled from the compressor then this is usually equal to the mass of fuel burned, the mass flow is then constant.

Modifications to Basic Cycle

Power Turbine Systems

In this case a 1st stage or H.P. turbine drives the compressor and a 2nd stage or L.P. turbine provides the power output as shown.
If each turbine has its own isentropic efficiency then the cycle can be shown on a T~ s diagram as shown below.





Line 1-2 represents irreversible adiabatic compression between P_1 and P_2
Line 2-3 represents constant pressure heat addition in the combustion chamber
Line 3-4 represents irreversible adiabatic expansion between P_2 and the turbine inter-stage pressure P_i
Line 4-5 represents irreversible adiabatic expansion between the inter-stage pressure P_i and P_1

Line 1-2_s represents ideal isentropic compression between P_1 and P_2
Line 3-4_s represents ideal isentropic expansion between P_2 and P_i
Line 4-5_s represents ideal isentropic expansion between P_i and P_1

$$\text{Work input to the compressor} = \mathrm{H_2 - H_1} = \mathrm{m_{air}c_{p\,air}}\left(T_2 - T_1\right)$$

$$\text{Heat input in combustion chamber} = \mathrm{H_3 - H_2} = \mathrm{m_{air}c_{p\,air}}\left(T_3 - T_2\right)$$

$$\text{Work output from the High Pressure Turbine} = \mathrm{H_3 - H_4} = \mathrm{m_{gas}c_{p\,gas}}\left(T_3 - T_4\right)$$

$$\text{Work output from the Low Pressure Turbine} = \mathrm{H_4 - H_5} = \mathrm{m_{gas}c_{p\,gas}}\left(T_4 - T_5\right)$$

$$\text{Compressor Work} = \text{High Pressure Turbine Work}$$

$$\text{Thermal efficiency} = \frac{\textit{Net Work Output}}{\textit{Heat Supply}} = \frac{\text{Low Pressure Turbine work}}{\text{Heat supplied}}$$

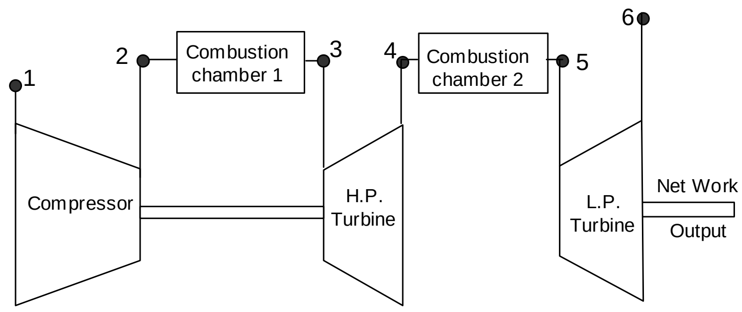
$$\text{Work Ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$\text{Work Ratio} = \frac{\text{Low pressure turbine work}}{\text{Work output from both turbines}}$$

$$\text{Work Ratio} = \frac{\mathrm{m_{gas}c_{p(gas)}}\left(T_4 - T_5\right)}{\mathrm{m_{gas}c_{p(gas)}}\left[\left(T_3 - T_4\right) + \left(T_4 - T_5\right)\right]}$$

Reheat

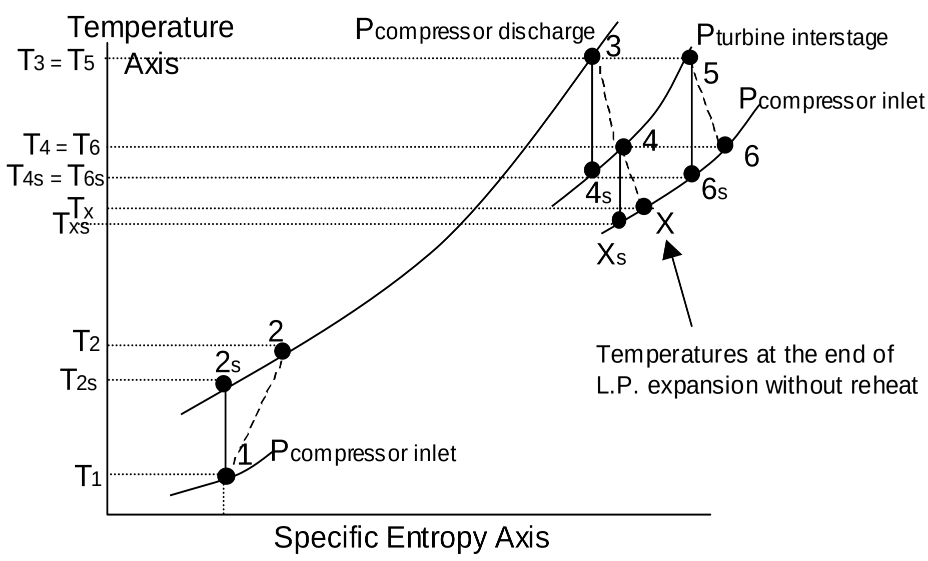
If the expansion process takes place in two separate turbines, then the output of the LP set can be increased by raising the stage inlet temperature. This can be done by placing a second combustion chamber between the turbine stages as shown below.



The thermodynamic cycle is shown below on the T~s diagram which has been drawn for the following conditions.

- The reheat temperature is the same as the temperature at inlet to the high pressure stage.
- The pressure ratio across the low pressure turbine is the same as the high pressure turbine therefore the isentropic temperature drop across both turbines is the same.
- The isentropic efficiency in both turbines is the same therefore the actual temperature at exit is the same.

The mass flow is relatively unchanged therefore the work output from both turbines is the same.
However should any of the above conditions change then the T-s diagram would need to be drawn for the actual conditions.



The HP turbine work must match the compressor work input i.e.
$c_{p(air)}(T_2 - T_1) = c_{p(gas)}(T_2 - T_1)$

The net work output is the LP turbine work given by

$$\text{Net work output } = c_{p(gas)}(T_5 - T_6)$$

If re-heating is not used $\quad \text{Net work output } = c_{p(gas)}(T_4 - T_X)$

Since the pressure lines on the T~s diagram diverge ($T_5 - T_6$) is always greater than ($T_4 - T_X$)

So that re-heating also increases the work output.

Also $\quad$ Work Ratio = 1- $\dfrac{\text{Compression work}}{\text{Expansion work}}$

If the expansion work is increased then the work ratio is increased if the compressor work remains the same.

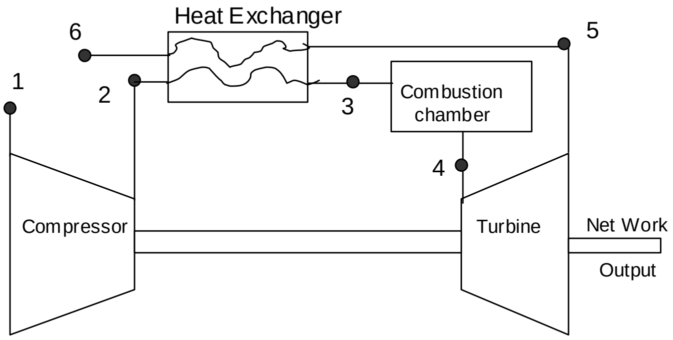
However the increased heat supply can reduce the thermal efficiency

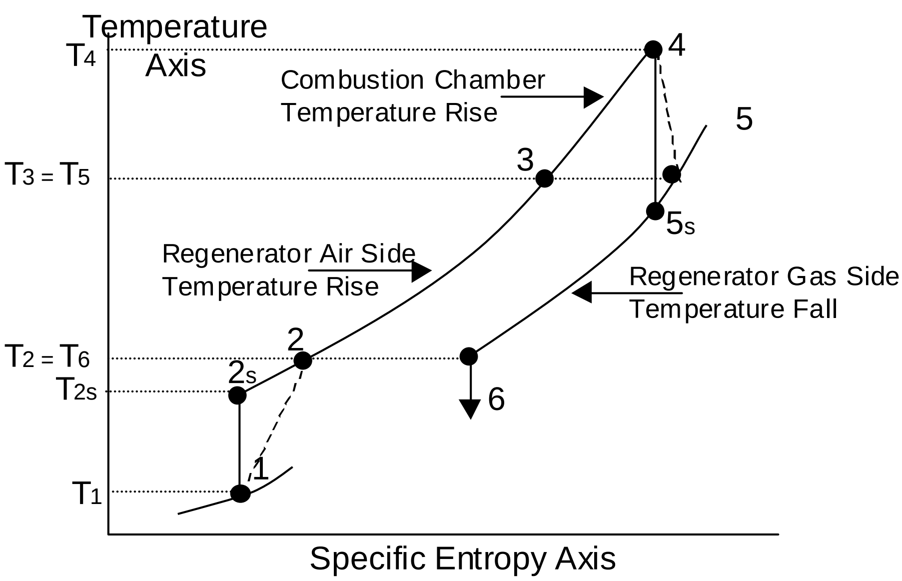
$$\text{Heat supply } = c_{p(air)}(T_3 - T_2) + c_{p(gas)}(T_5 - T_4)$$

The exhaust gasses leaving the LP turbine are much hotter when using reheat T_6 compared to T_X, some of this energy can be reclaimed in a heat exchanger .

Heat Exchanger

The exhaust gasses leaving the turbine have a relatively high temperature and hence enthalpy. Some of this energy can be recovered by transferring some of this heat to the air leaving the compressor using a heat exchanger or regenerator.
A simple plant and T~s diagram are shown below.





In an ideal heat exchanger the air would be heated from T_2 to T_3 which would be the same as T_5 and the gasses cooled from T_5 to T_6 which would be the same as T_2 .
If no heat is lost to the atmosphere then the heat lost by the gas is the same as the heat received by the air therefore.

$$\mathrm{m}_{(\mathrm{air})}\, c_{p(air)}\left(T_3 - T_2\right) = m_{(gas)} c_{p(gas)}\left(T_5 - T_6\right)$$

Heat exchanger effectiveness

This is discussed in the section concerning heat exchangers, however it comes down to a definition as follows

$$\text{Heat exchanger effectiveness} = \frac{\text{Heat received by the air}}{\text{Maximum possible heat which could be transferred from the gas}}$$

$$\text{Effectiveness } \varepsilon = \frac{m_{(air)} c_{p(air)} (T_3 - T_2)}{m_{(gas)} c_{p(gas)} (T_5 - T_2)}$$

Thermal ratio

A more convenient way of expressing heat exchanger efficiency is to use a Thermal ratio.

$$\text{Heat exchanger thermal ratio} = \frac{\text{Temperature rise of the air}}{\text{Maximum temperature difference available}}$$

$$\text{Thermal ratio} = \frac{(\mathrm{T}_3 - \mathrm{T}_2)}{(\mathrm{T}_5 - \mathrm{T}_2)}$$

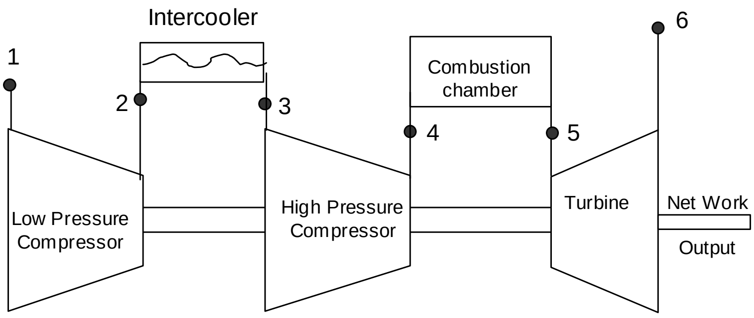
This is the same as the effectiveness when $m_a Cp_a$ is equal to $m_g Cp_g$

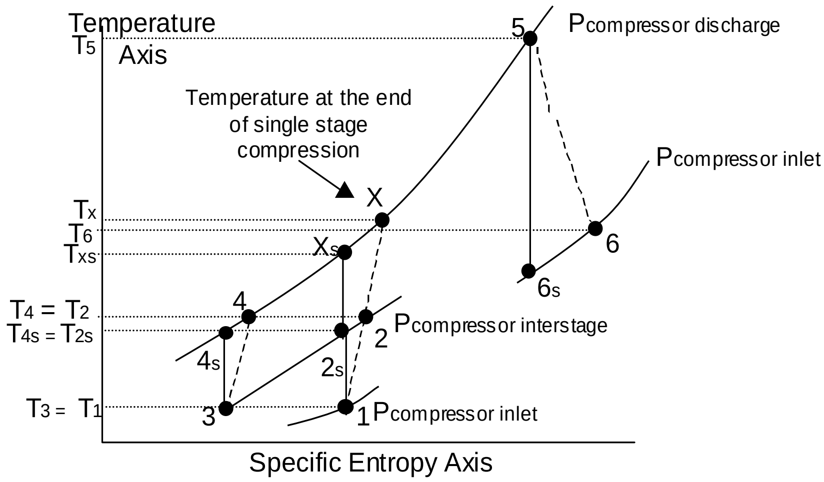
When a heat exchanger is used the heat supplied in the combustion chamber is reduced, for the same maximum cycle temperature, the work output is unchanged so the cycle efficiency is increased.

A heat exchanger can only be used when there is a large difference between the turbine outlet temperature and the compressor outlet temperature.
When this difference is small a large heat exchanger would be required to give a reasonable thermal ratio, hence they tend to be only found in large plants although the waste heat in the gas could be used for other purposes such as steam production.

Inter-cooling

When the compression is performed in two stages with an inter-cooler between the stages, then the work input for a given pressure ratio and mass flow is reduced.
A typical system and T~s diagram are shown





The cycle has been drawn for the condition where the pressure ratio, compression exponent and isentropic efficiency is the same in each compressor, while cooling in the intercooler is complete in that the air inlet to the second stage is the same as that for the first stage.

Single stage compression without inter-cooling is shown as line 1-X in the actual case and 1-X_s in the ideal case.

$$\text{Work Input with intercooling} = m_{(air)}c_{p(air)}(T_2 - T_1) + m_{(air)}c_{p(air)}(T_4 - T_3)$$

$$\text{Work Input with intercooling} = m_{(air)}c_{p(air)}(T_X - T_1)$$

$$= m_{(air)}c_{p(air)}(T_2 - T_1) + m_{(air)}c_{p(air)}(T_X - T_2)$$

If the isentropic efficiencies of the two compressors are equal to that of the single compressor without inter-cooling.
Then since the pressure lines are diverging (T_4-T_3) is less than(T_X-T_2) thus the work input with inter-cooling is less than that with no inter-cooling.
The work input required is a minimum when the pressure ratio in each stage is the same and when the air is cooled back to the inlet temperature.

$$\text{Work Ratio} = \frac{\text{Expansion work} - \text{Compression work}}{\text{Expansion work}}$$

If the compressor work is reduced then the work ratio is increased.
However the heat supplied in the combustion chamber is greater with inter-cooling than without it if the same maximum cycle temperature is used.

$$\text{Heat supply with intercooling} = m_{(air)} c_{p(air)} (T_5 - T_4)$$

$$\text{Heat supply without intercooling} = m_{(air)} c_{p(air)} (T_5 - T_X)$$

Although the work output is increased by inter-cooling the increased heat supply causes the cycle efficiency to decrease.

Solution to gas turbine Problems

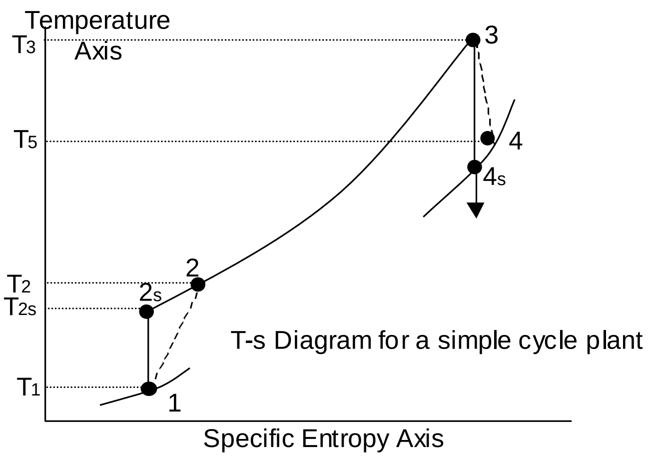
1. Sketch the plant layout diagram.
2. Sketch the temperature specific-entropy diagram for the plant given.
3. Identify key points for example, the single turbine drives the compressor and provides net work such as a turbo-generator, or the single turbine only drives the compressor such as an aircraft jet engine.
4. Match what the question is asking you to find with the information you have.
5. Calculate the cycle temperatures starting at the compressor inlet and methodically working your way round the cycle.

Gas turbine cycles Example 1

A simple gas turbine plant consists of a compressor and turbine, both having a pressure ratio of 4:1. The inlet to the compressor is at 15ºC, the compressor isentropic efficiency is 80% while that of the turbine is 70%.
The air is burnt in a combustion chamber with fuel having a calorific value of 43.5 MJ/kg.
The outlet from the combustion chamber and turbine inlet temperature, are both 650ºC.
For air and the combustion products; $\gamma = 1.4$, $c_p = 1000$ J/kgK.
Calculate

a) The temperature at the compressor outlet.
b) The temperature at the turbine outlet.
c) The mass of fuel burned per kg of air.

For gas turbine problems we shall mainly use the isentropic relationship for temperature and pressure because we need the temperatures to calculate the enthalpy rise and drop across the compressor and turbine and all we tend to know is the pressure ratio across the machines.
We also need to sketch the T-s diagram to picture what is happening.



The cycle is shown on the diagram above, we can calculate the isentropic temperature rise and then use the isentropic efficiency to calculate the actual temperature.

For the compressor

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad T_{2s} = 288(4)^{\frac{1.4-1}{1.4}} = 427.96K$$

This gives the isentropic temperature rise but we need the actual temperature rise due to the irreversible processes in the compressor.

$$\text{isentropic efficiency } \eta_{is} = \frac{T_{2s} - T_1}{T_2 - T_1}$$

$$T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_{is}}$$

$$T_2 = 288 + \frac{427.96 - 288}{0.8} = 462.96K$$

$$\text{compressor outlet tem perature } \mathrm{T_2} = 462.96K$$

For the Turbine

$$\frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \qquad T_{4s} = \frac{T_3}{\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{923}{(4)^{\frac{1.4-1}{1.4}}} = 621.13K$$

$$\text{isentropic efficiency } \eta_{\text{is}} = \frac{T_3 - T_4}{T_3 - T_{4s}}$$

$$T_4 = T_3 - \eta_{is}\left(T_3 - T_{4s}\right)$$

$$T_4 = 923 - 0{,}7(923 - 621.13) = 711.69K$$

$$\text{Turbine outlet temperature } \mathrm{T_4} = 711.69K$$

For the combustion chamber

We do not have the mass flow of air, however we can still calculate the mass of fuel but it will be related to the mass of air.

$$\text{Heat supply} = mc_p\left(T_3 - T_2\right)$$

$$\text{Heat supply} = 1000(923 - 462.96)$$

$$\text{Heat supply} = 460 \frac{kJ}{kg \text{ of air}}$$

$$\text{Heat supply} = \text{mass of fuel} \times \text{Calorific value}$$

$$\mathrm{m_f} = \frac{460 \times 10^3}{43.5 \times 10^6} = 0.0105 \frac{kg}{\text{kg of air}}$$

Mass of fuel supplied 0.0105 kg/kg of air

Gas turbine cycles Example 2

A gas generator consists of a simple cycle gas turbine plant with some of the gas being extracted from the cycle before it enters the turbine. The compressor takes in air at 1 bar 20ºC and compresses it to 6 bar.

The gas leaves the combustion chamber at 750ºC at which point gas is extracted at the rate of 2.5 kg/sec, the remaining gas being expanded in the turbine to a pressure of 1 bar.

The compressor isentropic efficiency is 80% while that of the turbine is 88%.

The air fuel ratio is 60 to 1.

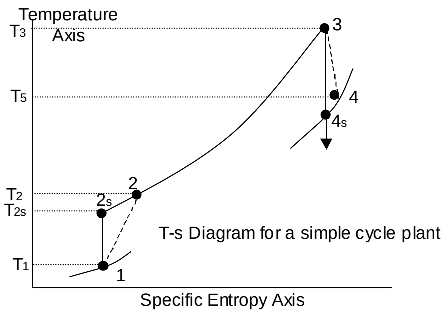
For air $\gamma = 1.4$, $c_p = 1005$ J/kgK.

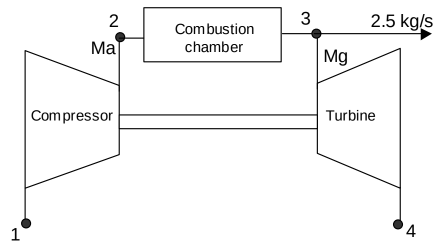
For the combustion products $\gamma = 1.33$, $c_p = 1150$ J/kgK.

Calculate

a) The mass flow of air through the compressor

b) The mass flow of gas through the turbine





In this particular case the plant is not designed to produce a work output it only produces hot gas for a particular process.

We can tackle this problem in exactly the same way as all the other problems, in other words methodically work your way around the cycle calculating the isentropic and actual temperatures.

Starting at the compressor inlet point 1

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad T_{2s} = 293(6)^{\frac{1.4-1}{1.4}} = 488.87K$$

$$\text{isentropic efficiency } \eta_{is} = \frac{T_{2s} - T_1}{T_2 - T_1} \qquad T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_{is}}$$

$$T_2 = 293 + \frac{488.87 - 293}{0.8} = 537.84K$$

$$\text{compressor outlet temperature } \mathrm{T}_2 = 537.84K$$

For the turbine $\dfrac{T_3}{T_{4s}} = \left(\dfrac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$ $\quad T_{4s} = \dfrac{T_3}{\left(\dfrac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}} = \dfrac{1023}{(6)^{\frac{1.33-1}{1.33}}} = 655.84K$

isentropic efficiency $\eta_{is} = \dfrac{T_3 - T_4}{T_3 - T_{4s}}$ $\quad T_4 = T_3 - \eta_{is}(T_3 - T_{4s})$

$$T_4 = 1023 - 0.88(1023 - 655.84) = 699.9K$$

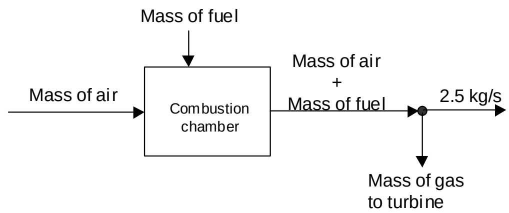
$$\text{Turbine outlet temperature } T_4 = 699.9K$$

We know that

$$\text{Compressor Work} = \text{Turbine Work}$$

$$m_{air} \times c_{p(air)} \times (T_2 - T_1) = m_{gas} \times c_{p(gas)} \times (T_3 - T_4)$$

The mass of air is the flow through the compressor and the mass of gas is the flow through the turbine.
We now need to link these values using the gas drawn off and the air fuel ratio.
The gas flow through the combustion chamber is shown below.



In this particular case the mass of air plus the mass of fuel is the mass of gas plus 2.5 kg/s.
Since the air / fuel ratio is 60:1 the mass of fuel is 1/60 of the mass of air.

$$m_{air} + 0.0167m_{air} = m_g + 2.5$$

$$1.0167m_{air} - 2.5 = m_g$$

We can calculate the work output for both compressor and turbine

$$\text{Compressor Work } = m_a \times 1.005 \times (537.84 - 293)$$

$$\text{Compressor Work } = 246.1m_a kW$$

$$\text{Turbine Work } = (1.0167m_a - 2.5) \times 1.15 \times (1023 - 699.9)$$

$$\text{Turbine Work } = 377.77m_a - 928.9$$

We can now equate turbine and compressor work

$$246.1m_a = 377.77m_a - 928.9$$

$$928.9 = 377.77m_a - 246.1m_a$$

$$7.05 = m_a$$

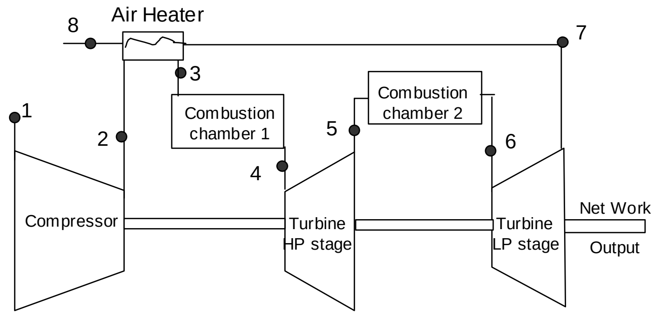
The mass of air through the compressor is 7.05 kg/s while the mass of gas through the turbine is 4.67 kg/s

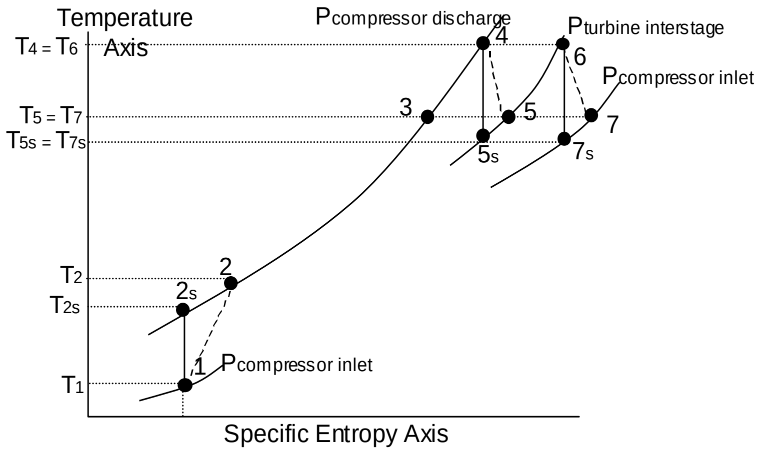
Gas turbine cycles Example 3

In a gas turbine plant air enters the compressor at a pressure and temperature of 1 bar 15ºC and is compressed through a pressure ratio of 6:1.
The air passes through a perfect heat exchanger before entering the combustion chamber where it is heated to 750ºC.
The gas is then expanded in two stages of equal pressure ratio with reheat to 750ºC between the stages.
Pressure loss throughout the cycle may be ignored.
The mass flow of fuel may be ignored.
The compressor isentropic efficiency is 80% while that of both turbine stages is 88%.
For air $\gamma = 1.4$, $c_p = 1005$ J/kgK.
For the combustion products $\gamma = 1.33$, $c_p = 1150$ J/kgK.
Calculate

a) The work output per kg of air flow
b) The work ratio
c) The cycle efficiency

For this example a plant diagram and T-s diagram are essential





In this example we can follow the same format as all other examples in that we can methodically work around the cycle taking account of the information we have.
With a perfect heat exchanger T_3 will be the same as T_7 and T_2 will be the same as T_8.
With equal pressure ratios and inlet temperatures T_{5s} will be the same as T_{7s} and with the same isentropic efficiency T_5 will be the same as T_7.
This cuts out a lot of the calculation.

Starting at the compressor inlet point 1

$$\frac{T_{2s}}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad T_{2s}=288(6)^{\frac{1.4-1}{1.4}}=480.53K$$

$$\text{isentropic efficiency } \eta_{is}=\frac{T_{2s}-T_1}{T_2-T_1} \qquad T_2=T_1+\frac{T_{2s}-T_1}{\eta_{is}}$$

$$T_2=288+\frac{480.53-288}{0.8}=528.66K$$

$$\text{compressor outlet temperature } \mathrm{T}_2=528.66K$$

For the turbine $\frac{T_4}{T_{5s}}=\left(\frac{P_4}{P_5}\right)^{\frac{\gamma-1}{\gamma}}$

Since the pressure drop across both stages is 6 and both stages have the same pressure ratios then the total pressure ratio equals the 1st stage ratio times the 2nd stage ratio

$$\text{Overall pressure ratio} = \text{HP pressure ratio} \times \text{LP pressure ratio}$$

$$\text{Overall pressure ratio} = (\text{HP pressure ratio})^2$$

$$\sqrt{\text{Overall pressure ratio}} = \text{HP pressure ratio}$$

$$\sqrt{6} = \text{HP pressure ratio}$$

$$2.4495 = \text{HP pressure ratio} = \text{LP pressure ratio}$$

$$T_{5s}=\frac{T_4}{\left(\frac{P_4}{P_5}\right)^{\frac{\gamma-1}{\gamma}}}=\frac{1023}{(2.4495)^{\frac{1.33-1}{1.33}}}=819.1K$$

$$\text{isentropic efficiency } \eta_{is}=\frac{T_4-T_5}{T_4-T_{5s}} \qquad T_5=T_4-\eta_{is}(T_4-T_{5s})$$

$$T_5=1023-0.88(1023-819.1)=843.569K$$

$$\text{Turbine outlet temperature } \mathrm{T}_5=843.569K$$

If the mass flow of fuel is ignored then the flow of gas through the turbine is the same as the air flow through the compressor.
If both turbines are identical then the total expansion work is twice the work of one turbine.
The net work output is given by

$$\text{Net Work Output} = \text{Total Turbine Work - Compressor Work}$$

$$\text{Net Work Output} = \left[2\times\left(m_{gas}\times c_{p(gas)}\times(T_4-T_5)\right)\right]-\mathrm{m}_{\mathrm{air}}\times c_{p(air)}\times(T_2-T_1)$$

The mass of air is the flow through the compressor and the mass of gas is the flow through the turbine.
Both stages of the turbine have been included since the question does not say the second stage is a free power turbine but implies that it is a single shaft turbine with two stages, hence the plant diagram showing this.
A further clue is that it is possible to calculate all the temperatures without equating compressor and HP turbine work.

$$\text{Net Work Output} = \left[2\times\left(m_{air}\times c_{p(gas)}\times\left(T_4 - T_5\right)\right)\right] - \mathrm{m_{air}}\times c_{p(air)}\times\left(T_2 - T_1\right)$$

$$\text{Net Work Output} = \left[2\times\left(m_{air}\times 1.15\times\left(1023 - 843.569\right)\right)\right] - \mathrm{m_{air}}\times 1.005\times\left(528.66 - 288\right)$$

$$\text{Net Work Output} = \left[2\times\left(206.34 m_{air}\right)\right] - 241.8633\mathrm{m_{air}}$$

$$\text{Net Work Output} = 412.69 m_{air} - 241.8633\mathrm{m_{air}}$$

$$\text{Net Work Output} = 170.828 m_{air}\ \text{kJ}$$

$$\text{Net Work Output} = 170.828\ \frac{\text{kJ}}{\text{kg}_{\text{air}}}$$

We can note that the work output from one turbine is less than that of the compressor.

$$\text{Work Ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$\text{Work Ratio} = \frac{\text{Net work output}}{\text{Work output from HP and LP turbines}}$$

$$\text{Work Ratio} = \frac{170.828 m_{air}}{412.69 m_{air}}$$

$$\text{Work Ratio} = 0.4139$$

The cycle efficiency is net work divided by the heat supply.
In this case the heat supply is in the two combustion chambers from T_3 to T_4 and T_5 to T_6. In the first combustion chamber the gas is mainly air hence the c_p for air has been used, in the second chamber, although there is still a lot of air the fluid will be mainly gas hence the c_p for gas has been used. We also used c_p for gas in the HP turbine so it would be reasonable to have continuity in the solution.

$$\text{Heat supply} = \mathrm{m_{air}}\times c_{p(air)}\times\left(T_4 - T_3\right) + m_{air}\times c_{p(gas)}\times\left(T_6 - T_5\right)$$

$$\text{Heat supply} = \mathrm{m_{air}}\times 1.005\times\left(1023 - 843.569\right) + m_{air}\times 1.15\times\left(1023 - 843.569\right)$$

$$\text{Heat supply} = 180.328\mathrm{m_{air}} + 206.345 m_{air}$$

$$\text{Heat supply} = 386.674\mathrm{m_{air}}\ \text{kJ}$$

$$\text{Thermal Efficiency } \eta_{\text{thermal}} = \frac{\text{Net work output}}{\text{Heat supply}}$$

$$\text{Thermal Efficiency } \eta_{\text{thermal}} = \frac{170.828\mathrm{m_{air}}}{386.674\mathrm{m_{air}}}$$

$$\text{Thermal Efficiency } \eta_{\text{thermal}} = 0.442$$

The thermal efficiency is therefore 44.2%